The University of Hong Kong
Department of Physics

Physics Laboratory
PHYS3850 Wave and Optics
Experiment No.3850-1: Wave and Resonance

Name: **University No.:**

A. AIMS

1. Experimentally determine the relationship between the length of a stretched string and the frequencies at which resonance occurs at a constant tension;
2. Verify the variation of the wave frequency with the tension in the stretched string with a constant length;
3. Investigate the resonance modes of a stretched string.
4. Determine the velocity of the wave

B. INTRODUCTION

The standard qualitative sonometer experiments can be performed by varying the tension, length, and linear density a string and observing the effects on the pitch of the plucked string. Also, by adding the Driver/Detector Coils, a function generator capable of delivering 0.5A of current, and an oscilloscope, the quantitative experiments can be performed for verifying the equations for wave motion on a string:

$$\lambda = \frac{2L}{n}$$

$$v = \sqrt{\frac{T}{\mu}}\ ^{1}$$

$$f = \left(\frac{n}{2L}\right)\sqrt{\frac{T}{\mu}}$$

where λ = wavelength (m)
L = length of string (m)
n = number of antinodes
v = velocity of wave propagation (m/s)
T = string tension (N)
μ = linear density of string or mass per unit of string (kg/m)
f = frequency of wave (Hz)

This experiment will be conducted using a Sonometer to investigate how the frequencies of vibrating wires vary with length, density, and tension. The driver and detector coil can be placed anywhere along the string. The driver coil drives string vibrations at any frequency the function generator will produce. The detector coil allows the vibration of the string to be viewed on the oscilloscope. With a dual trace oscilloscope, the phase differences between the driving frequency and the string vibrations can even be examined.

C. PRE-LAB READING MATERIAL: Theory - Background Information

1. Standing Wave on a Stretched String:

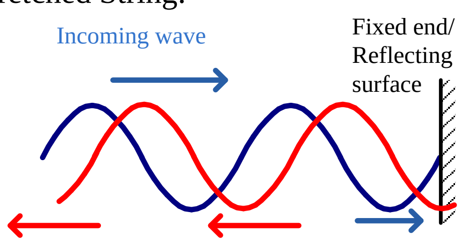


Figure 1. Standing Wave on a Stretched String

[1] Proof: http://hyperphysics.phy-astr.gsu.edu/hbase/waves/wavsol.html#c2

A simple sine wave traveling along a taut string can be described by the equation $y_1 = y_m \sin\left[2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)-\phi\right]$[2]. If the string is fixed at one end, the wave will be reflected back when it strikes that end. The reflected wave will then interfere with the original wave. The reflected wave can be described by the equation $y_2 = y_m \sin\left[2\pi\left(\frac{x}{\lambda}+\frac{t}{T}\right)\right]$. Assuming the amplitudes of these waves are small enough so that the elastic limit of the string is not exceeded, the resultant waveform will be just the sum of the two waves:

$$y_{\text{resultant}} = y_1 + y_2 = y_m \sin\left[2\pi\left(\frac{x}{\lambda}-\frac{t}{T}\right)-\phi\right] + y_m \sin\left[2\pi\left(\frac{x}{\lambda}+\frac{t}{T}\right)\right]$$

Using the trigonometric identity: $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{B-A}{2}\right)$,

the equation becomes $y_{\text{resultant}} = 2y_m \sin\left[2\pi\left(\frac{x}{\lambda}\right)-\frac{\phi}{2}\right]\cos\left[2\pi\left(\frac{t}{T}\right)-\frac{\phi}{2}\right]$.

This equation has some interesting characteristic. At a fixed time t_o, the shape of the string is a sine wave with a maximum amplitude of $2y_m \cos\left[2\pi\left(\frac{t_o}{T}\right)-\frac{\phi}{2}\right]$.

