**Laboratory Manual**
**NE03 - Faraday's Law of Induction**
**Department of Physics**
**The University of Hong Kong**

Aims

To demonstrate various properties of Faraday's Law such as:

1. Verify the law.
2. Demonstrate the lightly damped oscillation of the hall probe as a simple pendulum.
3. Find the amount of energy lost due to lightly damped oscillation.

Self-learning material:   Theory - Background Information

Magnetic field strength, magnetic flux density, magnetic flux and magnetic flux linkage

| | Magnetic field strength (B-field) | Magnetic flux density | Magnetic flux | Magnetic flux linkage |
|---|---|---|---|---|
| Definition | $B = \dfrac{F_{magnetic}}{il} = \dfrac{F_{magnetic}}{Qv}$ (1) | $B = \dfrac{\Phi}{A_\perp}$ (2) | $\Phi = \int \vec{B} \cdot d\vec{A} = BA_\perp$ (3) | $\Phi_{linkage} = N\Phi$ (4) |
| | Where $F_{magnetic}$ is the magnetic force acting on a long straight wire or on a charge; $v$ is velocity of the charge in the B-field; $Q$ is no. of charge; $i$ is current of the long straight wire; $l$ is length of the wire perpendicular to B-field | Where $\Phi = \int \vec{B} \cdot d\vec{A} = BA_\perp$ for a magnetic field ($\vec{B}$) which is constant over the cross-sectional area ($\overrightarrow{A_\perp}$) and perpendicular to the area; $N$ is the number of turns of wire in the coil. | | |
| SI Unit | Tesla[1] ($T$) | Weber[2] ($Wb$) | | |

Hall effect and Hall probe

Hall probe is a device to detect a constant magnetic field strength, $B$ by making use of Hall effect.

The Hall voltage, $V_{Hall}$, developed by a Hall probe is given by

$$V_{Hall} = \frac{Bi}{nQt} \qquad (5)$$

Where $B$ is the magnetic field perpendicular to the hall probe;
$i$ is amount of current passing through the hall probe
$n$ is number of charge-carrier per unit volume in the hall probe
$Q$ is amount of charge in charge-carrier

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[1] 1 Tesla ($T$) = 10,000 Gauss ($G$) which is a non-SI unit and it's commonly used

[2] 1 Weber ($Wb$) = 1 tesla meter square meter ($Tm^2$)

Faraday's Law of Induction

    (a)  Statement

        Faraday's law states that

> The induced e.m.f. is directly proportional to the rate of change of flux-linkage or rate of flux cutting.

    (b)  Mathematical treatment

        According to Faraday's Law of Induction, a changing magnetic flux through a coil induces an

electromotive force (e.m.f.) given by

$$\xi_{induced} = -N\frac{d\Phi}{dt} \tag{6}$$

        where $\Phi = \int \vec{B} \cdot d\vec{A} = BA_{\perp}$ for a magnetic field ($\vec{B}$) which is constant over the cross-sectional

area ($\overrightarrow{A_{\perp}}$) and perpendicular to the area. $N$ is the number of turns of wire in the coil.

        For finding average e.m.f., the equation (6) could approximately equal to

$$\overline{\xi_{induced}} = -N\frac{d\Phi}{dt} \approx -N\frac{\Delta\Phi}{\Delta t} \quad \Rightarrow \overline{\xi_{induced}} = -N\frac{\Delta(BA_{\perp})}{\Delta t} \quad \because \Phi = BA_{\perp} \text{ and } A_{\perp} \text{ is a constant and}$$

independent of time ($t$).

