Interplay between proton-neutron pairing and deformation in $N=Z$ medium mass nuclei

Danilo Gambacurta

INFN Sezione di Catania, Italy

2nd International Workshop and 12th RIBF Discussion on Neutron-Proton Correlations July 6 - 9, 2015
Outline

Proton-Neutron Pairing

- Isoscalar and Isovector PN Pairing
- Difficulties in mean field approaches

A beyond mean (MF) field approach for PN pairing

Approach combining MF and shell-model techniques

\[a\]


Applications, Correlation Energies: pf-shell Z\(=\)N nuclei

- Interplay of isoscalar and isovector PN correlations
- Interplay of PN correlations with deformation effects

Some Preliminary results

- Low-Lying Spectra
- PN pair transfer
Proton Neutron (PN) Pairing

Active in the two channels, $T=0$ and $T=1$, of the nuclear interaction
- Expected to be more important in $Z=N$ nuclei
- Renewed interest, high-intensity radioactive beams

Mean Field (MF) models and PN pairing
- MF (i.e., BCS or HFB) approaches designed to treat pp and nn pairing
- Particle number and isospin symmetries are broken
- BCS and pn pairing: *pn pairing does not coexist with the like-particle one*, (J. Engel *et al.* PRC 55, 1781 (1996))
- HFB needs to use complex wavefunctions and no imposed spatial symmetries, (J. Terasaky *et al.* PLB 437 1998,1)

Beyond MF approaches are needed
- Projected BCS not good as for the particle-like pairing $^a$
- Projection on N and T $^b$ or Quartet Formalism $^c$

---

$^a$N. Sandulescu *et al*., PRC 80, 044335 (2009)
$^b$A. A. Raduta *et al.* PRC 68 (2012) 054314,
$^c$N. Sandulescu and D. Negrea, PRC 85, 061303(R) (2012)
Combining mean field (MF) and shell-model (SM) techniques

**Step I: MF calculations**

- **Self-Consistent Skyrme-HF + BCS calculations**
  
  (ev8 code\(^a\): three-dimensional mesh, deformation accounted for)

- **SLy4 Interaction + Pairing contact interaction**
  
  \[ V(r, r') = -v_0 \left( 1 - \eta \frac{\rho(r)}{\rho_0} \right) \delta(r - r') \]

- MF calculations provide: s.p. basis and (T=1 and T=0) paring matrix elements (consistently calculated)

  \(^a\)P. Bonche, H: Flocard, P.H. Heenen


**Step II: SM calculations**

- Full diagonalization of the pairing Hamiltonian

- Pairing window (5 MeV) around the Fermi energy

- \( n = z = 8 \) active neutrons and protons
Pair Operators and Hamiltonian

Isovector and Isoscalar pairs

- Isovector $T=1$ pairs $P^\dagger$
  \[ P_1^\dagger(k) = \nu_k^\dagger \nu_k^\dagger, \quad P_{-1}^\dagger(k) = \pi_k^\dagger \pi_k^\dagger, \quad P_0^\dagger(k) = (\nu_k^\dagger \pi_k^\dagger + \pi_k^\dagger \nu_k^\dagger) / \sqrt{2} \]

- Isoscalar $T=0$ pairs $D^\dagger$
  \[ D_0^\dagger(k) = (\nu_k^\dagger \pi_k^\dagger - \pi_k^\dagger \nu_k^\dagger) / \sqrt{2} \]

Isoscalar Spin 1 and -1 pairs not included (yet)

\[ D_1^\dagger(k) = \nu_k^\dagger \pi_k^\dagger, \quad D_{-1}^\dagger(k) = \nu_k^\dagger \pi_k^\dagger. \]

Symmetries assumed in the ev8 code!
Pair Operators and Hamiltonian

**Hamiltonian** $H = H_{s.p.} + V$

$H_{s.p.} = \sum_k \epsilon_k (\pi_k^\dagger \pi_k + \pi_k^\dagger \pi_k^\dagger + \nu_k^\dagger \nu_k + \nu_k^\dagger \nu_k)$

**Particle like pairing, i.e. only pp and nn**

$V = \sum_{i \neq j, T_z = \pm 1} V_{ij}^{T=1, S=0} P_{T_z}^\dagger(i) P_{T_z}(j)$

Label: $|T_z = 1|$

**Full T=1 pairing, pp, nn and pn**

$V = \sum_{i \neq j, T_z = \pm 1, 0} V_{ij}^{T=1, S=0} P_{T_z}^\dagger(i) P_{T_z}(j)$

