Experiment NE05
Inductor -Resistor-Capacitor (LRC) Series Circuit
Laboratory Manual
Department of Physics
The University of Hong Kong

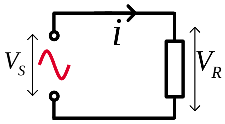
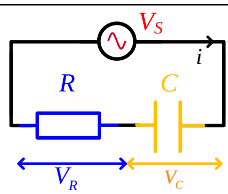
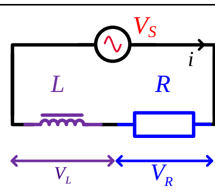
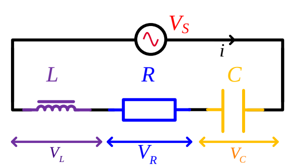
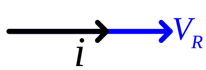
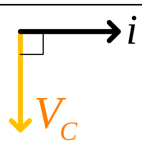
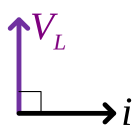
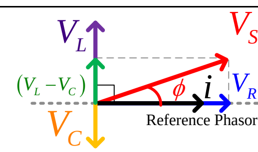
Aims

To demonstrate various properties of an LRC series circuit such as:

1. Phase difference between potential difference across the circuit elements and current of the circuit.
2. Resonant frequency of a LRC series circuit.
3. Impedance of the circuit under different alternating circuit (A.C.) frequencies.

Self-learning material: Theory - Background Information

Summary:

	R	RC	LR	LRC
Circuit				
Impendence, Z	R	$\sqrt{\left(R^2+X_C^2\right)}$	$\sqrt{\left(R^2+X_L^2\right)}$	if $X_L > X_C$ $\sqrt{\left(R^2+\left(X_L-X_C\right)^2\right)}$
Phase difference with i	0°	i leads V_C by 90°	i lags on V_L by 90°	i leads V_C by 90° and i lags on V_L by 90°
Phasor diagram				

Definition:

A.C.

(a) a.c. and d.c.

In a direct current (d.c.) the drift velocity is in one direction only. Common source of d.c. is a battery. In an alternating current (a.c.), the direction of the drift velocity reverses, usually many times a second. A.c. is commonly generated by an a.c. generator.

The effect of a.c. are essentially the same as those of d.c. Both are satisfactory for heating and lighting purposes.

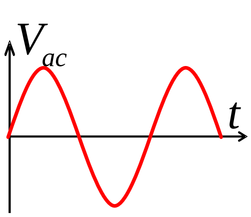


Figure 1:.Sinusoidal a.c.

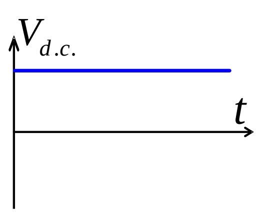


Figure 2:.constant d.c.

(b) Terms

1. Period, frequency and angular frequency

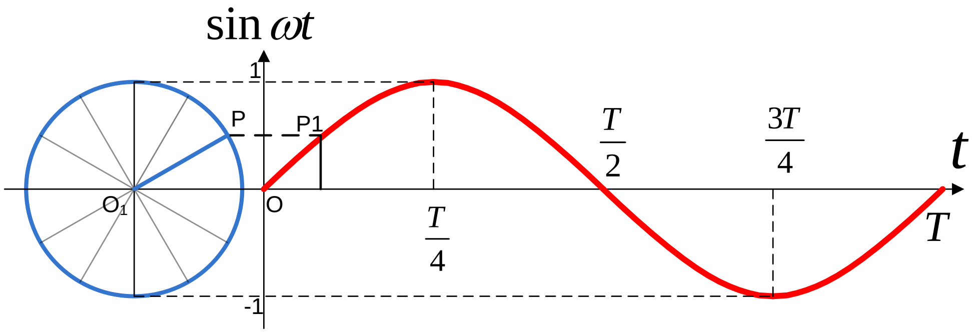
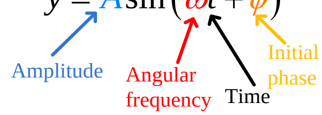


Figure 3: sine function of time graph

An *alternating current* or e.m.f. varies periodically with time in magnitude and direction. The simplest a.c. function is a sine function and is shown in Figure 3. The general equation of a sine function is shown below.

$$y = A\sin(\omega t + \phi)$$



One complete alternation is called a cycle and the number of cycles occurring in one second is termed the frequency (f) of the alternating quantity. The SI unit of frequency is the hertz (Hz) and was previously the cycle per second. The frequency of the electricity supply in Hong Kong is 50Hz which means that the duration of one cycle, known as the period (T), is 1/50 = 0.02s. In general $f = \frac{1}{T}$.

Relationships between period, frequency and angular frequency:

$$\boxed{T = \frac{1}{f} = \frac{2\pi}{\omega}} \qquad (1) \qquad \text{or} \qquad \boxed{f = \frac{1}{T}} \qquad (2)$$

$$\boxed{\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f} \qquad (3)$$

2. Simplest form of a.c.

The simplest and most important alternating e.m.f. can be represented by a sine curve and is said to have a *sinusoidal waveform*. It can be expressed by the equation:

$$\boxed{\zeta = \zeta_0 \sin \omega t} \qquad (4)$$

where ζ is the e.m.f. at time t, ζ_0 is the peak or maximum e.m.f. and ω is a constant which equals $2\pi f$ where f is the frequency of the e.m.f.

Similarly, for a sinusoidal alternating current we may write

$$\boxed{i = i_0 \sin \omega t} \qquad (5)$$

3. 4 important values of A.C.

a. Instantaneous value, V_t is defined as the e.m.f., potential difference, current and power of a.c. at certain time. Its mathematical expression at time t for potential difference: $V_t = V_o \sin \omega t$; for Current: $i_t = i_o \sin \omega t$.

a. Peak value, V_o is defined as maximum or minimum instantaneous value of a.c. Its mathematical expression for potential difference and current are $V_t = V_o$ and $i_t = i_o \because -1 \le \sin\omega t \le 1$

a. Averaged value/ Mean value[1], $\overline{P}$ is defined as the mean value of a certain periodic quantity P is represented by the notation $\overline{P}$ .It s mathematical expression is $\overline{P} = \frac{\int_0^T P \cdot dt}{T}$

i) Root-mean-squared value (r.m.s. value)[2]

a. Definition:

The *Root-mean-squared (r.m.s.) value* of an alternating current (also called the effective value) is the steady direct current which converts electrical energy to other forms of energy in a given resistance at the same rate as a.c.