$$\Rightarrow \overline{\xi_{induced}} = -NA_{\perp}\frac{\Delta B}{\Delta t}$$

        As a result, for this experiment, the area of the coil $A_{\perp} = \pi r^2 = \pi\frac{d^2}{4}$ is a constant where $r$ is the

radius of coil and $d$ is the diameter of the coil. and as the coil passes into or out of the magnetic field,

there is an average e.m.f. given by

$$\overline{\xi_{induced}} = -\frac{N\pi d^2}{4}\frac{\Delta B}{\Delta t}. \tag{7}$$
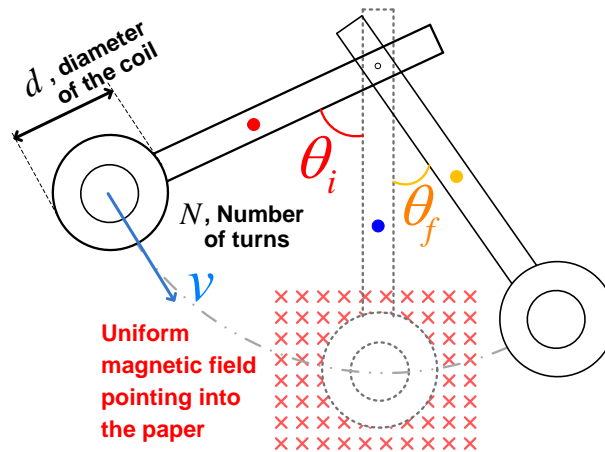


*Figure 1.* Position of center of mass

Center of mass of a body

 (a)  Definition:

        The terms "center of mass" and "center of gravity" are used synonymously in a uniform gravity field to represent the unique point in an object or system which can be used to describe the system's response to external forces and torques. The concept of the center of mass is that of an average of the masses factored

by their distances from a reference point. In one plane, that is like the balancing of a seesaw about a pivot point with respect to the torques produced.
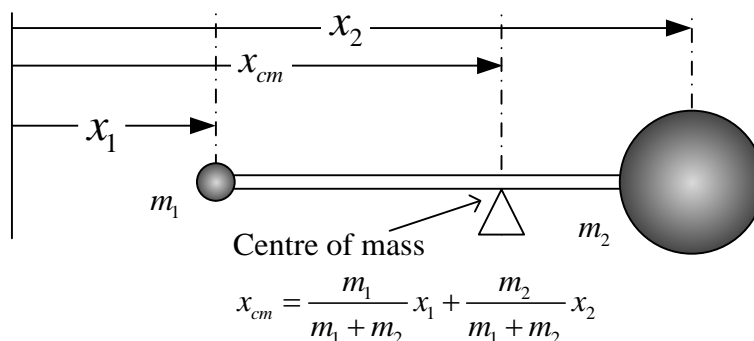


$$x_{cm} = \frac{m_1}{m_1 + m_2} x_1 + \frac{m_2}{m_1 + m_2} x_2$$

*Figure 2.* Definition center of mass

(b) Experimental method



*Figure 3.* Experimental method to find the position of center of mass

Once the coil wand is balanced by the index finger similar to the situation in *Figure 3*, the position of the finger is the location of the center of mass.

Simple harmonic motion (S.H.M.)

A. Definition of S.H.M.

If $x$ is displacement, and $a_x$ is the acceleration (in the $x$ direction), this can be expressed mathematically: $\boxed{a = -\omega^2 x}$ (8)     where $\omega$ is the angular frequency.

The minus sign indicates that a is always in the opposite direction $x$.

As the force is always in the direction towards the center, the force is called the restoring force.

B. Properties of S.H.M.

| Displacement | $x = A\cos \omega t$ |
|---|---|
| Velocity | $v_x = -\omega A \sin \omega t$ |
| Acceleration | $a_x = -\omega^2 A \cos \omega t$ |
| Period | $T = \dfrac{2\pi}{\omega}$ |
| Frequency | $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$ |
| Angular frequency | $\omega = \dfrac{2\pi}{T} = 2\pi f$ |

| | |
|---|---|
| | $\omega^2 = \dfrac{k}{m}$ mass-spring system |
| | $\omega^2 = \dfrac{g}{l}$ simple pendulum |

