

Laboratory Manual
Experiment NE04 - Resistor-Capacitor (RC) Circuit
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Aims:

To demonstrate various properties of a RC circuit such as:

1. Analyze the transient behavior of a series RC circuit.
2. Predict and measure the response V_C of an RC circuit.
3. Predict and measure the time constant τ of an RC circuit.
4. Determine the experimental value of capacitance of an RC circuit.

Self-learning material: Theory - Background InformationDefinition

A capacitor is a fundamental electronic device capable of storing charges/ electrical energy. In its simplest form, a capacitor consists of two parallel conducting plates that are separated by a dielectric material (i.e. air, polymers, quartz and glass...etc). When a capacitor is subjected to a potential difference, say by being connecting to a battery (d.c. source) as shown in figure 1 below, charges will be pushed to one of the plates until the accumulated charges produce a potential difference across the capacitor enough to oppose the electromotive force of the battery.

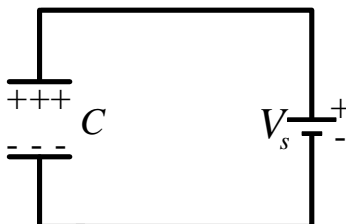


Figure 1: Schematic of a parallel plate capacitor.

The potential difference across the capacitor V_C , is proportional to the total charge of each of the plates $\pm Q$.

This proportionality constant is known as the capacitance, C , which is defined in (1) below.

$$\boxed{C \equiv \frac{Q}{V_C}} \quad (1)$$

In SI units, *capacitance* is measured in farads (F) or $J^{-1}C^2$. *Capacitance* is a measure of the capacity to store charge and thereby electrical energy*.

With the mechanics behind capacitors well understood, it is perhaps appropriate to examine the dynamic behavior of this electronic device. Qualitatively, it is perceivable that charges are able to flow freely at the instant when the capacitor is subjected to a potential difference. As charges, hence a back-voltage, build up, it will be increasingly difficult for the battery to move additional charges to the plates until the flow of charges stops altogether when the time tends towards infinity. In this experiment, the RC circuit will be employed to quantify such phenomenon.

For *parallel plates* only, the potential difference across the plates, V_C and electric field, $\left| \vec{E} \right|$ across the capacitor with separation d and surface area A_{surface} are:

$$V_C = \left| \vec{E} \right| \cdot d \quad (2)$$

* Although not probed in this experiment, it is useful to note that the electrical energy stored in the electric field of

the capacitor is given by $U = \frac{1}{2} QV_C = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV_C^2$.

According to Coulomb's law and considering the spherical surface,
$$|\mathbf{E}| = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = \frac{Q}{4\pi r^2} \frac{1}{\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

$$\Rightarrow |\mathbf{E}| = \frac{Q}{A_{\text{surface}} \epsilon_0} \quad (3)$$

$$\therefore C = \frac{Q}{V_C} = \epsilon_0 \frac{A_{\text{surface}}}{d} \quad (4)$$

Where $\sigma = \frac{Q}{A_{\text{Surface}}}$ is the surface charge density and ϵ_0 is the permittivity of free space and equals to

$$8.85418782 \times 10^{-12} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2$$

Charging behavior of capacitor in RC circuit

As apparent in its name, an RC circuit consists of a resistor and a capacitor coupled *in series* – apart from the potential difference of course. Consider a typical RC circuit with electromotive force V_s whose schematic is displayed in figure 2. With switch S in the *charging position* (position 1),

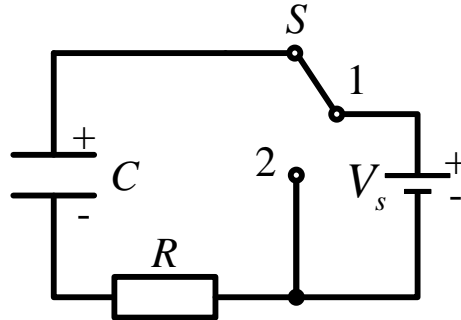


Figure 2: Schematic of charging RC circuit.