Although the instantaneous value (current or e.m.f.) is varying,. The average rate at which it supplies electrical energy to the lamp equals the steady rate of supply by the d.c. ($i_{d.c.}$) and in practice it is this aspect which is often important.

b. Proof:

In general, considering the energy supplied to a resistor with resistance R,

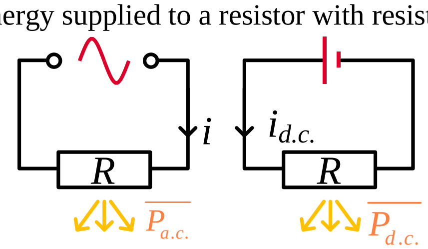


Figure 4: Schematic diagrams for a.c. and d.c. with same average power output

If the average power output of circuits in Figure 4 is equal, i.e.

$$\overline{P_{d.c.}} = \overline{P_{a.c.}}$$

$$i_{d.c.}^2 R = \left(\text{mean value of } i^2\right) \times R \qquad (6)$$

$$i_{d.c.} = \pm\sqrt{\left(\text{mean value of } i^2\right)} \qquad (7)$$

$$\therefore i_{d.c.} = \text{square root of the mean value of the square of the current}$$

$$\therefore i_{d.c.} = i_{r.m.s.} \qquad (8)$$

If the a.c. is sinusoidal (i.e. the source is a sine function) then $i = i_0 \sin\omega t$ (9)

$$\therefore i_{r.m.s.} = \pm\sqrt{\left(\text{mean value of } i_o^2 \sin^2 \omega t\right)} \qquad (10)$$

$$\therefore i_{r.m.s.} = \pm i_o\sqrt{\left(\text{mean value of } \sin^2 \omega t\right)} \qquad (11)$$

$$\boxed{\therefore i_{r.m.s.} = \pm\frac{i_o}{\sqrt{2}}} \qquad (12)$$

[1] Usually, the symbol of mean value or average value of a quantity x is donated by $\overline{x}$ or $< x >$

[2] Usually, the symbol of root-mean-squared of a quantity x is donated by $\sqrt{\overline{x^2}}$ or $\sqrt{< x^2 >}$

Graphically, the area of $P_{a.c.} - t$ curve is equal to that of $P_{d.c.} - t$ curve as shown in Figure 5.

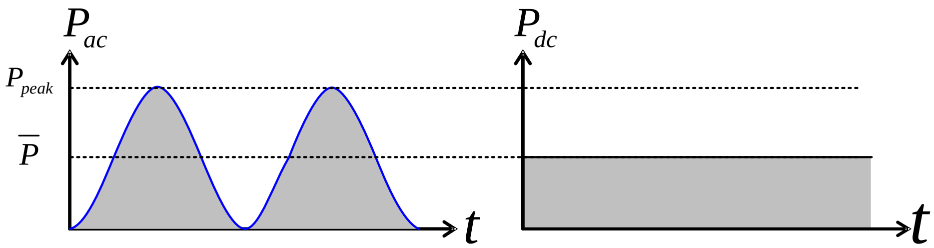


Figure 5: Power generated vs. time graphs for a.c. and d.c. sources

c. Why mean-squared-value of sine function, $\overline{\sin^2\theta} = \frac{1}{2}$?

Method 1:

$$\overline{\sin^2\theta} = \frac{\int_0^T \sin^2\omega t \cdot dt}{T} = \frac{\int_0^{2\pi/\omega} \sin^2\omega t \cdot dt}{2\pi/\omega}$$

$$= \frac{\int_0^{2\pi} \sin^2\omega t \cdot d(\omega t)}{2\pi}$$

$$= \frac{\int_0^{2\pi} \left[(1-\cos 2\omega t)/2\right] \cdot d(\omega t)}{2\pi}$$

$$= \frac{\int_0^{2\pi} \cdot d(\omega t)}{4\pi} - \frac{\int_0^{2\pi} \cos 2\omega t \cdot d(\omega t)}{4\pi}$$

$$= \frac{2\pi}{4\pi} - 0 = \frac{1}{2}$$

Method 2:

$$\because \overline{\sin^2\theta} = \overline{\cos^2\theta}$$

$$\because \sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \overline{\sin^2\theta + \cos^2\theta} = 1$$

$$\Rightarrow \overline{\sin^2\theta} + \overline{\cos^2\theta} = 1$$

$$\Rightarrow 2\overline{\sin^2\theta} = 1$$

$$\Rightarrow \overline{\sin^2\theta} = \frac{1}{2}$$

<u>Vector diagram or Phasor diagram</u>

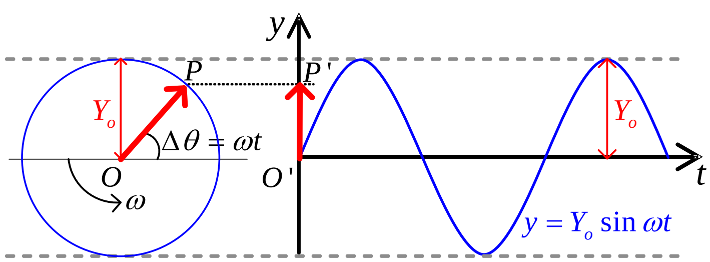


Figure 6: Rotating vector of an alternating quantity

A sinusoidal alternating quantity can be represented by a rotating vector (often called a Phasor) as shown in Figure 6. It is called a Phasor diagram. An alternating quantity $y = Y_o \sin \omega t$ where Y_o is the peak value of the quantity and its frequency $f = \frac{\omega}{2\pi}$. If the line OP has length Y_o (i.e. radius of an imaginary circle) and rotates in an anticlockwise direction about O with uniform angular velocity ω, the projection $O'P'$; of OP on $O'y$ at time t (measured from the time when OP passes through OO') is $Y_o \sin \omega t$. If OP is directed as shown Figure 6 by the arrow on it, then we can say that the projection on $O'y$ of the rotating vector OP gives the value at any instant of the sinusoidal quantity y. (Simple harmonic motion, being sinusoidal, can be derived similarly from uniform motion in a circle.)

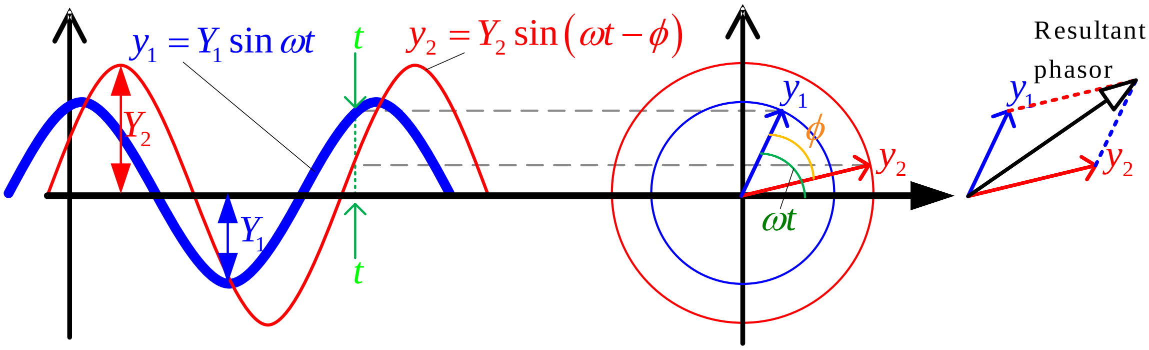


Figure 7: Phase difference between two alternating quantities

The Phasor diagram is very useful for representing two sinusoidal quantities which have the same frequency f or same angular frequency ω but are not in phase (i.e. $\phi \neq 0$) The wave forms of two such quantities y_1 and y_2 and the corresponding vector diagram are shown in Figure 7 for time t. The phase difference between them is ϕ , with y_2 lagging, and this phase angle is maintained between them as the vectors rotate. Being vectors they can be added by the parallelogram law if they represent similar quantities, e.g. p.ds. or currents

Algebraically y_1 and y_2 are expressed by the equations: $y_1 = Y_1 \sin \omega t$ and $y_2 = Y_2 \sin(\omega t - \phi)$

<u>Resistor:</u>

(a) <u>Resistance in a.c. circuit</u>

The ***resistance***, denoted by R, measures the ability of a resistor to oppose the passage of current through it. The formula for the resistance of a conductor is

$$\boxed{R = \rho \frac{l}{A}} \qquad (13)$$

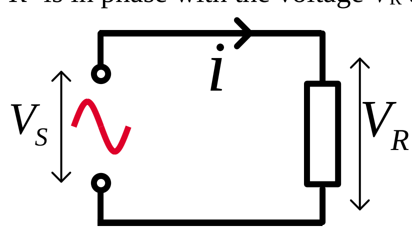
where ρ is the electrical resistivity of the material, l is the length, and A is cross-section area of the conductor.