For a uniform circular motion, the period is independent of radius. As a result, for a simple harmonic motion, the period is independent of amplitude. (i.e. called isochronous motion)
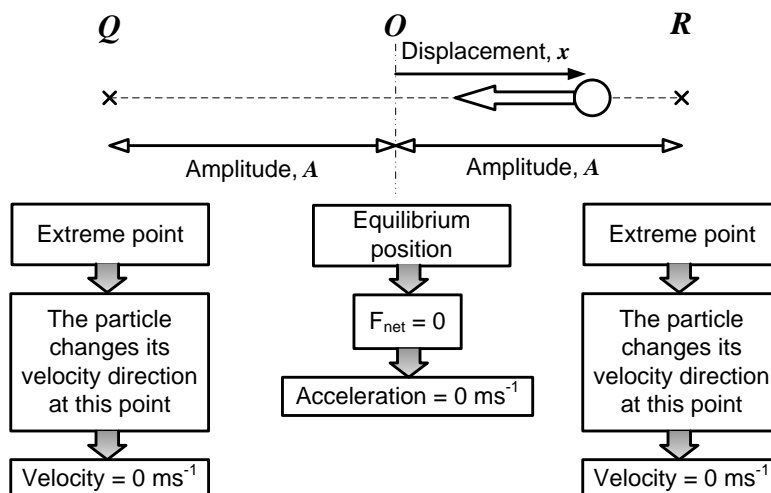


*Figure 4.* Properties of S.H.M. at extreme point and equilibrium position

C. Simple pendulum as an example of S.H.M.

i) The restoring force (
   For very small θ,

   $F_{net} = ma \Rightarrow -mg\sin\theta = ma$

   Since the force is in opposite direction to the displacement)

   $-mg\theta = ma$  $\quad \because \sin\theta \cong \theta = \dfrac{x}{\ell}$  (9)

   $-mg\dfrac{x}{l} = ma$

   $\Rightarrow a = -\left(\dfrac{g}{l}\right)x$  (10)

   $\therefore$ S.H.M.



*Figure 5.* Simple pendulum

   Comparing with $a = -\omega^2 x$, $\omega^2 = \dfrac{g}{l}$

   $\Rightarrow \omega = \dfrac{2\pi}{T} = 2\pi f = \sqrt{\dfrac{g}{l}}$

   $T = 2\pi\sqrt{\dfrac{l}{g}}$  (11)  or  $f = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{l}}$
   (12)
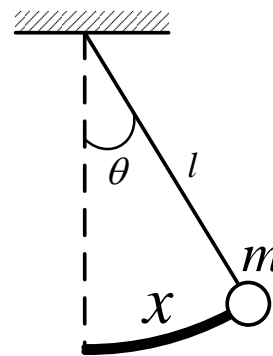
ii) The period $T$ depends on the length of string and not the amplitude. The length of pendulum can

be adjusted so that the period is 1 second. (Clock)This is one of the way to find the gravitational acceleration g by the measuring the period and the length of the string.

D. Damping of S.H.M. or Damped harmonic motion

i) Now, consider the case of simple pendulum. The amplitude of the oscillation of a pendulum gradually decreases to zero owing to the air resistance. (Energy is lost)

ii) Suppose the air resistance is directly proportional to the speed $v$ (i.e. $f_{air} = -bv$, where $b$ is a constant and the negative sign means air resistance is always in the opposite direction to the motion))

Then equation $a = -\omega^2 x$ becomes

$a = -\omega^2 x - \gamma v$     (13) where $\gamma = \dfrac{b}{m}$ is a

constant and $m$ is the mass of object

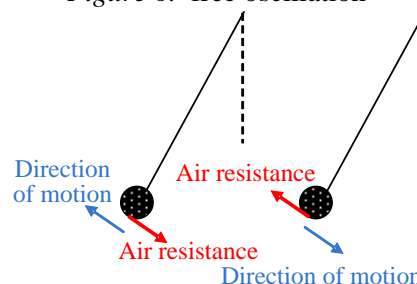$a + \omega^2 x + \gamma v = 0$     (14)
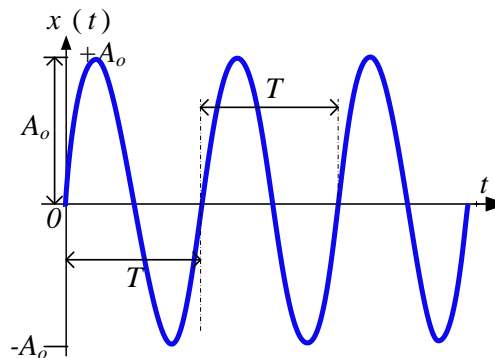