At a fixed position on the string x_o, the string is undergoing simple harmonic motion, with an amplitude $2y_m \sin\left[2\pi\left(\frac{x_o}{\lambda}\right)-\frac{\phi}{2}\right]$. When $\left|\sin\left[2\pi\left(\frac{x_o}{\lambda}\right)-\frac{\phi}{2}\right]\right| = 1$, resultant amplitude of oscillation is maximum. i.e. At points of the string where $x_o = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}, \ldots$ etc.,

When $\left|\sin\left[2\pi\left(\frac{x_o}{\lambda}\right)-\frac{\phi}{2}\right]\right| = 0$, resultant amplitude of oscillation is zero. i.e. At points of the string where $x_o = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda, \ldots$ etc., This wavelength is called a standing wave because there is no propagation of the waveform along the string. A time exposure of the standing wave would show a pattern something like the one in Figure 2. This pattern is called the envelope of the standing wave. Each point of the string oscillates up and down with its amplitude determined by the envelope. The points of maximum amplitude are called antinodes. The points of zero amplitude are called nodes.

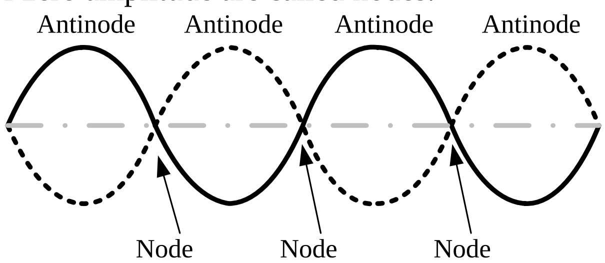


Figure 2. The Envelope of a standing wave

The phase of the standing wave depends on the sign of $\sin\left[2\pi\left(\frac{x_o}{\lambda}\right)-\frac{\phi}{2}\right]$, the phase will be changed after passing through each node or antinode.

[2] For more details: http://hyperphysics.phy-astr.gsu.edu/hbase/waves/wavsol.html#c4

For a standing wave on two fixed points with effective length L, it obeys:

$$y_{\text{resultant}} = y_1 + y_2\big|_{x=0} = 0 \Rightarrow \sin\left[-\frac{\phi}{2}\right] = 0$$

and

$$y_{\text{resultant}} = y_1 + y_2\big|_{x=L} = 0 \Rightarrow \sin\left[2\pi\left(\frac{L}{\lambda}\right) - \frac{\phi}{2}\right] = 0 \Rightarrow 2\pi\left(\frac{L}{\lambda}\right) = n\pi \text{ where } n = 0, 1, 2, 3....$$

As a result, the frequency of the standing wave:

$$f_{\text{stationary wave}} = \frac{v}{\lambda} = n\frac{v}{2L}$$

and

$$f_{\text{n}} \equiv nf_1 \equiv n\frac{v}{2L} = \left(\frac{n}{2L}\right)\sqrt{\frac{T}{\mu}} = \left(\frac{n}{2L}\right)\sqrt{\frac{T}{\frac{m}{L}}} = \left(\frac{n}{2}\right)\sqrt{\frac{T}{mL}}$$

2. Resonance:

The analysis above assumes that the standing wave is formed by the superposition of an original wave and one reflected wave. In fact, if the string is fixed at both ends, each wave will be reflected every time it reached either end of the string. In general, the multiply reflected waves will not all be in phase, and the amplitude of the wave pattern will be small. However, at certain frequencies of oscillation, all the reflected waves are in phase, resulting in a very high amplitude standing wave. These frequencies are called resonant frequencies.

