<u>**The University of Hong Kong**</u>

<u>**Department of Physics**</u>

Physics Laboratory

PHYS3760 Physics Laboratory

Experiment No.3760-7: Measurement of dynamic response of a second order system (system described by a second order differential equation)

Name:

University No:

Aim: Learn about dynamic processes for mechanical phenomena. Get familiar with Wheatstone bridge and its application in measurement to enhance signal-to-noise ratio.

Background:

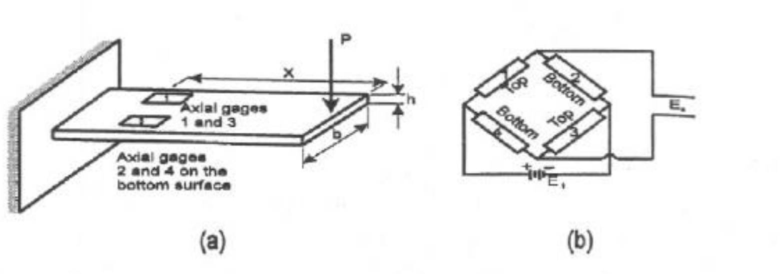

Fig. 1 Beam type load cell (a) elastic element and strain gages (b) connection into the Wheatstone bridge.

Strain gages are resistive sensors whose resistance changes depend on the strain. They can be used to measure force and pressure. Beam type load cell (Fig. 1 a), consisting of a cantilever beam with four strain gages, two at the top surface and two at the bottom surface, is commonly used for measurement of the force. The strain gages are connected to Wheatstone bridge, as shown in Fig. 1b. The bridge can contain 1, 2, or 4 gages, which determines the sensitivity of the measurement. In case of single strain gage with resistance R_g, the connection into the bridge is shown in Fig. 2. When the bridge is at equilibrium, the condition $R_gR_3=R_1R_4$ is satisfied. When the *Rg* is stretched, its resistance will increase ($R+\Delta R$). The potential at C will be lower than that at D by ΔV. The

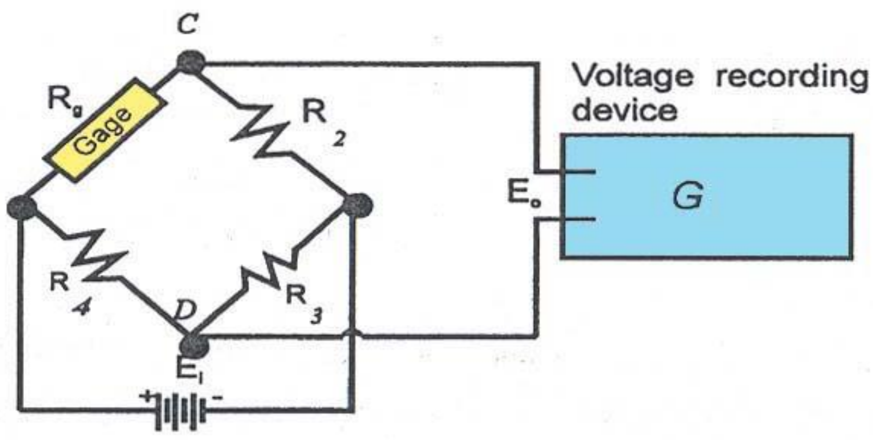

Fig. 2 Wheatstone bridge with a single gage

change of the resistance is proportional to the strain:

$$\frac{\Delta R}{R} = S_g \varepsilon \qquad (1)$$

where S_g is the gage factor or calibration constant for the gage, and the strain is given by:

$$\varepsilon = \frac{6FL}{Ebh^2} \qquad (2)$$

where E is the Young modulus, b is the width of the cantilever, h is the height of the cantilever. Therefore, the output voltage of a bridge with one gauge will be:

$$E_0 = US_g \frac{6FL}{4Ebh^2} \qquad (3)$$

With all four gages connected into the bridge, the voltage will be 4 times higher. From the equations above, we can observe that the output voltage E_o is proportional to the strain ε. By measuring E_o as a function of the load, we can determine elastic properties of the cantilever. Measurement can be performed with either two gauges and two resistors in the bridge, or with 4 gauges.

During the measurement it can be observed that it takes time for the signal to stabilize. This is because the response of any practical measurement system is not instantaneous. The dependence of the output voltage on time as a function of force acting on the cantilever F(t) can be described with a second order differential equation

$$\frac{d^2E_0}{dt^2} + 2\xi\omega_s \frac{dE_0}{dt} + \omega_s^2 E_0 = \omega_s^2 KF(t) \qquad (4)$$

where K is the sensor sensitivity, ω_s is the resonant frequency and ξ is the damping coefficient. After the excitation with short duration force pulse with intensity J (integral of F(t) over time), the output voltage of the bridge can be found as the solution to the above equation:

$$E_0(t) = KJ \frac{\omega_s}{\sqrt{1-\xi^2}} e^{-\xi\omega_s t} \sin\left(\sqrt{1-\xi^2}\,\omega_s t\right) \qquad (5)$$

The period T of these damped oscillations is then given by:

$$T = \frac{2\pi}{\omega_s \sqrt{1-\xi^2}} \qquad (6)$$

The ratio of the amplitudes of the first and n_{th} maximum (minimum, with even numbers denoting maxima and odd numbers denoting minima), a_0 and a_n is given by

$$\frac{a_0}{a_n} = \exp\left(\omega_s \xi n T/2\right) \qquad (7)$$

From Eqs. (6) and (7), we can obtain

$$\xi = \frac{\ln(a_0/a_n)}{\sqrt{\pi^2 n^2 + \ln^2(a_0/a_n)}} \qquad (8)$$

The ω_s can then be found from Eq. (6). Alternatively, we can find the logarithm of Eq.(7) and plot it as a function of time. The slope of the equation b=ω_s ξ. Combining this expression with Eq. (6), we can find that

$$\omega_s = \frac{1}{T}\sqrt{4\pi^2 + T^2 b^2} \qquad (9)$$

Once ω_s is known, from b we can determine ξ.

<u>Experimental setup:</u>
<u>Apparatus:</u>

- Stainless steel board with strain gauges x 1
- Stand x 1
- Connection boardx1 for connection strain gauges and resistors
- Multimeter x 1
- GPIB interface card & cable x 1
- DC power supply x 1
- Set of weights
- Ruler, Vernier calipers

Procedure

In this experiment, the damping characteristics of a stainless steel cantilever would be investigated.

- Measure the dimensions L, b, and h. A voltage of U=5V is applied to the bridge while the output voltage E_o is monitored using the Multimeter which is connected to the PC. Loads will be placed on the end of the cantilever. Measure the output voltage for different loads for two gages and 4 gages in the bridge. Plot the graphs, and determine the ratio of gage sensitivity to the Young modulus of the cantilever S_g/E.
- Remove the loads from the end of cantilever and connect all four gages. Apply short force pulse (one hit at the end of cantilever) and record the response. Determine the period of oscillations and the damping using both methods described above (approach using Eq. (8) and approach using Eq.(9)), and comment on the differences.

References

- J. G. Webster Ed., “The measurement, instrumentation, and sensors handbook”, CRC Press, 1999;
- J. Travis, “LabVIEW For Everyone”, Prentice-Hall PTR, 2002