For ohmic material, the current passing through it is directly proportional to potential difference across that material. i.e. Obeys *Ohm's law*. $V_R \propto i_R$

$$\Rightarrow V_R = Ri_R, \qquad \text{where } R \text{ is a constant.}$$

(b) Pure resistive circuit

The current *I* through a resistor *R*[3] is in phase with the voltage V_R applied to it.



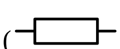
[3] Symbol of resistor could be IEC-style () or American-style ().

Figure 8: Schematic diagram of pure resistance circuit

Referring to the Figure 8, the a.c. source is a sinusoidal voltage. i.e. $V_S = V_R$

Let applied potential difference across resistor,

$$V_R = V_{Ro} \sin \omega t \qquad (14)$$

Where V_{Ro} is the peak value of potential difference across the resistor.

According to Ohm's law, the current is also a sinusoidal function. and in phase with voltage V_R applied

$$i = \frac{V_R}{R} = \frac{V_{Ro}}{R} \sin \omega t \qquad (15)$$

Let $i_o = \dfrac{V_{Ro}}{R}$ (16)

Where i_o is the peak value of current passing through the resistor. As a result,

$$i = i_o \sin \omega t \qquad (17)$$

Hence i and V_R are in phase, also

$$\boxed{R \equiv \frac{V_{Ro}}{i_o} = \frac{V_{R,r.m.s}}{I_{r.m.s.}} = \frac{V_R}{i}} \qquad (18)$$

Like the case in d.c., the peak current i_o passing through a resistor depends on both the resistance R and potential difference across it V_R.

<u>Capacitance in a.c. circuits</u>

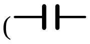
(a) Definition

A simple *capacitor*[4] consists of two parallel rectangular conducting plates separated by a dielectric (i.e. air, polymers, quartz and glass…etc), where a dielectric is a kind of insulator which can be polarized by applying an electric field. When a capacitor is completely charged by a battery (i.e. d.c. source), it contains equal but opposite charges on the two plates, i.e. one plate gains electrons and the other loses the same amount of electrons through the circuit. If the charge stored by a certain capacitor is Q, it actually means that one plate stores charge $+Q$, and the other plate stores charge $-Q$. Please noted that there is no current pass through the capacitor by using d.c. source. But there is a virtual "current" passing through the capacitor by using a.c. source.

The *capacitance*, denoted by C, is defined as the charge stored per unit voltage applied to the capacitor. In mathematical form, we have

$$\boxed{C \equiv \frac{Q}{V_C}} \qquad (19)$$

where Q is the charge stored and V_C is the potential difference across the plates. In SI units, *capacitance* is measured in farads (F) or $J^{-1}C^{2}$.

[4] Symbol of capacitor could be ().

Capacitance is a measure of the capacity to store charge and thereby electrical energy[5]. The capacitance is an intrinsic property that could be calculated by knowing the dimension and material used.

For *parallel plates* only, the potential difference and electric field across the capacitor with separation d and surface area $A_{surface}$ are:

$$V_C = \left|\vec{E}\right| \cdot d \qquad (20)$$

$$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2} = \frac{Q}{4\pi r^2}\frac{1}{\varepsilon_o} = \frac{\sigma}{\varepsilon_o} \Rightarrow \left|\vec{E}\right| = \frac{Q}{A_{surface}}\frac{1}{\varepsilon_o} \qquad (21)$$

By eliminating $\left|\vec{E}\right|$ in the above equations, for a parallel-plate capacitor, the capacitance is

$$\boxed{C = \varepsilon\frac{A}{d}} \qquad (22)$$

where ε is the permittivity of the dielectric material separating the two plates, d is the separation between the plates, and A is the overlapping area of two plates.

(b) Flow of a.c. 'through' a capacitor

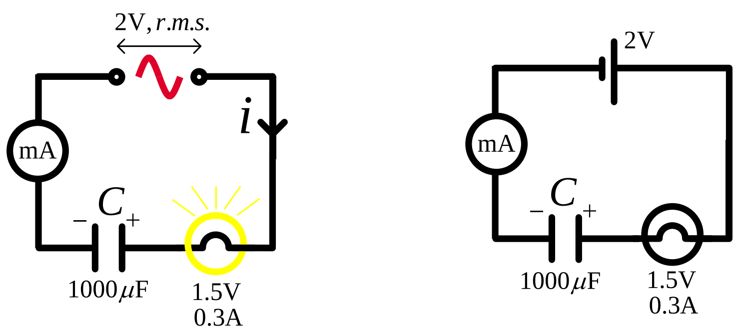


Figure 9: Phase difference between V_C and i

As shown in Figure 9, if a 1000μF capacitor is connected in series with a 1.5V, 0.3 A lamp, and a 2V d.c. supply, the lamp as expected, does not light. Is there any current flow? With a 2V r.m.s. 50 Hz supply, it is nearly fully lit.

The a.c. is apparently flowing through the capacitor. In fact, the capacitor is being charged, discharged, charged in the opposite direction and discharged again, fifty times per second (the frequency of the a.c.), and the charging and discharging currents flowing through the lamp light it. No current actually passes through the capacitor (since its plates are separated by an insulator(i.e. dielectric material) but it appears to do so and we talk as if it did. A current would certainly be recorded by an a.c. milliammeter.

(c) Phase relationships

When a.c. flows through an ideal resistor (having no capacitance or inductance) the current and potential difference reach their peak values at the same instant, i.e. they are in phase. This is not so for a capacitor.

For a *capacitor*, the current through it is seen to lead the potential difference across it by *one-quarter of a cycle*, i.e. the current reaches its maximum value one-quarter of a cycle before the potential difference reaches its peak value.