Figure 6. free oscillation



Figure 7. Direction of motion and air resistance

iii) Equation (14) is a **differential equation of damped oscillation** and $x$ will depend on $\gamma$ and $\omega$

iv) According to the solution of equation(14), 3 cases could be resulted.

1. $\left(\dfrac{\gamma}{2}\right) - \omega^2 > 0$          ($k$ is large, great damping) ➜ Heavy damped motion

2. $\left(\dfrac{\gamma}{2}\right) - \omega^2 = 0$          (critical value) ➜ Critical damped motion

3. $\left(\dfrac{\gamma}{2}\right) - \omega^2 < 0$          ($k$ is small, small damping) ➜ slightly damped motion
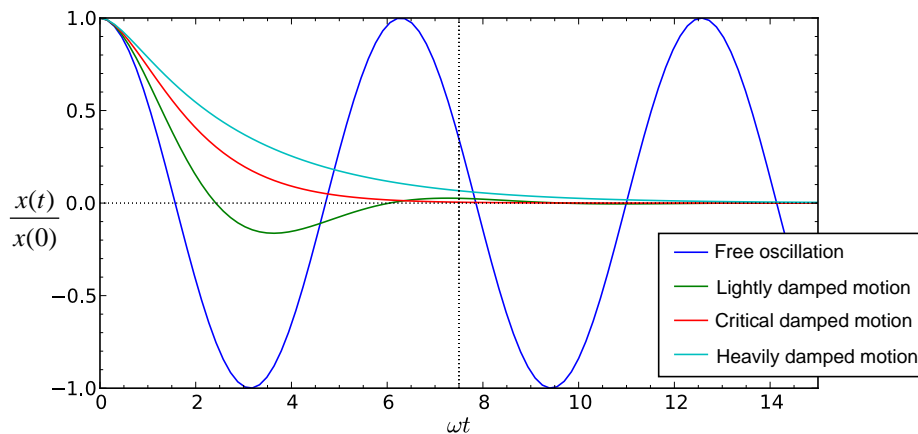
*Figure 8.* Amplitude ratio of free oscillation and different damped motion

v)   In this experiment, only the slightly damped motion is concerned

Slightly damped motion (i.e. $\left(\dfrac{\gamma}{2}\right) - \omega^2 < 0$)

a.   $x = e^{-\frac{\gamma}{2}t}\left(A_1 \cos qt + B_1 \sin qt\right) = C e^{-\frac{\gamma}{2}t} \cos\left(qt + \varphi\right)$          (15)

  where $A_1, B_1, C$ are constants and $q = \sqrt{\omega^2 - \left(\dfrac{\gamma}{2}\right)^2}$

b.   The period $T$ is constant ($\dfrac{2\pi}{q}$) and is slightly smaller.