Label: $T = 1$

**pn pairing with in the T=1 and T=0 channels (S_z = 0)**

$V = \sum_{i \neq j} V_{ij}^{T=1, S=0} P_0^\dagger(i) P_0(j) + V_{ij}^{T=0, S=1} D_0^\dagger(i) D_0(j)$

Label: $T_z = 0$
Merits and Disadvantages

**Merits**

- Deformation “realistically“ described, i.e., from Skyrme HF+BCS
- Single particle levels and (T=1 and T=0) two body matrix elements consistently described
- Isospin and particle number symmetries restored in SM solution
- Effect of different kinds of pairing can be studied

**Disadvantages**

- Rotational symmetry is broken, (states have not good angular momentum)
- T=0 spin aligned pairs not included
MF Skyrme HF+BCS Results, Energy vs deformation
MF Skyrme HF+BCS Results, Mean Gap vs deformation

$$\Delta = \sum_i \Delta_i v_i^2 / \sum_i v_i^2, \quad \Delta_i = \sum_j V_{ij}^{T=1,S=0} u_j v_j$$
S.P. energies evolution

Proton-Neutron Pairing
The approach
Applications and Results

$^52\,\text{Fe}$

$^56\,\text{Ni}$
Schematic Results

Restricted spaces and Constant Interaction

- $f_{7/2}^7$ and $f_{5/2}^5$ shells
- Only $T=1$ pairing
- Constant paring $V_{ij}^{T=1, S=0} = V_{T=1, S=0} = -0.5\text{MeV}$, average $\Delta \simeq 1\text{MeV}$,
- Different Effects:
  - BCS vs SM, (particle number fluctuations)
  - Proton-Neutron $T=1$ pairing only
  - Spin Orbit suppression
- Correlation Energy $E_{corr}$, with respect to the unperturbed case
Proton-Neutron Pairing

The approach

Applications and Results

Schematic results

BCS vs SM

proton-neutron T=1 pairing

Spin Orbit suppression

BCS

\[ E_{\text{corr}} \text{[MeV]} \]

N

\(|T_z|=1\)

|T_z|=1

T=1 [S.O.=0]

T=1 [S.O.=1]
More realistic results

Matrix Elements Average

Realistic calculations
- 4 particle and 4 hole states and $n = 8$ active particles (for neutron and proton)
- Pairing matrix elements in the two channels calculated from MF $\Rightarrow$ state dependent
Proton-Neutron Pairing

More realistic results

Matrix Elements Fluctuations

"Realistic" calculations

- 4 particle and 4 hole states and \( n = 8 \) active particles (for neutron and proton)
- Pairing matrix elements in the two channels calculated from MF \( \Rightarrow \) state dependent
- Strong suppression of \( T=0 \) matrix elements, see also:
  - A. Poves and G. Martinez-Pinedo, PLB 430, 203 (1998);
  - S. Baroni et al. PRC81, 064308 (2010)
  - H. Sagawa et al. PRC87, 034310 (2013)
Proton-Neutron Pairing T=1 channel

Skyrme HF+BCS Results vs Shell Model

- $^44$Ti
- $^48$Cr
- $^52$Fe
- $^56$Ni
- $^60$Zn
- $^64$Ge
Skyrme HF+BCS Results vs Shell Model

Proton-Neutron pairing $T=1$ and $T=0$ channel

$\nu_0^{T=0} = x \nu_0^{T=1}$
Skyrme HF+BCS Results vs Shell Model

Isospin decomposition, $x=1.6$

- $^{44}\text{Ti}$
- $^{48}\text{Cr}$
- $^{52}\text{Fe}$
- $^{56}\text{Ni}$
- $^{60}\text{Zn}$
- $^{64}\text{Ge}$
Proton-Neutron Pairing
Competition of isoscalar and isovector pairing on deuteron transfer

Deuteron transfer probability

\[ \hat{Q}_{10} = \sum_{ij} \hat{P}_{0}^\dagger(i) \hat{P}_{0}(j), \quad P_0^\dagger(k) = (\nu_k^\dagger \pi_k^\dagger + \pi_k^\dagger \nu_k^\dagger) / \sqrt{2} \]

\[ \hat{R}_{01} = \sum_{ij} \hat{D}_{0}^\dagger(i) \hat{D}_{0}(j), \quad D_0^\dagger(k) = (\nu_k^\dagger \pi_k^\dagger - \pi_k^\dagger \nu_k^\dagger) / \sqrt{2} \]

\[ Q \equiv \langle N, Z| \hat{Q}_{10}|N, Z \rangle \]

\[ = \sum_{\alpha} |\langle N + 1, Z + 1, \alpha| \sum_{i} \hat{P}_{0}^\dagger(i)|N, Z \rangle|^2 \]

\[ = \sum_{\alpha} |\langle N - 1, Z - 1, \alpha| \sum_{i} \hat{P}_{0}(i)|N, Z \rangle|^2. \]

