

Laboratory Manual
Experiment NE02 - Rotary Motions
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In this set of experiments, the moment of inertia of rotating objects of different shapes and the law of conservation of angular momentum will be investigated.

Introduction

Moment of inertia is a quantity to measure the reluctance of an object against rotation when an external torque is applied on the object. In general, the moment of inertia I is given by

$$I = \int r^2 \rho(r) dV \quad \text{--- (1)}$$

where ρ is the density of the object at a point on the object which has a distance r from the axis of rotation.

When a torque is applied to an object, the object rotates with an angular acceleration. The magnitude of the torque τ can be related to the resulting angular acceleration α by

$$\tau = I\alpha \quad \text{--- (2)}$$

where the angular acceleration α can be calculated from

$$\alpha = \frac{a}{r} \quad \text{--- (3)}$$

where a is the linear acceleration of the object and r is the distance between the object and the axis of rotation.

For a rotating rigid object, its angular momentum L is proportional to its moment of inertia by

$$L = I\omega \quad \text{--- (4)}$$

where the angular velocity ω can be calculated from

$$\omega = \frac{v}{r} \quad \text{--- (5)}$$

where v is the linear velocity of the object and r is the distance between the object and the axis of rotation, and its rotationally kinetic energy is expressed as

$$E_r = \frac{1}{2}I\omega^2 \quad \text{--- (6)}$$

Apparatus

- 1 x CI-6538 Rotary Motion Sensor
- 1 x CI-6691 Mini-Rotational Accessory
- 1 x ME-9348 Mass and Hanger Set
- 1 x ME-9355 Base and Support Rod
- 1 x *ScienceWorkshop*® 500 Interface
- Serial port to USB adaptor
- Some paper clips (for masses < 1 g)
- Beam balance
- Calipers

Experiment 1: Moment of inertia of a point mass

Purpose

The purpose of this experiment is to calculate the moment of inertia of a point mass experimentally and to verify this experimental value with the calculated theoretical value.

Theory

The moment of inertia I of a point mass m can be expressed as

$$I = mr^2 \quad \text{--- (6)}$$

where r is the distance between the mass and the axis of rotation.

In this experiment, a pulley system is used to find the moment of inertia of the target object. A schematic diagram of the pulley system is shown in Fig.1. Two masses are attached to the two ends of a rod such that they are equidistant from the axis of rotation. Then, the total moment of inertia of two masses is

$$I_{total} = (m_1 + m_2)R^2 \quad \text{--- (7)}$$

where m_1 and the m_2 are the corresponding mass of two masses, and R is the distance of the masses from the axis of rotation (i.e. the distance between the mass at one end of the rod and the center of the rod).

To find the moment of inertia experimentally, a known torque τ , which is resulted by the hanging

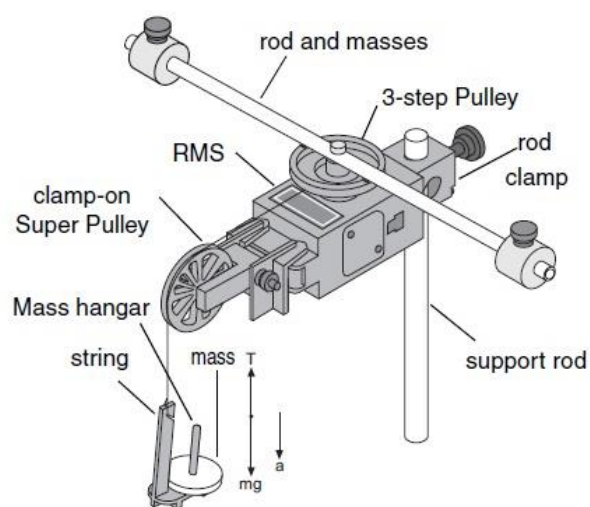


Fig.1 : Rotary motion sensor, the pulley system, and the force diagram of the mass hanger system.

mass from the thread, is applied to the 3-step Pulley to rotate the rod, and the magnitude of the torque can be calculated from

$$\tau = rT \quad \text{--- (8)}$$

where r is the **radius of the pulley** about which the thread is wound, and T is the tension in the thread.