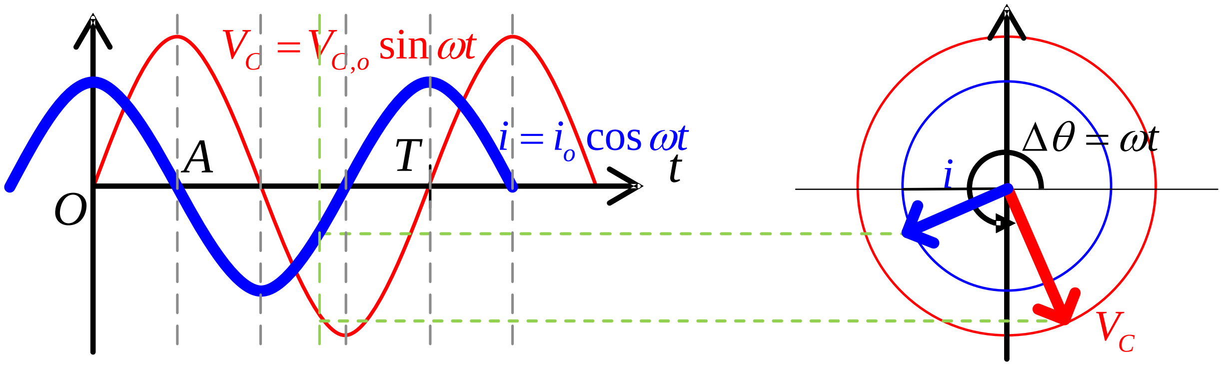


Figure 10: Phase difference between V_C and i

Considering a pure capacitance circuit as shown in Figure 11, current i and applied potential difference V_C are out of step because current i flow is a maximum immediately an uncharged capacitor is connected to an a.c. supply. There is as yet no charge on the capacitor to oppose the arrival of charge. Thus at O the applied potential difference, V_C though momentarily zero, is increasing at its maximum rate (the slope of the tangent at O to the potential difference graph is a maximum) and so the rate of flow of charge – the current, i – is also a maximum.

Between O and A the potential difference V_C is increasing but at a decreasing rate, the charge on the capacitor is increasing $(\because Q = CV_C)$ but less quickly, which means that the charging current i is less. At A the applied potential difference V_C is a maximum and for a brief moment is constant. The charge on the capacitor will also be a maximum and constant. The rate of flow of charge is therefore zero, i.e. the current $i = \dfrac{dQ}{dt}$ is zero. The phase difference between V_C and i can thus be explained.

(d) Mathematical treatment

Considering a *pure capacitance circuit* as shown in Figure 11, let a potential difference V_C be applied across a capacitance C and let its value at time t be given by

$$\boxed{V_C = V_{C,o}\sin\omega t} \qquad (23)$$

where $V_{C,o}$ is its peak value and $\omega = 2\pi f$ where f is the frequency of the supply.

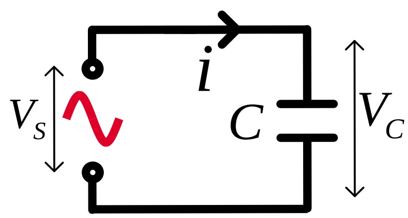


Figure 11: Schematic diagram of pure capacitance circuit

The charge Q on the capacitance at time t is

$$Q = V_C C \qquad (24)$$

For the current i flowing 'through' the capacitor we can write

$$i = \text{rate of change of charge} = \frac{dQ}{dt} \qquad (25)$$

$$\because Q = CV_C \text{ (Definition of capacitance)}$$

$$\because \frac{d}{dt}(uv) = u\frac{dv}{dt} + v\frac{du}{dt} \text{ (quotient rule) where } u \text{ and } v \text{ are function of } t \Rightarrow i = \frac{d}{dt}\left(CV_C\right) \qquad (26)$$

$$\because \frac{d}{dt}(C) = 0 \text{ as capacitance is a constant and independent of } t$$

$$\Rightarrow i = C\frac{dV_C}{dt} + V_C\frac{dC}{dt} \qquad \Rightarrow i = C\frac{dV_C}{dt} \qquad (27)$$

$$\because V_C = V_{C,o}\sin\omega t \qquad \Rightarrow i = C\frac{d}{dt}\left(V_{C,o}\sin\omega t\right) \qquad (28)$$

$$\because \frac{d}{dt}(\sin\alpha t) = \frac{d(\alpha t)}{dt}\left[\frac{d}{d(\alpha t)}(\sin\alpha t)\right] \text{ where } \alpha \text{ and } V_{C,o} \text{ are constant and independent of } t$$

$$\Rightarrow i = CV_{C,o}\frac{d}{dt}(\sin\omega t) \qquad (29)$$

$$\because \left[\frac{d}{d(\alpha t)}(\sin\alpha t)\right] = \cos\alpha t \quad \text{and} \quad \frac{d(\alpha t)}{dt} = \alpha \qquad \text{where } \alpha \text{ is a constant and independent of } t$$

$$\Rightarrow i = CV_{C,o}\cos\omega t\frac{d}{dt}(\omega t) \qquad (30)$$

$$\boxed{\therefore i = CV_{C,o}\omega\cos\omega t} \qquad (31)$$

Mathematically, since $\cos\omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$

Equation (31) becomes, $$i = CV_{C,o}\omega\cos\omega t = CV_{C,o}\omega\sin\left(\omega t + \frac{\pi}{2}\right) \qquad (32)$$

By comparing $V_C = V_{C,o}\sin\omega t$ with $i = CV_{C,o}\omega\sin\left(\omega t + \frac{\pi}{2}\right)$, the current 'through' capacitor (a cosine function) thus leads the applied potential difference (a sine function) by one quarter of a cycle or, as is often stated, by $\frac{\pi}{2}$ radians[6] or $90°$ (1 cycle being regarded as 2π radians or $360°$) as shown in Figure 10.

Let $i_o = \omega CV_{C,o}$ (33)

$$\boxed{i = i_o\cos\omega t} \qquad (34)$$

According to equation (33),

[6] Without specifically mentioned, all angle should be in terms of radians rather than degree.

$$\therefore \frac{V_{Co}}{i_o} = \frac{1}{\omega C} = \frac{1}{2\pi fC} \qquad (35)$$

The ratio of r.m.s. of potential difference to that of current equal to the ratio of peak value of potential difference to that of current.

$$\frac{V_{r.m.s.}}{i_{r.m.s.}} = \frac{V_{C,o}}{i_o} \qquad (36)$$

By substituting equation (35) into equation (36) and $\omega = 2\pi f$

$$\therefore \frac{V_{r.m.s.}}{i_{r.m.s.}} = \frac{1}{\omega C} = \frac{1}{2\pi fC} \qquad (37)$$

This expression resembles $\frac{V}{i} = R$ which defines resistance, $\frac{1}{2\pi fC}$ replacing R. The quantity $\frac{1}{2\pi fC}$ is taken as a measure of the opposition of a capacitor to a.c. and is called ***capacitive reactance*** X_C. Hence,

$$\boxed{X_C \equiv \frac{V_{r.m.s.}}{i_{r.m.s.}} = \frac{1}{2\pi fC}} \qquad (38)$$

The ohm is the SI unit of X_C if the unit of f is s^{-1} (hertz) and that of C is CV^{-1} The term $\frac{1}{fC}$ then has units $\frac{\mathrm{V}}{\mathrm{Cs}^{-1}} = \mathrm{VA}^{-1} = \Omega$. It is clear that X_C decreases as f and C increase.

Reactance is not to be confused with resistance; in the latter electrical power is dissipated, whereas it is not in a reactance.

<u>Inductor</u>

(a) Definition

An *inductor*[7] is a device storing energy in magnetic field, for example coils or solenoids (i.e. a series of coils). The inductance, denoted by L, is the ability that an inductor to store energy in a magnetic field. In particular, the term "self-inductance" is used to describe the behavior of generating an opposing electromotive force (or called back e.m.f.) proportional to the rate of change in current in a circuit. It is Faraday's Law[8], i.e. $\zeta_{back} \propto -\frac{di}{dt}$ (39) or $\zeta_{\text{back}} = -L\frac{di}{dt}$ (40)

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[7] Symbol of an inductor/ a solenoid could be with ferromagnetic materials () or without ferromagnetic materials ().