Kirchhoff loop rule reads:
$$V_s = V_R + V_C \quad (5)$$

According to Ohm's law (i.e. $V_R = IR$), equation (5) and equation (1), becomes

$$V_s = V_R + V_C = IR + \frac{Q}{C} \quad (6)$$

Since the current is defined as $I = \frac{dQ}{dt}$, rearranging equation (6) becomes a first order non-homogeneous ordinary differential equation:

$$V_s = \frac{dQ}{dt} R + \frac{Q}{C}$$

$$R \frac{dQ}{dt} + \frac{1}{C} Q - V_s = 0 \quad (7)$$

As detailed in the *appendix*, the solution of equation (7) means the charges stored in the capacitor at time t which is given by

$$Q(t) = CV_s \left(1 - e^{-\frac{t}{RC}} \right) \quad (8)$$

One can verify this result by plugging it back into equation(7). Substitution of equation (8) into equation (1) yields the potential difference across the capacitor, namely

$$V_C(t) = V_s \left(1 - e^{-\frac{t}{RC}} \right) \quad (9)$$

and taking the time derivative of equation (8) (i.e. $\frac{dQ(t)}{dt}$) then substituting into $V_R = IR = \frac{dQ}{dt} R$ gives the

$$\text{potential difference across the resistor}^\dagger, \text{ namely } V_R(t) = V_s e^{-\frac{t}{RC}} \quad (10)$$

Equations (8), (9) and (10) invites one to define a *time constant*,

$$\boxed{\tau \equiv RC} \quad (11)$$

The SI unit of *time constant* is second (s). At a time $t = RC$, the current has decreased to $\frac{1}{e}$ (about 0.3679) of its initial value and the charge has increased to *within* $\frac{1}{e}$ of its final value. The product RC is called the *time constant*, or the *relaxation time*, of the circuit. Generally, Having the units of time, the *time constant* represents the time for the exponential term to drop to $\frac{1}{e}$ or 36.79% of its original value. Each subsequent time constant will decrease it by the same fraction.

Physically, one can interpret this relation by imagining that the capacitor is a bucket being filled by water, which represents the charge. The more water the bucket can hold, the longer it takes to fill the bucket. On the other hand, the resistor resists the flow into the bucket; the slower the flow, the longer it takes to fill the bucket.

In particular, during the *charging phase*, the currents according to the differentiation of equation (8) with respect to t is

$$I(t) = \frac{dQ}{dt} = \frac{V_s}{R} e^{-\frac{t}{RC}} \quad (12)$$

$$I(t) = I_o e^{-\frac{t}{RC}} \quad (13)$$

where $I_o = \frac{V_s}{R}$ is the initial current in the circuit

Discharging behavior of capacitor in RC circuit

The behavior of the RC circuit during the capacitor's discharge can also be examined in a similar manner. Once again, the Kirchhoff loop rule can be employed in figure 3 but now with the switch in the *discharge position* (position 2).

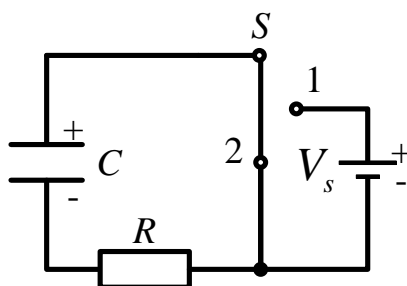


Figure 3: Schematic of discharging RC circuit.

Referring to Figure 3, according to the Kirchhoff loop rule: $V_C = V_R$ (14)

[†] Details of the calculation are featured in the appendix.

Substituting equation (1) and Ohm's law (i.e. $V_R = IR$) into equation (14), it results $\Rightarrow \frac{Q}{C} = IR$

According to the definition of current (i.e. $I = \frac{dQ}{dt}$),

$$\begin{aligned} \Rightarrow \frac{1}{C}Q &= \frac{dQ}{dt}R & \Rightarrow \frac{1}{RC}Q &= \frac{dQ}{dt} \\ & \Rightarrow \frac{1}{RC}dt &= \frac{dQ}{Q} & \quad (15) \end{aligned}$$