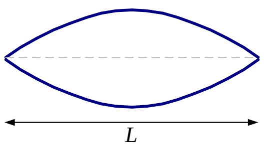
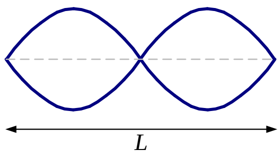
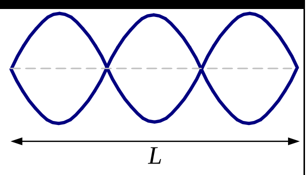
In this experiment, the relationship between the length of the string and the frequencies at which resonance occurs is investigated. It is shown that the conditions for resonance are more easily understood in terms of the wavelength of the wave pattern, rather than in terms of the frequency. In general, resonance occurs when the wavelength (L) satisfies the condition:

$$\boxed{\lambda = \frac{2L}{n}}$$

where n is the number of antinode on the string and equal to 1, 2, 3, 4…

Another way of stating this same relationship is to say that the length of the string is equal to an integral number of half wavelengths. This means that the standing wave is such that a node of the wave pattern exists naturally at each fixed end of the string.

2.1. Lowest Three Resonance Frequency:

Mode of resonance	First harmonic	Second harmonic	Third harmonic
Graphical representation			
Length	$L = \frac{\lambda}{2}$	$L = \lambda$	$L = \frac{3}{2}\lambda$
Wavelength	$\lambda = 2L$	$\lambda = L$	$\lambda = \frac{2}{3}L$

3. Velocity of Wave Propagation

Assuming a perfectly flexible, perfectly elastic string, the velocity of wave propagation (v) on a stretched string depends on two variables: the mass per unit length or linear density of the string (μ) and the tension of the string (T). The relationship is given by the equation:

$$\boxed{v = \sqrt{\frac{T}{m}}}$$

Without going into the derivation of this equation, its basic form can be appreciated. The equation is analogous to Newton's Second law, providing a relationship between a measure of force, a measure of inertia, and a quantity of motion. With this analogy in mind, it makes sense that the velocity should depend on the tension and linear density of the string. Hence, the forms of the two equations are not the same is to be expected. The motion of the string is considerably different from the motion of a simple rigid body acted on by a single force.

D. SETUP

1. Experimental Apparatus

Sonometer base with tensioning lever

- Two bridges
- 6 wires (guitar strings), two for each of the following diameters (linear densities):
    a. 0.010" (0.39g/m)
    b. 0.017" (1.12g/m)
    c. 0.022" (1.84g/m)
- Driver/Detector Coils
- A function generator and a power amplifier capable of delivering 0.5A into a low impedance load
- An oscilloscope, preferably dual trace

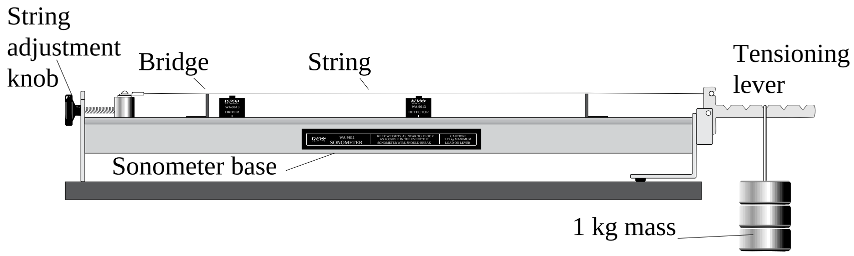
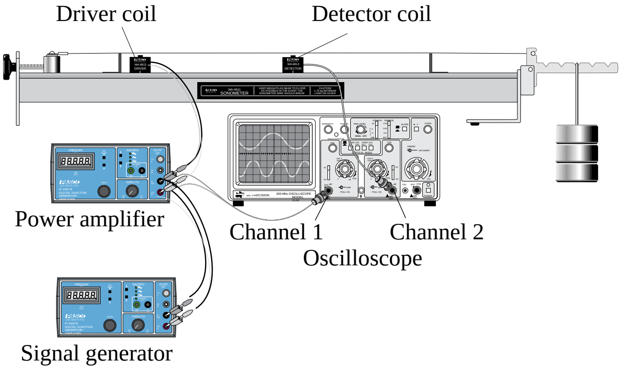


Figure 3. The Sonometer and Suggested Accessories

2. Set-up

- The experimental set up is shown in below.



E. EXPERIMENT 1: Resonance Modes of a Stretched String

Safety Issue: Please wear the goggle glass during experiment!!!!!!