c.   The amplitude is decreasing at a constant rate $e^{-\frac{\gamma}{2}T}$ i.e. $\dfrac{A_o}{A_1} = \dfrac{A_1}{A_2} = ... = \dfrac{A_{n-1}}{A_n} = e^{-\frac{\gamma}{2}T}$   (16)
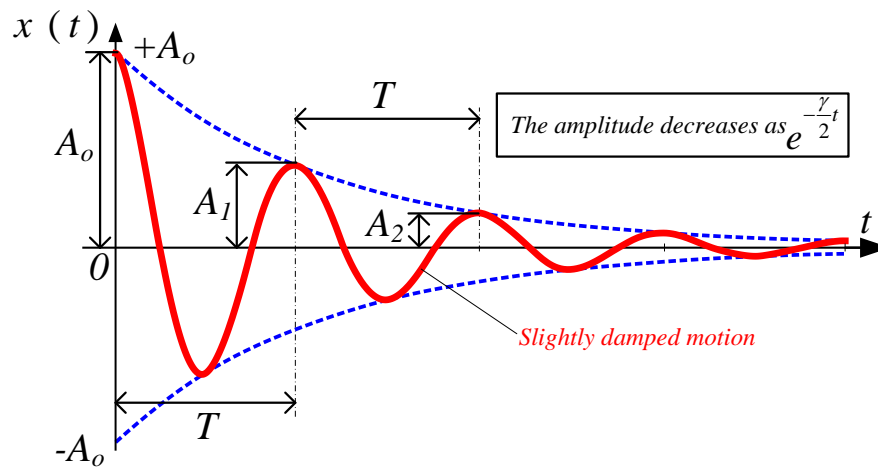


*Figure 9.* Amplitude of slightly damped motion

The blue dotted curves in the graph above are the pair of functions $+e^{-\frac{\gamma}{2}T}$, $-e^{-\frac{\gamma}{2}T}$ often referred to

as the envelope of the oscillation curve, as they "envelope" it from above and below.

Eddy current

(a) Definition

Any piece of metal or coil moving in a magnetic field, or exposed to a changing one, has e.m.fs induced in it, as we might expect. These can cause currents, called eddy currents, to flow inside the metal and they may be quite large because of the low resistance of the paths they follow.

(b) Magnetic effect of eddy current

According to Lent's law, eddy currents will circulate in directions such that the magnetic fields they create oppose the motion (of flux change) producing them. This acts as a brake on the moving body.
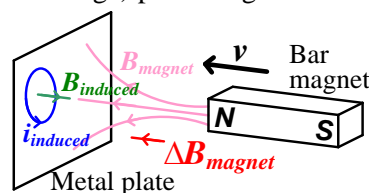
*Figure 10.* Eddy current in a piece of metal plate

Energy and Power

      If the center of mass of the pendulum starts from rest at an initial height $h_i$, its potential energy is

$$U_i = mgh_i \qquad (17)$$

      As the pendulum swings and passes through the magnet, some energy is lost to mechanical frictional heat and some energy is converted to electrical energy and then to thermal energy in the resistor. Thus the *center of mass* of the pendulum does not rise to the same height but rather to a lower final height, $h_f$. See *Figure 11*.

      The total energy lost by the pendulum is equal to its change in potential energy:

$$\text{Total Energy Lost} = \Delta U = mg(h_i - h_f) \qquad (18)$$

or

$$\text{Total Energy Lost} = \Delta U = mgl\left(1 - \cos\theta\right) \qquad (19)$$

where $l$ is the distance from the pivot point to the center of mass of the coil wand. and $\theta$ is the angular position of the coil wand.
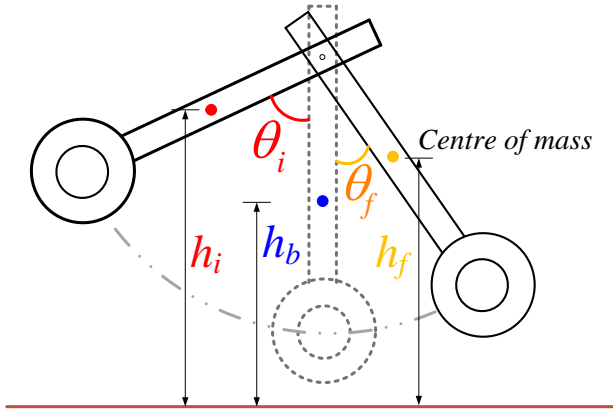