According to Newton's second law of motion, the tension can be calculated from the below expression:

$$F_{net} = mg - T = ma \quad \text{--- (9)}$$

where m is the **mass of the hanging mass**.

From the experimental value of angular acceleration obtained by the rotary motion sensor, the torque and the linear acceleration can then be obtained from Equation (3), (8) and (9). Using these values, the experimental value of the moment of inertia of the point masses can hence be found from Equation (2).

Experimental procedures

1. Open the DataStudio® working file 'NE02_EXP1_UID.ds'. Click 'Save Activity As' function to save your own file where UID should be your/your partner University no.(e.g. NE02_EXP1_3333123456.ds)
2. Read the total mass of two masses on Table 1.1 in the worksheet.
3. Attach the two masses on each end of the rod (part of Mini-Rotational Accessory) equidistant from the center of the rod at any radius (please refer to Fig. 1).
4. Read the distance between the point mass and the rotational axis on Table 1.1 in the worksheet.
5. Tie one end of the thread, about 80 cm, to the Mass Hanger, and another end to one of the pulleys on the Rotary Motion Sensor.
6. Measure the diameter of the pulley about which the thread is wound using calipers. Record the value on Table 1.2.
7. Mount the rod on the Rotary Motion Sensor.
8. Mount the Rotary Motion Sensor to the Support Rod in a way that the rotation of the rod would not be interfered, and connect it to a computer.
9. Mount the clamp-on Super Pulley onto the Rotary Motion Sensor (RMS).
10. Align the thread with the level of the pulley about which the thread is wound by adjusting the height of the Super Pulley.
11. Put the 50 g mass on the Mass Hanger and wind up the thread.(Please refer to Fig.2 for the physical set-up)

12. Start DataStudio to measure the angular velocity of the rod and masses. Release the hanging mass. Stop DataStudio at end the data collection.
13. The slope of linear fit of angular velocity represents the angular acceleration (α). Record the slope on Table 1.2.
14. Repeat steps (11) to (13) again for second trial in Table 1.2.
15. Take the masses off the rod and repeat Steps (11) to (13) in order to find the angular acceleration of the apparatus alone. Choose a hanging mass of different weight or another pulley if necessary so that the rod does not rotate so fast that the data-logging software cannot keep up with the rate. Record the mass of the hanging mass, the radius of the pulley, and the measured slopes on Table 1.3 in the worksheet.
16. Save the working file and exit the program.



Fig. 2: Physical set-up of the Rotary Motion Sensor and the Pulley for Experiment 1

Experiment 2: Moment of Inertia of Disk and Ring

Purpose

The purpose of this experiment is to find the experimental values of moment of inertia of a ring and a disk, and to verify them with the theoretical values. The pulley system used in Experiment 1 is used in this experiment.

Theory

The moment of inertia I of a disk M about its center of mass is given by

$$I_{disk} = \frac{1}{2}M_d R_d^2 \quad \text{--- (11)}$$

where M_d and R_d is the mass and radius of the disk respectively.

The moment of inertia I of a ring m about its center of mass can be expressed as

$$I_{ring} = \frac{1}{2}M_r(R_i^2 + R_o^2) \quad \text{--- (10)}$$

where M_r is the mass of the ring, and R_i and R_o is the inner and outer radii of the ring respectively.

In order to find the moment of inertia of the disk and the ring experimentally, we follow the same procedures as in Experiment 1. Firstly, a resultant torque τ by the tension of the thread is applied to the 3-step Pulley to rotate the rod. Then, the angular velocity of the system with ring and disk, and the system with ring alone is recorded respectively to obtain the angular acceleration of the system.

From the experimental value of angular acceleration obtained by the rotary motion sensor, the torque and the linear acceleration can then be obtained from Equation (3), (8) and (9). These values are useful in the calculation of the moment of inertia of the ring and the disk from Equation (2).