[8] $\zeta_{\text{back}} = -\frac{dN\Phi}{dt} = -NA\frac{dB}{dt} = -NA\frac{d}{dt}\left(-\mu\frac{N}{l}i\right)$ where $\Phi = BA$, $B = \mu\frac{N}{l}i$, N is the number of turns on coils in the solenoid, l is the length of the solenoid, A is the cross-sectional area of solenoid and i is the current passing through of the solenoid

$\Rightarrow \zeta_{\text{back}} = -\frac{\mu N^2 A}{l}\frac{di}{dt}$ and comparing $\zeta_{\text{back}} = -L\frac{di}{dt}$ $\qquad \therefore L = \mu\frac{N^2 A}{l}$

For *an infinitely long solenoid*, the inductance is

$$\boxed{L = \mu \frac{N^2 A}{l}} \quad (41)$$

, where μ is the permeability of the material inside the solenoid, N is the number of turns of coils in the solenoid, A is the cross-section area of the coil, l is the length of the coil.

The SI unit of inductance L is in henrys.

(e) Phase relationships

An *inductor* in an a.c. circuit as shown in Figure 13 behaves like a capacitor in that it causes a phase difference between the applied potential difference V_L and the current i. In this case, however, the current i lags on the potential difference V_L by one-quarter of a cycle (i.e. $90°$) as shown in Figure 12.

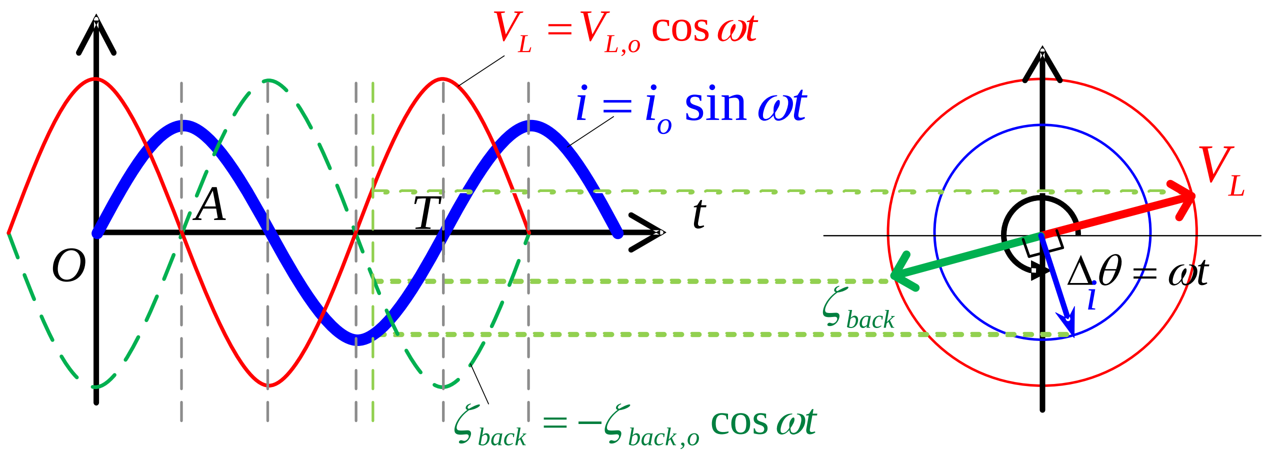

Figure 12: Phase difference between V_L and i

In Figure 12, at O, the current i is zero but its rate of increase is a maximum (as given by the slope of the tangent to the current graph at O or $\frac{di}{dt} = \max$) which means, for an inductor of constant inductance L, that the rate of change flux is also a maximum ($\frac{d\Phi}{dt} = A\frac{dB}{dt} = \max$)[9]. Therefore by Faraday's law the back e.m.f. (or called induced e.m.f.)is a maximum ($\zeta_{induced} = \max$) but, by Lenz's law, of negative sign since it acts to oppose the current change.

At A the current i and flux Φ are momentarily a maximum and constant. Their rate of change is zero (slope of tangent to current graph is zero at A or $\frac{di}{dt} = 0$ and $\frac{d\Phi}{dt} = A\frac{dB}{dt} = \max$) and so the back e.m.f. is zero ($\zeta_{induced} = 0$). If the inductor has negligible resistance, then at every instant the applied potential difference V_L must be nearly equal and opposite to the back e.m.f. (i.e. $|V_L| = |\zeta_{induced}|$). The potential difference V_L acts on the coil whilst the e.m.f. acts back upon the source, just like two forces acting on different bodies.

---

[9] For a long solenoid, $B = \mu \frac{N}{l} i \Rightarrow \frac{dB}{dt} \propto \frac{di}{dt}$

(f) Mathematical treatment

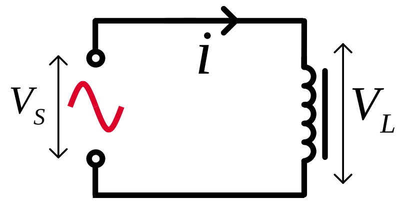

Figure 13: Schematic diagram of pure inductance circuit

In this case, it is simpler to start with a pure inductance current as shown in Figure 13. Consider an inductor with inductance L through which current i flows at time t where

$$\boxed{i = i_o \sin \omega t} \qquad (42)$$

where $i_{L,o}$ is its peak value and $\omega = 2\pi f$ where f is the frequency of the supply.

The back e.m.f. (or called induced e.m.f.) in the inductor due to changing current is

$$\boxed{\zeta_{back} = -L\frac{di}{dt}} \qquad (43)$$

Substituting equation (42) into equation (43)

$$\Rightarrow \zeta_{back} = -L\frac{d}{dt}\left(i_o \sin \omega t\right) \qquad (44)$$

$\because i_o$ is a constant and independent of t $\quad \Rightarrow \zeta_{back} = -Li_o \frac{d}{dt}\left(\sin \omega t\right) \qquad (45)$

$\because \frac{d}{dt}\left(\sin \alpha t\right) = \frac{d\left(\alpha t\right)}{dt}\frac{d}{d\left(\alpha t\right)}\left(\sin \alpha t\right) = \alpha \cos \alpha t$ and $\frac{d\left(\alpha t\right)}{dt} = \alpha$ where α is a constant and independent of t where α is a constant and independent of t

$$\Rightarrow \boxed{\zeta_{back} = -\omega L i_o \cos \omega t} \qquad (46)$$

Assuming the inductor has zero resistance , then for current to flow the applied potential difference must be equal and opposite to the back e.m.f., hence

$$V_L = -\zeta_{back} \qquad (47)$$

Substituting equation (46) into equation (47)

$$\therefore V_L = \omega L i_o \cos \omega t \qquad (48)$$

The applied potential difference is given by

$$\boxed{V_L = V_{L,o} \cos \omega t} \qquad (49)$$

where $\omega = 2\pi f$ where f is the frequency of the supply and $V_{L,o}$ is its peak value. According to equation (48) is given by

$$V_{L,o} = \omega L i_o \qquad (50)$$

The ratio of r.m.s. of potential difference to that of current equal to the ratio of peak value of potential difference to that of current.