Global information on the probability to transfer a deuteron from the initial g.s.
Competition of isoscalar and isovector pairing on deuteron transfer

Deuteron transfer probability, $x=1.6$
Some Preliminary Results

- Low Lying Spectra
- PN pair transfer
Low Lying Spectra

Dependence on the isoscalar strength, $\nu_0^{T=0} = x\nu_0^{T=1}$

![Graph showing low lying spectra for $^{54}$Co, $^{56}$Ni, and $^{58}$Cu]

- For $^{54}$Co:
  - $x=0$: Red line
  - $x=1$: Black line
  - $x=1.6$: Green line

- For $^{56}$Ni:
  - $x=0$: Red line
  - $x=1$: Black line
  - $x=1.6$: Green line

- For $^{58}$Cu:
  - $x=0$: Red line
  - $x=1$: Black line
  - $x=1.6$: Green line

Legend:
- [0] Red line
- [1] Black line
- [2] Green line

Energy (MeV) is plotted against the parameter $x$ for different isotopes.
Proton-Neutron Pairing

Applications and Results

Low Lying Spectra

Dependence on the isoscalar strength, $\nu_0^{T=0} = x\nu_0^{T=1}$

![Graphs showing dependence on isoscalar strength for isotopes of V, Cr, and Mn.](graph.png)
Dependence on the isoscalar strength, $\nu_0^{T=0} = x\nu_0^{T=1}$
pn pair transfer, \((N,Z)\equiv(\text{even},\text{even})\)

- Isovector pn \(T=1\) pairs

\[
P_0^\dagger(k) = \left(\nu_k^\dagger \pi_k^\dagger + \pi_k^\dagger \nu_k^\dagger\right)/\sqrt{2}
\]

- Isoscalar pn \(T=0\) pairs

\[
D_0^\dagger(k) = \left(\nu_k^\dagger \pi_k^\dagger - \pi_k^\dagger \nu_k^\dagger\right)/\sqrt{2}
\]
Proton-Neutron Pairing

The approach

Applications and Results

### pn pair transfer

The transfer of proton-neutron pairs is described by the equation

\[ v_0^{T=0} = xv_0^{T=1} \]

For different systems, the transition probabilities are shown in the graphs below.

1. **48 Cr, g.s. (T=0) \rightarrow 50 Mn, y**
   - Transition probability graph for the system 48 Cr, ground state (T=0) transitioning to 50 Mn, with y labels indicating the transition probabilities plotted against the variable x.
   - The graph shows two states: \(|y>=|1\text{ex,}(T=0)>\) and \(|y>=|\text{g.s.},(T=1)>\).

2. **56 Ni, g.s. (T=0) \rightarrow 58 Cu, y**
   - Transition probability graph for the system 56 Ni, ground state (T=0) transitioning to 58 Cu, with y labels indicating the transition probabilities plotted against the variable x.
   - The graph shows two states: \(|y>=|1\text{ex,}(T=0)>\) and \(|y>=|\text{g.s.},(T=1)>\).
Proton-Neutron Pairing

The approach

Applications and Results

pn pair transfer

$\nu_0^{T=0} = x \nu_0^{T=1}$

$$|^{48}\text{Cr, g.s. (T=0)}> \rightarrow |^{50}\text{Mn, y}>$$

$$|^{56}\text{Ni, g.s. (T=0)}> \rightarrow |^{58}\text{Cu, y}>$$

Trans. Prob. (arb. units)

Average number of pairs

$<\text{g.s. (T=1)}|\mathbf{P}^+\mathbf{P}|\text{g.s. (T=1)}>$

$<\nu_0 (T=0)|\mathbf{D}^+\mathbf{D}|\nu_0 (T=0)>$

$|\text{g.s. (T=1)}> \rightarrow |\text{P}^+\text{P}|\text{g.s. (T=1)}>$

$|\nu_0 (T=0)> \rightarrow |\nu_0 (T=0)>$

$|^{48}\text{Cr, g.s. (T=0)}> \rightarrow |^{50}\text{Mn, y}>
Summary and Conclusions

- We investigated proton-neutron correlations and deformation effects in Z=N pf shell nuclei, combining MF and SM approaches.
- Deformation effects are “realistically” described (Skyrme-HF + BCS) and strongly affect pairing correlations.
- $T=1$ pn pairing correlations typically much stronger than $T=0$ ones.
- Large deformation can favor competition between two channels.