Experimental procedures

1. Open the DataStudio® working file 'NE02_EXP2_UID.ds. Click 'Save Activity As' function to save your own file where UID should be your/your partner University no.(e.g. NE02_EXP2_33123456.ds)

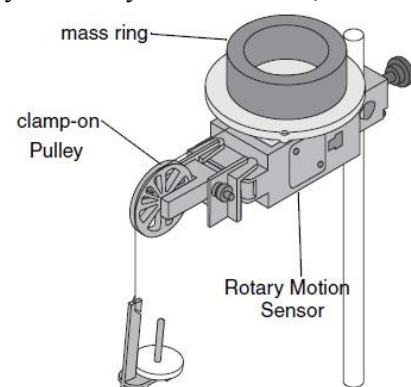


Fig. 3: Set-up of Rotary Motion Sensor with ring and disk

2. Read the mass of the ring and the disk by the beam on Table 2.1 in the worksheet.
3. Measure the internal jaw and external jaw diameters of the ring, and the diameter of the disk by calipers. Record the radii on Table 2.1 in the worksheet.
4. Measure the diameter of the pulley about which the thread is wound using calipers. Record the value on Table 2.2 in the worksheet.
5. Place the disk on the uppermost pulley of the Rotary Motion Sensor (see Fig.3).
6. Put the mass ring on the disk such that the two ring pins are in the holes on the disk.
7. Align the thread with the level of the pulley about which the thread is wound by adjusting the height of the Super Pulley.
8. Wind up the thread.(Please refer to Fig.5 for the physical set-up)
9. Click “Start” to measure the angular velocity of the ring and disk, and then release the hanging mass. Until the mass hanger landed on the table, Click “Stop” to stop data measurement.
10. Find the slope of linear fit of angular velocity represents the angular acceleration (α). Record the slope in the Table 2.2.
11. Repeat steps (6) to (10) again for second trial record in Table 2.2.
12. Remove the mass ring from the disk (refer to Fig. 4) and repeat Steps (6) to (9) in order to find the angular acceleration with the disk alone. (Please refer to Fig. 6 for the physical set-up) Choose a hanging mass of different weight or another pulley if necessary so that the rod does not rotate so fast that the data-logging software cannot keep up with the rate. Record the mass of the hanging mass, the radius of the pulley, and the measured slopes on Table 2.3 in the worksheet. Repeat this procedure again for second trial record in Table.2.3.
13. Save the working file and exit the program.

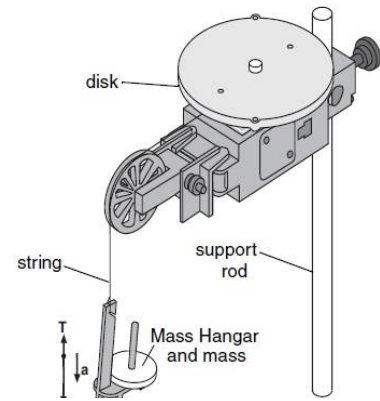


Fig. 4: Set-up of Rotary Motion Sensor with disk alone



Fig. 5: Physical set-up for Experiment 2 with both the disk and the ring

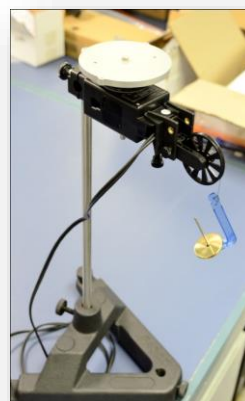


Fig. 6: Physical set-up for Experiment 2 with the disk alone

Experiment 3: Conservation of Angular Momentum

Purpose

The purpose of this experiment is to measure the angular velocity after the collision experimentally and to verify this with the value calculated theoretically.

Theory

The angular momentum L of an object can be expressed as

$$L = I\omega \quad \text{--- (12)}$$

where I is the moment of inertia of the object and ω is its angular velocity.