i.e. $$\because \frac{V_{r.m.s.}}{i_{r.m.s.}} = \frac{V_{L,o}}{i_o} \qquad (51)$$

Substituting equation (50) into equation (51) and $\omega = 2\pi f$

$$\therefore \frac{V_{r.m.s.}}{i_{r.m.s.}} = \frac{V_{L,o}}{i_o} = \frac{i_o \omega L}{i_o} = \omega L = 2\pi fL \qquad (52)$$

This expression resembles $\frac{V}{i} = R$ which defines resistance, $2\pi fL$ replacing R. The quantity $2\pi fL$ is taken as a measure of the opposition of an inductor to a.c. and is called ***inductive reactance*** X_L. Hence,

$$\boxed{X_L \equiv \frac{V_{r.m.s.}}{i_{r.m.s.}} = 2\pi fL} \qquad (53)$$

<u>Phasor diagrams for pure resistance, pure capacitance and pure inductance circuit</u>

The vector diagram for a pure resistance in an a.c. circuit. The current i is in phase with applied potential difference V_R (i.e. phase difference = $0°$ or $360°$) as shown in Figure 14.

The vector diagram for a pure capacitance (i.e.. infinite dielectric resistance) in an a.c. circuit. The current i leads the applied potential difference V_C by $90°$.(i.e. phase difference = $90°$) as shown in Figure 15.

For a pure inductance (i.e. zero resistance), in this case the current i lags on the applied potential difference V_L by $90°$.(i.e. phase difference = $90°$) as shown in Figure 16.

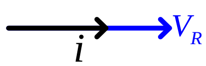

Figure 14: Phasor diagram of V_R and i

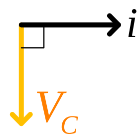

Figure 15: Phasor diagram of V_C and i

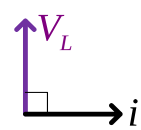

Figure 16: Phasor diagram of V_L and i

<u>RC series A.C. circuit</u>

Suppose an alternating potential difference V_C is applied across a resistance R and a capacitance C in *series* as shown in Figure 17. Because of in series arrangement. the same current i flows through each component and so the reference vector will be that representing i. The potential difference V_R across R is in phase with i, and V_C, that across C, lags on i by $90°$ (or $\frac{\pi}{2}$ radians) The vector diagram is as shown in Figure 18.

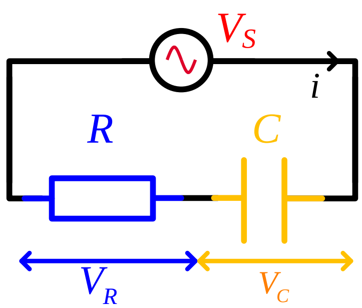

Figure 17: Schematic of RC series circuit.

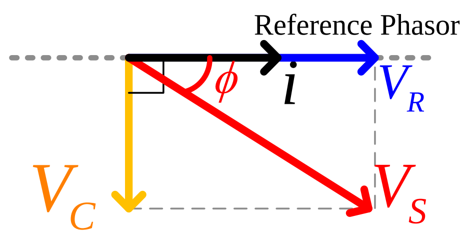

Figure 18: Phasor diagram of RC series circuit

The vector sum of V_R and V_C equals the applied potential difference V_S hence

$$V_S^2 = V_R^2 + V_C^2 \quad (54)$$

But $V_R = iR$ and $V_C = iX_C$ where X_C is the capacitive reactance of C and equals $\frac{1}{\omega C}$, hence

$$V_S^2 = i^2\left(R^2 + X_C^2\right) \qquad (55)$$

$$V_S = i\sqrt{\left(R^2 + X_C^2\right)} \qquad (56)$$

The quantity $\sqrt{\left(R^2 + X_C^2\right)}$ is called the impedance Z of the circuit and measures its opposition to a.c. It has resistive and reactive components and like both is measured in ohms. Hence

$$\boxed{Z \equiv \frac{V_S}{i} = \sqrt{\left(R^2 + X_C^2\right)}} \qquad (57)$$

Also, from the vector diagram we see that the current i leads by V_S a phase angle ϕ which is less than $90°$ (or less than $\frac{\pi}{2}$ radians) and is given by $\boxed{\tan\phi = \frac{V_C}{V_R} = \frac{iX_C}{iR} = \frac{X_C}{R}}$ (58)

LR series A.C. circuit

The analysis in Figure 19 is similar as RC series A.C. circuit but in this case the potential difference V_L across L leads on the current i and the potential difference V_L across L is again in phase with i. As before the applied potential difference V_S equals the vector sum of V_C and V_R, and so

$$V_S^2 = V_L^2 + V_R^2 \qquad (59)$$

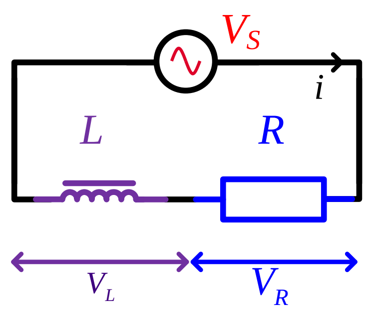

Figure 19: Schematic of LR series circuit.

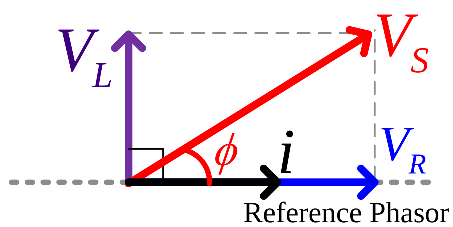

Figure 20: Phasor diagram of LR series circuit

But $V_R = iR$ and $V_L = iX_L$ where X_L is the reactance of L and equals ωL, hence

$$V_S^2 = i^2\left(R^2 + X_L^2\right) \qquad (60)$$

$$V_S = i\sqrt{\left(R^2 + X_L^2\right)} \qquad (61)$$

Hence the impedance Z is given by

$$\boxed{Z \equiv \frac{V_S}{i} = \sqrt{\left(R^2 + X_L^2\right)}} \qquad (62)$$

Also, from the vector diagram we see that the current i lags on V_S by a phase angle ϕ which is less than $90°$ (or less than $\frac{\pi}{2}$ radians) and is given by $\boxed{\tan\phi = \frac{V_L}{V_R} = \frac{iX_L}{iR} = \frac{X_L}{R}}$ (63)

LRC series a.c. circuit

A circuit as shown in Figure 21 consists of an inductor L, a resistor R and a capacitor C is called an LRC circuit. It can be connected in series or in parallel. In this experiment, only the series LRC circuit is focused. In any series circuit, all circuit elements share the same current at any point in the LRC circuit. i.e.

$$i_S(t) = i_L(t) = i_R(t) = i_C(t) \qquad (64)$$

The circuit forms a simple harmonic oscillator for current and resonance occurs under specific conditions.

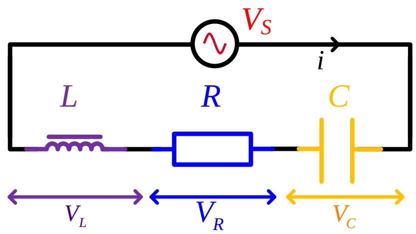

Figure 21: Schematic of LRC series circuit.