Outlook

- Excited States (in progress).
- PN-transfer probabilities in the two channels (in progress).
- Including all kinds of $T=0$ pairs.
- Self-Consistent procedure, e.g. SM solution iteratively done inside MF calculations (... next step).
Proton-Neutron Pairing

The approach

Applications and Results
Proton-Neutron Pairing

Applications and Results

**pn pair transfer**

**pn pair transfer, \( v_0^{T=0} = xv_0^{T=1} \)**

\[
|^{48}\text{Cr, g.s.}^{(T=0)} \rangle \rightarrow |^{50}\text{Mn}\rangle
\]

\[
|^{56}\text{Ni, g.s.}^{(T=0)} \rangle \rightarrow |^{58}\text{Cu}\rangle
\]

**Average number of pairs**

\[
<\text{g.s.}^{(T=0)}|P^+P|\text{g.s.}^{(T=0)}>
\]

\[
<\text{g.s.}^{(T=0)}|D^+D|\text{g.s.}^{(T=0)}>
\]

**Trans. Prob. (arb. units)**

\[
|y\rangle = |^{1}\text{ex.}^{(T=0)}\rangle
\]

\[
|y\rangle = |\text{g.s.}^{(T=1)}\rangle
\]
pn pair transfer

\[ ^{48}\text{Cr, g.s.}^{(T=0)} \rightarrow \rightarrow ^{50}\text{Mn,y} \]

\[ ^{56}\text{Ni,g.s.}^{(T=0)} \rightarrow \rightarrow ^{58}\text{Cu,y} \]

\[ \nu_0^{T=0} = xv_0^{T=1} \]

Proton-Neutron Pairing
The approach
Applications and Results
Proton-Neutron Pairing

The approach

Applications and Results

PN pairing and Symmetries, from A. L. Goodman NPA186 (1972) 475;

Axial symmetry

\[
R_z(\theta) \begin{pmatrix}
c_{jm\tau}^+ c_{jm\tau}^-
\end{pmatrix}
R_z^{-1}(\theta) = \begin{pmatrix}
c_{jm\tau}^+ c_{jm\tau}^-
\end{pmatrix}
\]

Signature symmetry

\[
R_x(\pi) \begin{pmatrix}
c_{i\gamma\tau}^+ c_{j,-\gamma,\tau}^-
\end{pmatrix}
R_x^{-1}(\pi) = \begin{pmatrix}
c_{i\gamma\tau}^+ c_{j,-\gamma,\tau}^-
\end{pmatrix}
\]
Matrix Elements

\[ |k\tau_k\rangle = \int d^3r \sum_{\sigma_k} \phi_k(\sigma_k, r) |r\sigma_k\tau_k\rangle \]

\[ \langle i\tau_i i\tau_i| V^{T,S} |j\tau_j j\tau_j\rangle = \langle i\tau_i i\tau_i|VP_S P_T |j\tau_j j\tau_j\rangle \]

\[ \langle i\tau_i, i\tau_i| V^{T=1,S=0} |j\tau_j, j\tau_j\rangle = \frac{1}{4} \left( \delta_{\tau_i\tau_j} \delta_{\tau_i\tau_j} + i \leftrightarrow i \right) \int d^3r V(r) \rho_i(r) \rho_j(r) \]

\[ \langle i\tau_i, i\tau_i| V^{T=0,S=1} |j\tau_j, j\tau_j\rangle = \frac{1}{4} \left( \delta_{\tau_i\tau_j} \delta_{\tau_i\tau_j} - i \leftrightarrow i \right) \int d^3r V(r) F_{i,j}(r) \]

\[ F_{i,j}(r) = 2\Re \left[ \phi_i^*(+, r)\phi_i(-, r)\phi_j^*(-, r) - \phi_i^*(+, r)\phi_i(-, r)\phi_j^*(-, r) \right] \]

\[ + \left[ (\phi_i^*(+, r)\phi_i(+, r) - \phi_i^*(-, r)\phi_i(-, r)) \left[ \phi_j(+, r)\phi_j^*(-, r) - \phi_j(-, r)\phi_j^*(+, r) \right] \right]. \]
Deformation effects

Deformation effects, Schematic

Deformation effects, Realistic