Upon integration on both sides of equation (15), equation (15) gives $\frac{1}{RC} \int dt = \int \frac{1}{Q} dQ$

$$Q \int \frac{1}{x} dx = \ln x + \text{constant where } \ln x = \log_e x \text{ is the Natural logarithm} \quad \Rightarrow \frac{1}{RC}t = \ln Q(t) + c_o$$

According to logarithmic identities, $Q(t) = A_o e^{-\frac{t}{RC}}$ (16)

where c_o and A_o are just arbitrary constants. Evaluating equation (16) at $t = 0$ while noting equation (1) (i.e. $C \equiv \frac{Q(t)}{V_C(t)}$) and $V_C(t=0) = \varepsilon$ give $A_o = CV_s$; hence, the solution to (16) is

$$Q(t) = CV_s e^{-\frac{t}{RC}} \quad (17)$$

Finally, utilizing the relations in equation (1), the potential difference across the capacitor and resistor are respectively

$$V_C(t) = V_s e^{-\frac{t}{RC}} \quad (18)$$

and substituting equation (18) into equation (14) gives, $V_R(t) = V_s e^{-\frac{t}{RC}}$ (19)

The main objective in this laboratory exercise is to investigate the RC circuit. In addition to the measurement of $V_C(t)$ and $V_R(t)$, it is important to point out that the current flowing through the circuit naturally falls out from the derivation of the potential difference in each circuit element.

In particular, during the *discharging phase*, the currents according to the differentiation of equation (16) with respect to t is

$$I(t) = \frac{dQ}{dt} = -\frac{V_s}{R} e^{-\frac{t}{RC}} \quad (20)$$

$$I(t) = -I_o e^{-\frac{t}{RC}} \quad (21)$$

where $I_o = \frac{V_s}{R}$ is the initial current in the circuit

As a reminder, the change of sign in (13) and (21) simply relates to the reversal of current direction.

Apparatus

- 1 x PASCO Scientific EM-8678 charge/ discharge circuit board
- 1 x PASCO Scientific CI-6760 *Science Workshop 500* data logger
- 1 x PASCO Scientific CI-6503 current probe
- 2 x PASCO Scientific CI-6503 voltage probe
- 2 x 1.5V AA batteries
- Miscellaneous wirings
- PASCO *DataStudio* computer interface

Displayed in figure 4 is the PASCO Scientific EM-8678 charge/discharge circuit board with the relevant components indicated accordingly. Circled in blue and positioned in the upper left corner is a capacitor of $C = 1.0\text{F}$; circled in red and positioned in the lower middle are three resistors of $R_1 = 10\Omega$, $R_2 = 33\Omega$ and $R_3 = 100\Omega$; boxed in orange and positioned in the lower right corner of the board is the switch S to toggle between *charge* and *discharge*; and finally boxed in green and positioned upper part are the battery holders. Since each battery is $\mathcal{E} = 1.5\text{V}$, connecting them in series will provide a theoretical total electromotive force, $\mathcal{E} = 3.0\text{V}$.

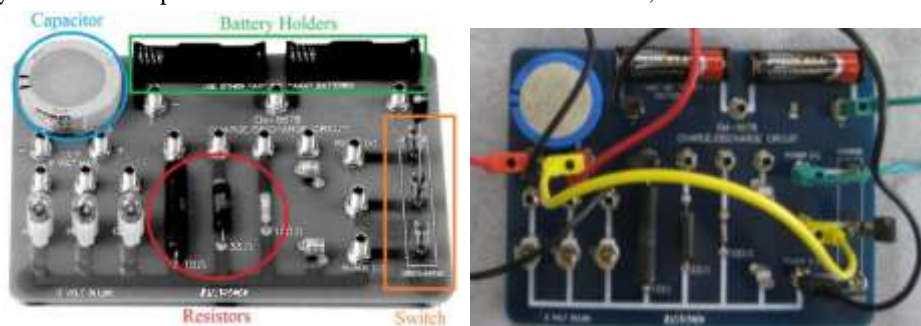


Figure 4: PASCO Scientific EM-8678 charge/ discharge circuit board.