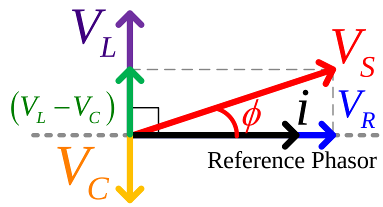

Figure 22: Phasor diagram of LRC series circuit

According to the analysis in LR, RC series circuits, V_L leads the current (reference) vector i by $90°$, V_C lags on it by $90°$, and V_R is in phase with it. V_L and V_C are therefore $180°$ (half a cycle) out of phase, i.e. in antiphase or out of phase.

If V_L is greater than V_C, or in other words, X_L is greater than X_C, their result $(V_L - V_C)$ is in the direction of V_L. The vector sum of $(V_L - V_C)$ and V_R equals the applied potential difference V_S, therefore

$$V_S^2 = V_R^2 + (V_L - V_C)^2 \qquad (65)$$

But $V_R = iR$, $V_C = iX_C$ and $V_L = iX_L$ where X_C is the reactance of C and equals $\frac{1}{\omega C}$ and X_L is the reactance of L and equals ωL, hence

$$V_S^2 = i^2\left(R^2 + (X_L - X_C)^2\right) \qquad (66)$$

$$V_S = i\sqrt{\left(R^2 + (X_L - X_C)^2\right)} \qquad (67) \qquad \text{if } X_L > X_C$$

Hence the total impedance Z_{Total} is given by

$$\boxed{Z_{Total} \equiv \frac{V_S}{i} = \sqrt{\left(R^2 + (X_L - X_C)^2\right)}} \qquad (68) \qquad \text{if } X_L > X_C$$

or

$$\boxed{Z_{Total} = \sqrt{\left(R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2\right)}} \qquad (69)$$

Also, from the vector diagram we see that the current i lags on V_S by a phase angle ϕ which is less than $90°$ (or less than $\frac{\pi}{2}$) radians and is given by

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{i\left(X_L - X_C\right)}{iR} = \frac{X_L - X_C}{R} \qquad (70) \qquad \text{if } X_L > X_C$$

$$\phi = \tan^{-1}\left(\frac{2\pi fL - \frac{1}{2\pi fC}}{R}\right) \qquad (71)$$

<u>Electrical resonance – Series resonance</u>

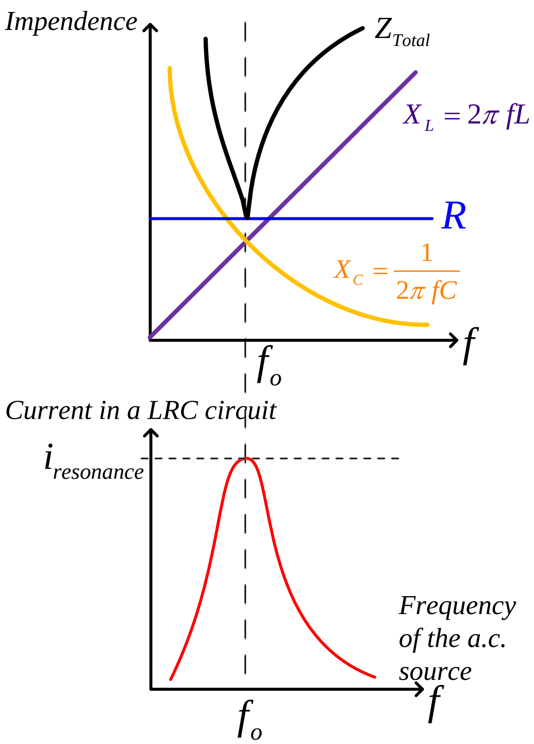

Figure 23: Figures of impendence vs. frequency and current resonance

LRC Circuit series resonance conditions:

$X_L = X_C$ $\qquad \phi = 0$

$Z_{Total} = R$ $\qquad f_o = \frac{1}{2\pi\sqrt{LC}}$

The equation (68) just derived for the total impedance Z_{Total} on a LRC series circuit shows that Z varies with the frequency f of the applied potential difference since X_L and X_C both depend on f .The relationships are shown in upper panel of Figure 23.

X_L and increases with f , ($\because X_L = 2\pi fL$) X_C and decreases with f ($\because X_C = \frac{1}{2\pi fC}$), R is assumed to be independent of f (but it can vary).

At a certain frequency, called the ***resonant frequency*** f_o when $X_L = X_C$ and Z_{Total} has its minimum value, being equal to R ($\because Z_{Total} = \sqrt{\left(R^2 + \left(X_L - X_C\right)^2\right)}$). The circuit behaves as pure resistance since the capacitive reactance and inductive reactance cancel each other and the current i has a maximum value (given by $i = \frac{V_R}{R}$). The phase angle ϕ (given by $\tan\phi = \frac{(X_L - X_C)}{R}$) is zero, the applied potential difference V_S and the current i are in phase and there is as said to be resonance.

According to equation(68), f_o is obtained from $Z_{Total} = R$, that is

$$X_L = X_C \qquad (72)$$

By substituting equations (38) and (53) into equation(72),

$$2\pi f_o L = \frac{1}{2\pi f_o C} \qquad (73) \qquad \text{or} \qquad 4\pi^2 f_o^2 LC = 1 \qquad (74)$$

$$\boxed{f_o = \frac{1}{2\pi\sqrt{LC}}} \qquad (75)$$

If L is in henrys and C in farads, f_o will be in hertz.

At ***resonant frequency*** f_o, the physical significant is that the energy transfer to LRC circuit from the a.c. source is maximum.

Experiment 1: Finding the resonant frequency through curves fitting

Setup Procedures

1. Set up the PASCO 850 Universal Interface and the computer.
2. Connect banana plug patch cords into the 'OUTPUTS' port on the PASCO 850 Universal Interface.

Figure 24. Connecting power source of the Interface to the Circuit board as LRC series circuit

3. Connect the following Voltage Sensors into the interface.

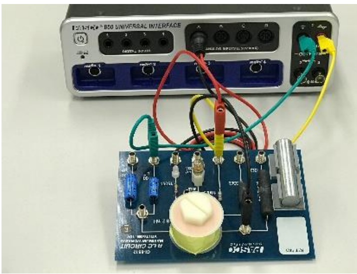

Figure 25. Voltage Sensor in Channel A for measuring p.d. across resistor, V_R

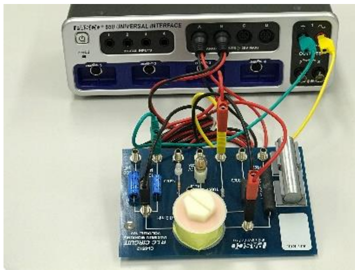

Figure 26. Voltage Sensor in Channel B for measuring p.d. across inductor V_L

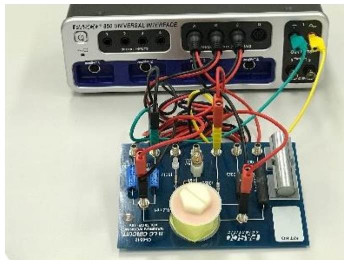

Figure 27. Voltage Sensor in Channel C for measuring p.d. across capacitor, V_C

4. Ensure the circuit is connected to the $R = 100\ \Omega$ resistor, $C = 100\ \mu\text{F}$ capacitor, $L = 8.2$ mH inductor in series and three voltage sensors with each electronic components