You have to wear safety goggle glass once you conduct this experiment. It protects your eyes get hurt from the vibrating string. You have to bear in mind that every stretched vibrating string could be broken suddenly and unexpectedly.

Procedure

1. Setup the Sonometer as shown in Figure 3.

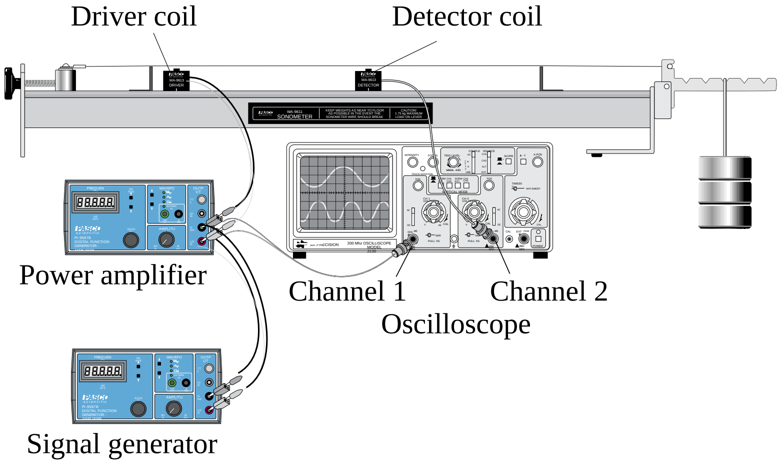


Figure 3. Equipment setup for experiment 1
(The exact apparatus may be different from the graph but its function is unchanged)

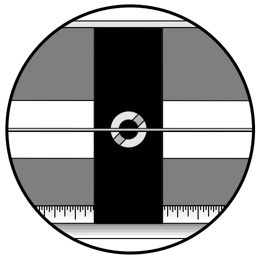


Figure 4. Top view from driver coil

2. Start with bridges 60cm apart. It is suggested that the left bridge should be located at 10 cm and the right bridge should be located at 70 cm.
3. Use the 0.01" (0.39 g/m) strings and hang a mass of approximately 1kg from the tensioning lever.

Caution: Put the hanging mass carefully and slowly. Otherwise, it will break the string and the broken string may hurt you.

4. Adjust the string adjustment knob so that the tensioning lever is horizontal. Position the driver coil approximately 5cm from left bridge (i.e. located at 15 cm) and position the detector near the center of the wire.
5. Record the length, tension (*mg*, where *g* is the acceleration due to gravity), and linear density of the string in Table 1.1
6. Turn on the signal generator and power amplifier.
7. Set the signal generator to produce a sine wave and set the gain of the oscilloscope to approximately 5mV/cm.
8. Slowly increase the frequency of the signal to the driver coil, starting at approximately 25Hz. Listen for an increase in the volume of the sound from the sonometer and/or an increase on the size of the detector signal on the oscilloscope screen. Frequencies that result in maximum string vibration are resonant frequencies. Determine the lowest frequency at which resonance occurs. This is resonance in the first, or fundamental mode. Measure this frequency and record it in Table 1.1.

Caution: Try with small driving power first and then increase gradually to obtain a reasonable resonant amplitude. Do not exceed the recommended power output or the driver could be damaged.

9. Start with the detector as close as you can get it to one of the bridges. Watch the oscilloscope as you slide the detector slowly along the string. Locate and record the locations of each node and antinode. Record your results in Table 1.1.
10. Continue increasing the frequency to find successive resonant frequencies for first order, second order, third order (optional) and fourth order (optional). Record the resonance frequency for each mode, and the locations of nodes and antinodes in Table 1.1.

Note: The driving frequency of the signal generator may not be the frequency at which the wire is vibrating. By using a dual trace oscilloscope, you can determine if the two frequencies are the same, or if the vibrating frequency is a multiple of the driving frequency, as shown in Figure 5.