Experiment 1: Measurement of time constant**Setup Procedures**

1. Connect a wire from positive terminal of the 3V battery to the “charge” position of switch. Also, connect another wire from negative terminal of the 3V battery to the “discharge” position of switch.



Figure 5: Connecting the wire from positive terminal of 3V battery to the switch

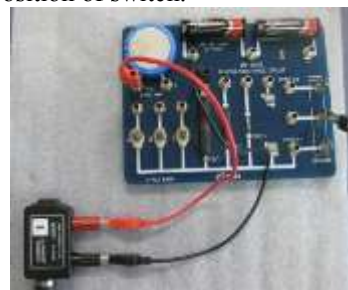


Figure 6: Connecting the current sensor to the capacitor and the resistor

2. Connect voltage sensor and positive terminal of the current sensor to the capacitor (**Beware of the polarities of the sensors**)



Figure 7: Connecting the wire from negative terminal of 3V battery to the switch

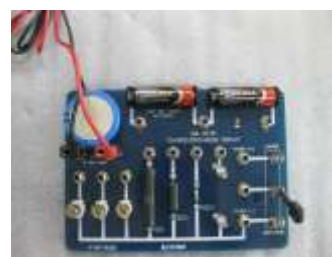


Figure 8: Connecting the voltage sensor to the capacitor

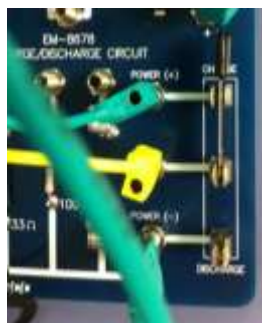


Figure 9: Switch at “Charge” position



Figure 10: Switch at “Middle” position



Figure 11: Switch at “Discharge” position

3. Connect negative terminal of the current sensor to $R_1 = 10\Omega$ resistor.
4. Ensure the switch is opened and it is in the “middle” position as shown in Figure 10. (If the switch is in the “charge” position, change the switch to “discharge” position about 50 seconds (i.e. 5RC).). If ones would like to discharge a capacitor quickly, he or she could simply make the capacitor short circuit by a wire as shown in Figure 13.



Figure 12: A wire for shorting the capacitor



Figure 13: Shorting the capacitor

Experimental Procedures

1. Open the DataStudio® working file 'NE04_EXP1_UID.ds'. Click 'Save Activity As' function to save your own file where UID should be your/your partner University no.(e.g. NE04_EXP1_3333123456.ds)
2. Click "start" to start measurement.
3. Put the switch to the "charge" position as shown in Figure 9.
4. A charging curve is observed. Wait for the potential difference across capacitor V_C is constant. Please refer to Pre-lab question 6 in your worksheet to estimate the charging time.
5. Click "stop" to stop measurement.
6. Click to Fit the charging curve by using *Inverse exponent fit* (i.e. $A(1 - e^{-Cx}) + B$).
7. Complete the table in the worksheet. (Table 1.1 for 10Ω)
8. Press the start button.
9. Put the switch to the "discharge" position as shown in Figure 11.
10. A discharging curve is observed. Wait for the potential difference across capacitor V_C is constant. Please also refer to Pre-lab question 6 in your worksheet to estimate the discharging time.
11. Press the stop button.
12. Fit the discharging curve by using *natural exponent fit* (i.e. $Ae^{-Cx} + B$).
13. Complete the table worksheet. (Table 1.2 for 10Ω)
14. Use the wire in Figure.12 to short the capacitor (in Figure.13) to make fast capacitor discharging process.
15. Repeat steps (2) to (14) with changing the resistor (33Ω) in Figure.14 to complete the Table 1.3 for charging experiment and Table 1.4 for discharging experiment respectively.
16. Repeat steps (2) to (14) with changing the resistor (100Ω) in Figure. 14 to complete the Table 1.5 for charging experiment and Table 1.6 for discharging experiment respectively.
17. Take one more set of data (i.e. 2nd trial for 10Ω and 33Ω except for 100Ω) through step (2) to step (16).
18. By clicking "Save Activity" to save the experimental result.