Experimental Procedure

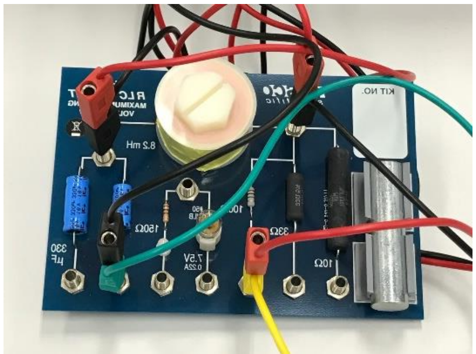

Figure 28. Configuration of the LRC circuit

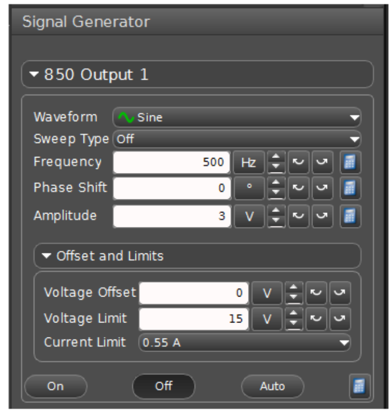

Figure 29(a) Output frequency Panel of signal generator

1. Open the Capstone® working file 'EXP05_UID.cap. Click 'Save Experiment As' function to save your own file which UID should be your/your partner University no.(e.g. EXP5_3333123456.cap)
2. Select Page 1 and then change the Signal Generator to 'On' mode as shown in Fig.29 (a) before starting measurement.
3. *Set* the Amplitude to 3.000 V and entry corresponding frequency as shown in the worksheet (Table 1.1).
4. *Click* 'Record' to start time duration measurement and wait until the end.
5. *Watch* the Graphs of potential difference across inductor V_L , resistor V_R and capacitor V_C versus time respectively. All curves are should be a sine function.
6. Highlight the portion of the data points starting from 0 second.
7. *Click* the Fit tool  and fit all curves as "sine fit"
8. Check whether the computed estimation curve is matching with your data point performance.
9. Repeat the procedure (8) to record the Amplitude and Phase of sine fitting curve of voltage across capacitor, resistor and inductor respectively in the worksheet (Table 1.1).

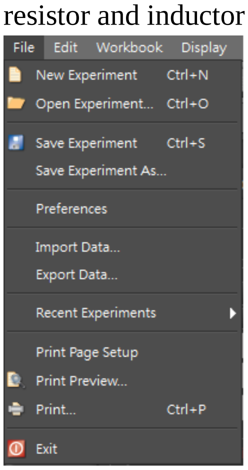

Figure 30 Save Activity As …

10. Click "snapshot " to record the picture and name the photo (i.e. 10Hz)

11. *Click* “Delete Last Run” and then select “Delete All Runs” to delete all previous recorded data.

Figure 31 Delete All/Last Data Runs Panel

12. Repeat the procedure from (2) to (11) with changing the frequency to complete all data record in the worksheet (Table 1.1).
13. Click “Save Experiment as” to save your group file.

Experiment 2: Data Analysis of experiment 1

<u>Experimental Procedure</u>

1. Click “Page 2” to activate the start the graph plotting work.
2. In the windows, two tables are shown.(i.e. Table of the potential difference across the inductor V_L versus frequency and the potential difference across the capacitor V_C versus frequency)

▲ Set	■ Set	▲ Set
Frequency (Hz)	VL (V)	VC (V)

Figure 32. Tables of V_L vs. frequency and V_C vs. frequency

3. Entry the Amplitude of V_L and V_C with different frequencies recorded in the **Table 1.1** of worksheet .
4. After entry all data, two set of data points will show in the graph paper accordingly. *Fit* the data points of p.d. across inductor by “Fit Tool” and *choose* “linear fit”; and the data points of p.d. across capacitor by “Fit Tool” and *choose* “inverse fit” as shown in figure 33.
5. Two best fit curves are shown and they intersect at a point. Zoom in the graph to find the detail of the intersect point.
6. Use the “Coordinate Tool” to read the x and y coordinates of the intersection point (i.e. (x, y)) as shown in Figure 34.
7. Click “Save Experiment As…” to save your group file.
8. Complete the Table 2.1 to Table 2.3 in the worksheet.

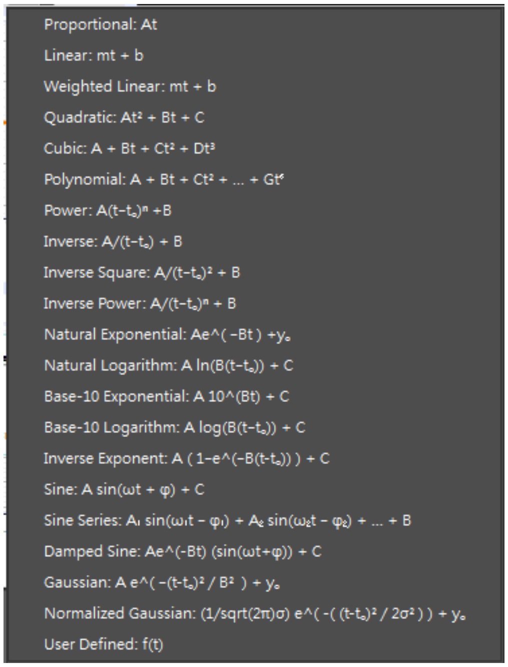

Figure 33. Fitting configuration

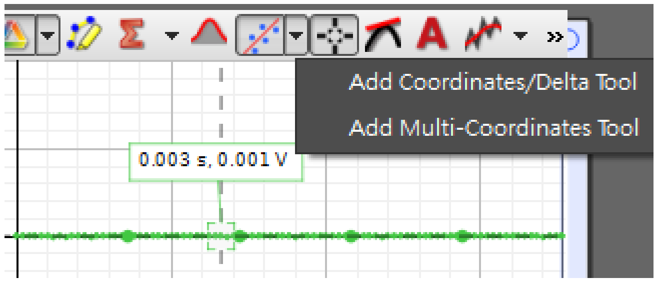

Figure 34. Coordinate Tool

Experiment 3: Knowing the properties of the LRC circuit at resonant frequency

<u>Experimental Procedure</u>

1. Repeat the procedure illustrated in Experiment 1 by using the resonant frequency from **Table 2.3**, and then record all properties of inductor, resistor and capacitor under resonant frequency in the **Table 3.1.**

References:

Capacitor:

1. Chapter 26, Physics for Scientists and Engineers with Modern Physics 8th Edition, John Jewett and Raymond Serway

Inductor:

2. Chapter 32, Physics for Scientists and Engineers with Modern Physics 8th Edition, John Jewett and Raymond Serway

Alternating Circuit and LRC circuit:

3. Chapter 33, Physics for Scientists and Engineers with Modern Physics 8th Edition, John Jewett and Raymond Serway
4. http://hyperphysics.phy-astr.gsu.edu/hbase/electric/rlcser.html

Inductance of a solenoid:

5. http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/indcur.html#c2