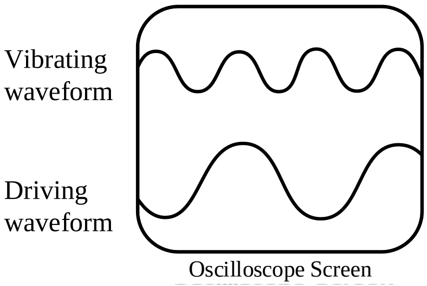


Figure 5. String vibrations at a multiple of the driving frequency

11. From your results, determine and record the wavelength of each resonance pattern you discovered. (Note that adjacent nodes are one half wavelengths apart.)
12. Change the string length to 40cm, repeat the measurements and record them in Table 1.2.
13. Change the string length to 60cm. Change the string tension by hanging the weight from a different notch, repeat the measurements and record them in Table 1.3.
14. Change the linear density of the string by changing using 0.017" (1.12g/m) string, repeat the measurements as Step 5 and record them in Table 1.4.

F. EXPERIMENT 2: Velocity of Wave Propagation

Procedure:

1. Set up the Sonometer as shown in Figure 6.

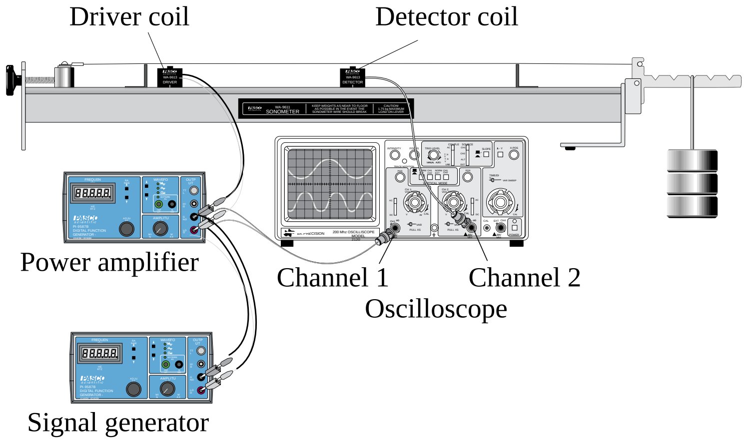


Figure 6. Equipment setup

(The exact apparatus may be different from the graph but its function is unchanged)

2. Set the bridges 60cm apart. Use any of the included strings and hang a mass of approximately 1kg form the tensioning lever. (It is suggested to start a string with heaviest linear density first)
3. Adjust the string adjustment knob so that the tensioning lever is horizontal. Position the driver coil approximately 5cm from one of the bridges and position the detector near the center of the wire.
4. Set the signal generator to produce a sine wave and set the gain of the oscilloscope to approximately 5mV/cm.
5. Slowly increase the frequency of the signal driving the driver coil, starting with a frequency of around 25Hz. Determine the lowest frequency at which resonance occurs. Record this value in Table 2.1.

Caution: Try with small driving power first and then increase gradually to obtain a reasonable resonant amplitude. Do not exceed the recommended power output or the driver could be damaged.

6. Repeat the above steps for two other different notches (different tension).

The tension is determined as shown in Figure 7. Just multiply the weight of the hanging mass by one, two, three, four or five, depending on which notch of the tensioning lever the mass is hanging from. The linear densities of the strings are given in the first page.

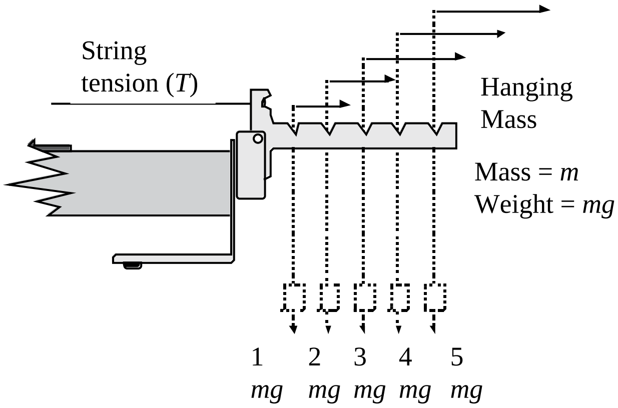


Figure 7. Setting the tension

7. Repeat the above steps by using two other different strings.

To be sure you have found the lowest resonance frequency, slide the detector coil of the string. The wave pattern should have just a single antinode located midway between the two bridges.