Figure 14: Changing the resistor / Changing the number of cell

Experiment 2: Measurement of capacitance

Setup Procedures

1. Changing the battery voltage from $3V$ to $1.5V$ by changing the cable connection as shown in Figure 14
2. Verify the cable connection is $1.0F$ capacitor and $R_3 = 100 \Omega$ resistor as experiment 1.
3. Ensure the switch is opened and it is in the “middle” position as shown in Figure 10. (If the switch is in the “charge” position, change the switch to “discharge” position about 50 seconds (i.e. $5RC$).). If ones would like to discharge a capacitor quickly, just make the capacitor short circuit by a wire as shown in Figure 13.

Experimental Procedures

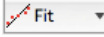


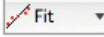
1. Open the DataStudio® working file ‘NE04_EXP2_UID.ds’. Click ‘Save Activity As’ function to save your own file where UID should be your/your partner University no.(e.g. NE04_EXP2_3333123456.ds)
2. Click “start” to start measurement.
3. Put the switch to the “charge” position as shown in Figure 9.
4. A charging curve is observed. Wait for the potential difference across capacitor V_C is constant. The charging time please refer to Pre-lab question 6 in your worksheet.
5. Click “stop” to stop measurement.
6.  Fit the charging curve by using Inverse exponent fit (i.e. $A(1 - e^{-Cx}) + B$).
7. Select the Area under the current-time curve as shown in the upper right panel of Figure 16. Find out the area of current-time curve by using “show selected statistic” , “Tick” the Area as shown in Figure 15.



Figure 15: Functions in .

8. Record the reading in the Table 2.1(a) in the worksheet.
9. Discharge the capacitor for few seconds through shorting the terminals of the capacitor by using a wire as shown in Figure 13.
10. Change the $1.5 V$ battery to $3.0V$ battery connection.
11. Repeat the steps (2) to (7) for $3.0V$ case.
12. Record the reading in the Table 2.1(b) in the worksheet.
13. Input the values of potential difference across the capacitor V_C and the area under the current-time curve into the table of DataStudio as shown in bottom right panel of Figure 16 and **Table 2.1** in the worksheet.(Note: Never using the theoretical values of $1.5V$ and $3.0V$ as the p.d. across capacitor). The first set of the data point is entered in the **Table 2.1** in the worksheet for you. i.e. (0,0)
14.  Fit the curve of potential difference across the capacitor and the area under the current-time curve by using proportional fit (i.e. Ax).
15. Complete the **Table 2.2** in the worksheet.
16. By clicking “Save Activity” to save the experimental result.

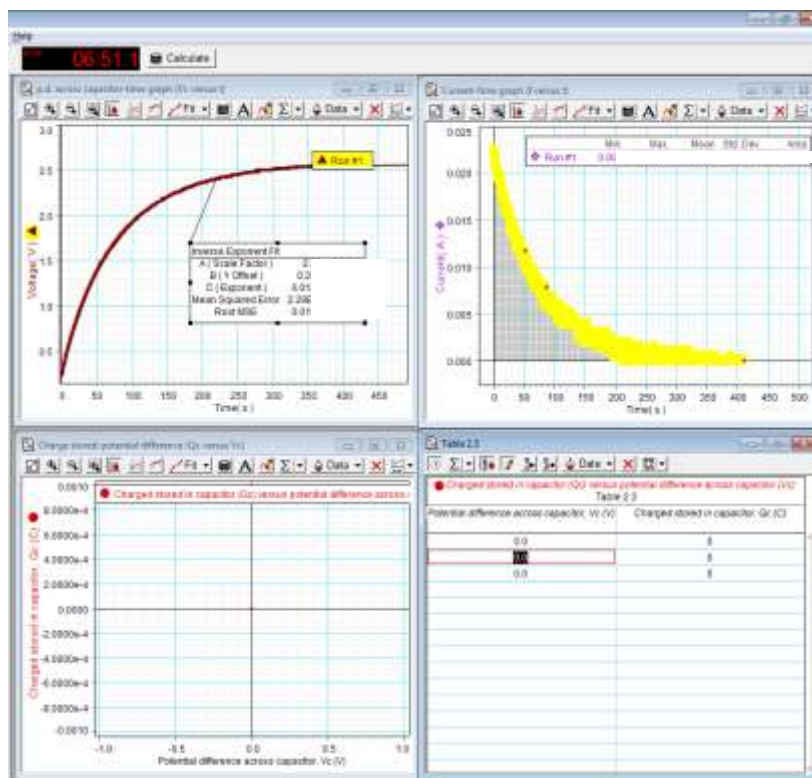


Figure 16: Table for inputting the values of p.d. across capacitor and charged stored in it.