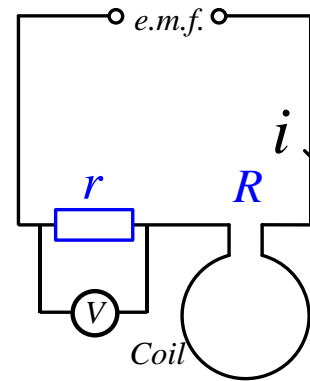Figure 11: Coil Height Decreases           Figure 12: Coil and Resistor Circuit

The thermal energy dissipated in the resistor ( $R$ ) is given by

$$E_{loss} = \int P_{loss}\,dt \approx \sum P_{loss}\Delta t$$

$$E_{loss} = \sum P_{loss}\Delta t = \text{Area Under Power against Time graph} \quad (20)$$
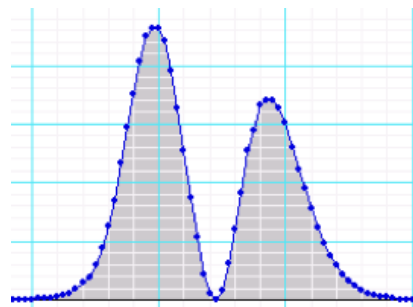
where $P_{loss}$ is the power dissipated and $P$ is time.



*Figure 13: Coil and Resistor Circuit*

Consider the circuit as shown in *Figure 12*, $V_r$ is the potential difference (p.d.) across the resistor ($r$), $i$ is the current through the coil, and $R$ is the resistance of the coil. According to Kirchhoff loop rule reads:

$$\zeta_{e.m.f.} = V_R + V_r \qquad (21)$$

Because of energy conservation, $P_{e.m.f.} = P_R + P_r$

$$P_{e.m.f.} = P_{output} = i^2 R + i^2 r$$

The p.d. across the resistor ($r$), $V_r$, is measured by voltmeter; according to Ohm's law,

$$V_r = ir \Rightarrow i = \frac{V_r}{r}$$

The power output of the circuit is given by $P_{output} = i^2(R+r) \qquad (22)$

$$\therefore P_{output} = \left(\frac{V_r}{r}\right)^2 (R+r) \qquad (23)$$

In this experiment, $R = 1.9\Omega$ and $r = 4.7\Omega$

Apparatus
- $1 \times$ PASCO Scientific EM-8099 Induction Wand
- $1 \times$ PASCO Scientific EM-8641 Variable Gap Lab Magnet
- $1 \times$ PASCO Scientific CI-6503 Voltage Sensor
- $1 \times$ PASCO Scientific CI-6520A Magnetic Field Sensor
- $1 \times$ PASCO Scientific CI-6538 Rotary Motion Sensor
- $1 \times$ PASCO Scientific CI-6400 *Science Workshop* 500 data logger
- PASCO *DataStudio* computer interface
- $1 \times$ large rod stand
- Miscellaneous wirings

## Experiment 1: Average induced e.m.f.
**Setup Procedures:**
1. *Put* a rod in the stand and clamp the cross-rod to it as shown in *Figure 14*. *Put* the Rotary Motion Sensor at the end of the cross-rod.
2. *Attac*h the coil wand to the Rotary Motion Sensor with the tabs on the 3-step pulley just to the sides of the wand as shown in *Figure 15*.
3. *Plug* the Voltage Sensor into Channel A of the ScienceWorkshop 500 interface.
4. *Plug* the Rotary Motion Sensor into Channels 1 and 2.
5. *Plug* the Magnetic Field Sensor into Channel B.
6. *Plug* the Voltage Sensor banana plugs into the banana jacks on the end of the coil wand.
7. Put the pole plates on the magnet as shown in Figure 16. (Caution: Hold the plates tight and approach slowly to the strong magnet; be careful of the plates' edges and watch out your fingers).
8. *Drape the Voltage* Sensor wires over the rods as shown in *Figure 14* so the wires will not exert a torque on the coil as it swings. It helps to hold the wires up while recording data.
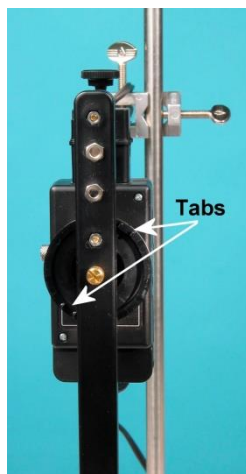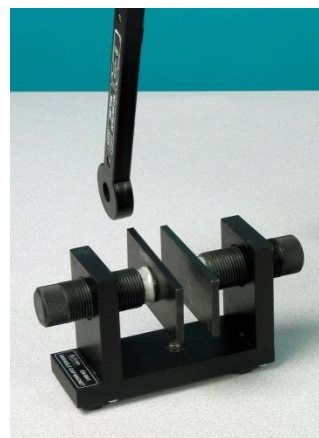
*Figure 14:* Rod Stand