G. DATA COLLECTION:

Table 1.1

Length: ________m Tension: _______N Linear density: _______ __g/m = ________ kg / m

	Resonant frequency (Hz)	Location of node (m)	Location of antinode (m)	Wavelength (m)
First order				
Second order				
Third order				

Table 1.2

Length: ________m Tension: _______N Linear density: _______ __g/m = ________ kg / m

	Resonant frequency (Hz)	Location of node (m)	Location of antinode (m)	Wavelength (m)
First order				
Second order				
Third order				

Table 1.3

Length: ________m Tension: _______N Linear density: _______ __g/m = ________ kg / m

	Resonant frequency (Hz)	Location of node (m)	Location of antinode (m)	Wavelength (m)
First order				
Second order				
Third order				

Table 1.4

Length: ________m Tension: _______N Linear density: _______ __g/m = ________ kg / m

	Resonant frequency (Hz)	Location of node (m)	Location of antinode (m)	Wavelength (m)
First order				
Second order				
Third order				

H. <u>DISCUSSION QUESTIONS:</u>

Answer the following questions in your report.

Analysis for Experiment 1 - Resonance Modes of a Stretched String

1. String length:
   Using your data, determine the shape of the successive resonance waveforms as the frequency is increased. How do the wave shapes depend on the length of the string? Sketch the resonance waveforms for an arbitrary string length. What relationship holds between the wavelength of the wave and the string length when resonance occurs? Can you state this relationship mathematically?

   For each string length, inspect the frequencies at which resonance occurred. Determine a mathematical relationship between the lowest resonant frequency (the fundamental frequency) and the higher frequencies (overtones) at which resonance occurred.

2. String tension:
   Do the frequencies at which resonance occurs depend on the tension of the wire? Do the shapes of the resonance patterns (locations of nodes and antinodes) depend on the tension of the wire?

3. String density:
   Do the frequencies at which resonance occurs depend on the linear density of the wire? Do the shapes of the resonance patterns (locations of nodes and antinodes) depend on the linear of the wire?

Analysis for Experiment 2 - Velocity of Wave Propagation

4. <u>Table 2.1</u>

   Length: _________m

Tension (N)	Linear density (g/m)	Lowest resonance frequency (Hz)	Velocity (m/s)

5. Use your measured string length, the fundamental frequency, and the equation $v = f\lambda$ to determine the velocity of the wave on the string for each value of tension and linear density that you used and record it in Table 2.1.

6. Determine the functional relationship between the speed of the wave (v) and the wire tension (T).

7. Determine the functional relationship of the speed of the wave (v) to the linear density of the string (μ).

8. Use your answer to question 1, and the expression $v = f\lambda$, to determine the resonant frequencies of a wire of length L.

9. Use your experimental results to write an expression for the resonant frequencies of a vibrating wire in terms of T, μ and L.

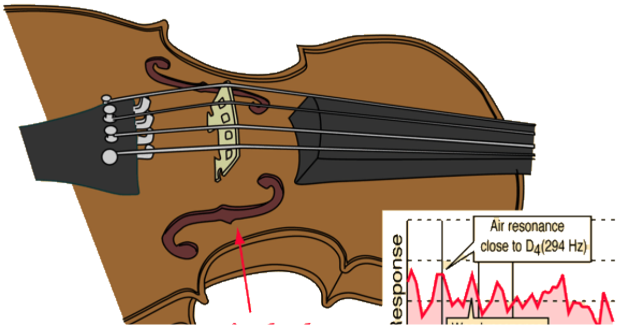
Applications of Stationary Wave and Resonance:

The frequencies produced by stretched strings are determined by the tension, mass and length of the strings, and consist of a fundamental frequency and all harmonics of that fundamental. Even though these frequencies are determined, the timbre of the sound produced by the string can vary considerably depending upon the method of excitation of the string. In the violin family the string may be bowed or plucked (pizzicato). In the piano the string is struck by a felt hammer, and in the harpsichord the strings are plucked by a "quill".