Appendix

Solving First Order inhomogeneous Ordinary Differential Equations:

In general, a first order inhomogeneous ordinary differential equation is of the form

$$A \frac{dy(x)}{dx} + By(x) + C = 0 \quad (\text{A1})$$

where A , B and C are constants (Don't confuse this C with the capacitance). The first step to tackling this equation is to find the homogeneous solution of the equation, which, by definition, is the solution if the equation were homogeneous (i.e. $C = 0$) as shown in (A2). Rearranging and integrating the homogeneous equation yields the homogeneous solution, y_h (A3):

$$A \frac{dy(x)}{dx} + By(x) = 0 \quad (\text{A2})$$

$$\Rightarrow -\frac{B}{A} \int dx = \int \frac{dy}{y}$$

$$\Rightarrow y_h = A_o e^{-\frac{B}{A}x} \quad (\text{A3})$$

$$\text{The particular solution}^\ddagger y_p, \text{ on the other hand, is simply } \Rightarrow y_p = -\frac{C}{B} \quad (\text{A4})$$

[‡] The particular solution is just some solution that satisfies the non-homogeneous equation.

which can easily be verified by plugging (A4) into (A1). The general solution, $y(x)$ is then the sum of the

homogeneous and particular solution; hence,

$$y(x) = y_h + y_p = A_0 e^{-\frac{B}{A}x} - \frac{C}{B'} \quad (\text{A5})$$

Comparing equation (A1) to equation (7) reveals that in the case of this experiment, $A = R$, $B = \frac{1}{C}$ and $C = -V_s$. The general solution of (7) is therefore $Q(t) = A_0 e^{-\frac{t}{RC}} + CV_s$ (A6)

Boundary condition $Q(t = 0) = 0$ can now be applied to rid (A6) of its arbitrary constant. Since $A_0 = -C\varepsilon$ according to the boundary condition, (A6) is successfully recovered.

Potential difference across the resistor during the charging phase:

The potential difference across a resistor is given by the Ohm's law:

$$V_R(t) = i(t)R = R \frac{dQ}{dt} \quad (\text{A7})$$

Taking the derivative of equation (9) with respect to t , namely

$$\frac{dQ}{dt} = \frac{d}{dt} \left[CV_s \left(1 - e^{-\frac{t}{RC}} \right) \right] = \frac{V_s}{R} e^{-\frac{t}{RC}} \quad (\text{A8})$$

then substituting into equation (A7) gives equation (10) as required.

References

Capacitor:

1. Chapter 26, Physics for Scientists and Engineers with Modern Physics 8th Edition, John Jewett and Raymond Serway
2. <http://ocw.mit.edu/courses/physics/8-02t-electricity-and-magnetism-spring-2005/lecture-notes/chap5capacitance.pdf>

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3. Chapter 28, Physics for Scientists and Engineers with Modern Physics 8th Edition, John Jewett and Raymond Serway
4. <http://hyperphysics.phy-astr.gsu.edu/hbase/electric/capdis.html>
5. <http://faculty.wvu.edu/vawter/PhysicsNet/Topics/DC-Current/RCSeries.html>
6. <http://youtu.be/LDtDNHMveBQ>

Time constant:

7. http://www.electronics-tutorials.ws/rc/rc_1.html

Natural logarithm:

8. A-10, Appendix B, Physics for Scientists and Engineers with Modern Physics 8th Edition, John Jewett and Raymond Serway
9. http://en.wikipedia.org/wiki/Natural_logarithm

Logarithmic identities:

10. http://en.wikipedia.org/wiki/List_of_logarithmic_identities