*Figure 15:* Tabs



*Figure 16:* Magnet Pole Plates

**Experimental Procedure:**
**How to measure magnetic field strength between the magnetic poles?**

1. Using caliper to *measure* the diameter of the coil and *record* it in the **Table 1.3** of the worksheet (Page4).
2. Click the DataStudio working file 'NE03_EXP1_UID.d
3. Click 'Save Activity As' to create your group which UID should be your/your partner university no.(e.g. NE03_EXP1_12345678.ds).
4. Keep the Magnetic Field Sensor away from the magnetic plates.
5. Click 'START' to start data measurement.
6. Press 'TARE' button on the sensor to set zero of the sensor.
7. Put the Magnetic Field Sensor inside the pole plates on the magnet to measure the constant magnetic field strength between the magnet poles.
8. *Hold* the Magnetic Field Sensor steady about 5 seconds.
9. Click 'STOP' to stop data measurement.
10. *Read* the magnetic field strength between the magnet poles in Tesla ($T$) from the magnetic field strength vs. time graph.
11. *Find* out the mean value of the magnetic field strength by using "show selected statistic" $\boxed{\Sigma\ \cdot}$, select the "mean" function as shown in *Figure 18.*
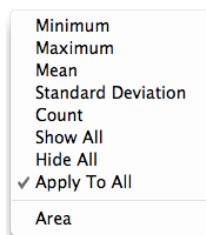


*Figure 17: Coil Passes through Magnet*

*Figure 18: Functions in* Σ▾.

12. Repeat the measurement of the magnetic strength field twice. *Record* the reading in **Table 1.1** in worksheet.

**How to measure the amplitude and the time duration of induced e.m.f.?**

1. *Adjust* the gap between the magnet poles so the coil wand will be able to pass through but put the magnet poles as close together as possible.
2. *Adjust* the height of the coil so it is in the middle of the magnet.
3. *Align* the wand from side-to-side so it will swing through the magnet without hitting it.
   **(Beware: the case of coil wand is plastic and fragile. Please avoid any collision.)**
4. Keep the induction wand in the steady position.
5. *Click* 'START' and pull the coil wand back about 40 degree and let it swing through the magnet.
6. When the coil wand stops swinging and stays at equilibrium position, *click* 'STOP' to end the measurement.
7. *Use* the 'Zoom select' to enlarge the portion of the voltage vs. time graph where the coil passed through the magnet.
8. *Use* the 'Smart Tool' to determine the amplitude of **first peak**
9. *Use* the 'Smart Tool' again to determine the time difference from the beginning to the end of the first peak.
10. *Repeat* steps (1) to step (9) twice and record the data in the **Table 1.2** in the worksheet.
11. Save the working file and then exit the program.