In this experiment, the law of conservation of angular momentum is investigated. A non-rotating mass ring is dropped onto a spinning disk. This action slows down the rotation of the disk. The final angular frequency of the system can be calculated from the conservation law expressed as below:

$$L = I_i\omega_i = I_f\omega_f \quad \text{--- (13)}$$

where I_i is the initial moment of inertia of the system, i.e. the moment of inertia of the disk alone (Eq. 11), I_f is the final moment of inertia, i.e. the combined moment of the inertia of both the disk and the ring, while ω_i and ω_f are the initial and final angular velocities of the rotating system.

Experimental procedures

1. Open the DataStudio® working file 'NE02_EXP3_UID.ds'.
Click 'Save Activity As' function to save your own file where UID should be your/your partner University no.(e.g. NE02_EXP3_3333123456.ds)
2. Detach the clamp-on Super Pulley and thread form the Rotary Motion Sensor and hold the mass ring just above the disk as shown in Fig. 7.
3. Align the center of the ring with the center of the disk.
4. Give a push to the disk to let the disk spin. Click "Start" to start measurement.
5. Drop the ring onto the spinning disk after around 25 data points have been taken in DataStudio.
6. Click "Stop" to stop measurement.
7. Click on the Smart Tool in DataStudio. Move the cursor to capture the data points immediately before and after the collision and record the angular velocities at the points on Table 3.2.

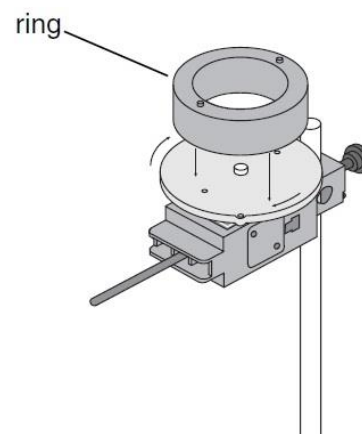


Fig. 7: The set-up for the drop-ring on the disk

8. Repeat steps (1) to (6) again for second trial record.
9. Read the mass of the ring and the disk on Table 3.1 in the worksheet
10. Measure their diameters. Record the masses and the radii on Table 3.1 in the worksheet.
11. Save the working file and exit the program.

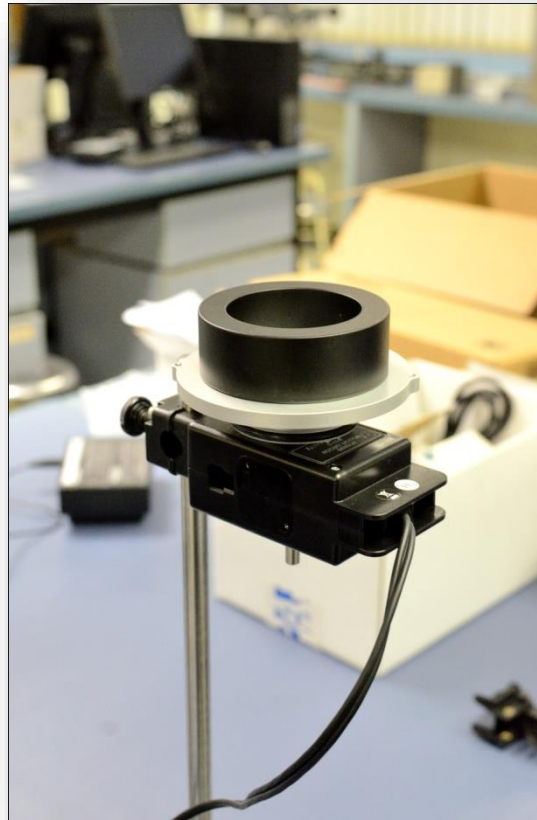


Fig. 8. Physical set-up of Experiment 3 with the ring dropped on the disk

References:

Morin, D. (1999). *Introduction to Classical Mechanics: With Problems and Solutions*. Cambridge University Press.

PASCO Scientific. (n.d.). Pasco CI-6538 Rotary Motion Sensor Manual. Roseville, CA: PASCO Scientific.