Even when the form of excitation is established, there are differences in harmonic content depending upon the location of excitation on the string. If a violin is bowed close to the bridge (sul ponticello) then the sound is brighter with more harmonic content. If bowed further from the bridge (sul tasto) then the sound is darker, more mellow with less harmonic content.

Air Resonance and the f-holes

The f-holes of a violin form the opening of a cavity resonator which in the resonance curve for the Stradivarius shown enhances frequencies close to the open string D4 at 294 Hz.



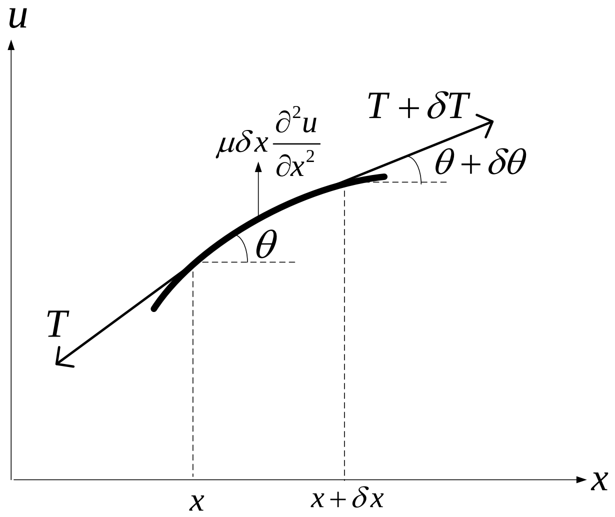
I. REFERENCES:

1. Hecht, E. (2001). *Optics 4th edition. Optics 4th edition by Eugene Hecht Reading MA AddisonWesley Publishing Company 2001* (Vol. 1, p. 122). Addison Wesley. Retrieved from http://adsabs.harvard.edu/abs/2001opt4.book.....H
2. Jenkins, F. A., White, H. E., & Jenkins, F. A. (1976). Fundamentals of optics. New York: McGraw-Hill.

J. APPENDIX:

Derivation of wave equation of transverse wave on a stretched string:

As shown in the figure, the transverse displacement is $u(x,t)$ and we will assume that this displacement is small.



Equations of motion:

Horizontal direction:

$$(T+\delta T)\cos(\theta+\delta\theta)=T\cos\theta$$

And small displacement u ➔ $\cos\theta \to 1$

Giving $T+\delta T=T$

i.e. $\delta T=0$, $T=\text{constant}$

Vertical direction:

$$(T+\delta T)\sin(\theta+\delta\theta)-T\sin\theta=ma_y$$

$\because \delta T=0$ and $\sin\theta\cong\theta$

$$T\sin(\theta+\delta\theta)-T\sin\theta=(\mu\delta s)\frac{\partial^2 u}{\partial t^2}$$

$$T(\theta+\delta\theta)-T\theta=(\mu\delta x)\frac{\partial^2 u}{\partial t^2}$$

$$T\frac{\partial\theta}{\partial x}=\mu\frac{\partial^2 u}{\partial t^2}$$

$$T\frac{\partial^2 u}{\partial x^2}=\mu\frac{\partial^2 u}{\partial t^2} \qquad \because \tan\theta\approx\theta\approx\frac{\partial u}{\partial x}$$

The wave equation is

$$\frac{\partial^2 u}{\partial t^2}=v^2\frac{\partial^2 u}{\partial x^2}$$

where speed of wave is $v=\sqrt{\frac{T}{\mu}}$.