## Experiment 2: Lightly damped oscillation

**Lightly damped oscillation due to friction and air resistance, eddy current and power loss of the resistance in the coil**

## Experimental Procedure:

1.  Open the DataStudio file called "NE03_EXP2_UID.ds".
2.  Click 'Save Activity As' to create your group which UID should be your/your partner university no.(e.g. NE03_EXP1_12345678.ds).
3.  Now, the amount of energy lost to friction, air resistance and eddy current will be measured by letting the pendulum swing with **the coil connected in a complete circuit and presence of magnetic field.**
4.  *Connect* the resistor with both plugs in the coil wand. This completes the series circuit of the resistor and coil as shown in the *Figure 12*.
5.  *Click* 'START' with the coil at rest in its equilibrium position between the coils. Then rotate the wand to an initial angle of 25 degrees and let it go. The angle is shown in digital meter
6.  *Click* STOP 'after the coil stops swinging.
7.  *Measure* the initial angular position and angular position to which the pendulum firstly rises to another side after it passes once through the magnet. Record it in **Table 2.3** in the worksheet for the 1$^{st}$ trial measurement.
8.  *Calculate* the initial and final heights by using the distance from the center of mass to the pivot and the initial and final angular position in **Table 2.2** in the worksheet. Record it in **Table 2.3** in the worksheet for the 1$^{st}$ trial measurement.
9.  Calculate the total energy lost using equation (18).
10. *Use* the 'Zoom select' to enlarge the portion of the angular position vs. time graph where at least five periods of curve is observed.
11. *Use* the 'Smart Tool' to determine the periods and amplitudes from the Angular position vs. time graph. *Record* the data in **Table 2.4**.
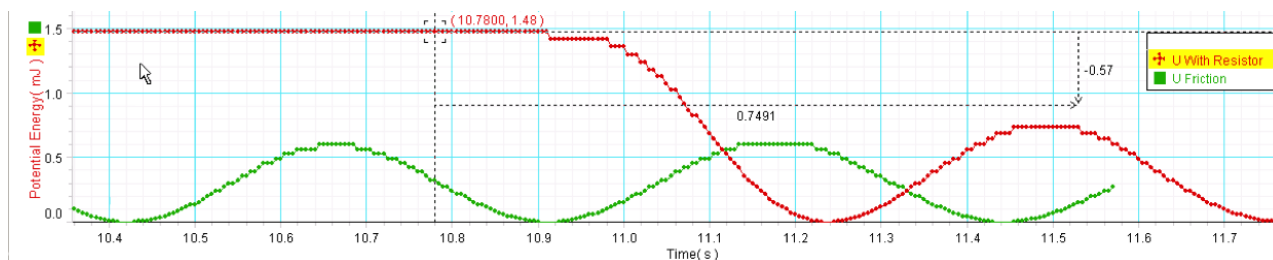


*Figure 23* Potential Energy vs. Time graph

12. *Highlight* both peaks on the power vs. time graph and find the area by using "show selected statistic" , "Tick" the Area as shown in *Figure 18*. This area is the energy dissipated by the resistor.
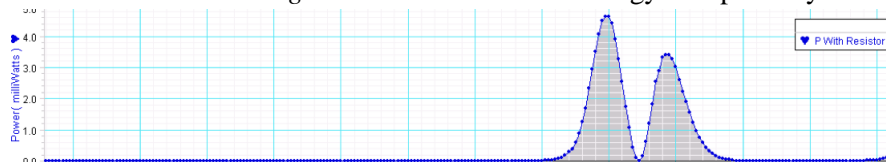


*Figure 24* Power vs. Time graph

13. Save the working file and then exit the program.

References:

Physical pendulum and Center of mass
1. http://hyperphysics.phy-astr.gsu.edu/hbase/pendp.html
2. Chapter 10, Physics for Scientists and Engineers with Modern Physics 8th Edition, John Jewett and Raymond Serway

S.H.M.and Damped motions
3. http://hyperphysics.phy-astr.gsu.edu/hbase/shm.html#c1
4. Chapter 15, Physics for Scientists and Engineers with Modern Physics 8th Edition, John Jewett and Raymond Serway

Magnetic field and Hall effect
5. Chapter 29, Physics for Scientists and Engineers with Modern Physics 8th Edition, John Jewett and Raymond Serway

Faraday's law and Eddy Current
6. Chapter 31, Physics for Scientists and Engineers with Modern Physics 8th Edition, John Jewett and Raymond Serway