THE UNIVERSITY OF HONG KONG

Department of Physics

PHYS2250 Introductory mechanics

Laboratory manual 2250-2: Projectile and ballistic pendulum

Projectile motion, along with wedge problems and simple pendulum, has always been a popular topic to be introduced in introductory mechanics. Its pedagogical advantage is hard to be overstated when we have been accummulating so many related life experiences ranging from basketball games to the *Angry Birds*. Most often, students' first encounter with physics analysis of projectile motion took place in high schools, but only the theoretical derivation - typically includes decomposition of velocity vector and kinematic equations concerning uniformly accelerated motion - was emphasized. In this laboratory session, students will be asked to review these concepts from an experimental perspective. All formulae are derived by assuming the absence of air resistance.

1 Determining initial velocity of projectile

In most typical projectile problems, the initial velocity from a launcher is given as a known parameter. Experimentally, this information is usually missing until we make some measurements. In this particular experiment, a plastic ball will be put into the launcher shown in Figure 1. The ball then acquires its kinetic energy during the launch when the compressed spring is being released. In any case, the initial velocity will not be known until we come up with some way to measure or deduce it. We will be trying out two ways to determine the initial velocity.

Figure 1: This is a projectile launcher that can be set at any angle ranging between 0° to 90°. A plumb bob made up of a hex bolt and a string can assist you in reading off the projectile angle θ.

Task 1: Determining initial velocity by shooting off horizontally

Suppose we set up the launcher at some height h, pointing at $\theta = 0°$, then the horizontal range is given by

$$x = v_0 t \ ,$$

where t is the time of flight. Use this relation to determine v_0. When measuring t, decide if you should use a stopwatch or deduce it from the vertical height. Discuss your decision and back it up with error analysis/estimation in your report. Remember to repeat your measurements for several times until the uncertainty becomes reasonably small.

The second method is done by colliding a metal ball into a pendulum. The setup is shown in Figure 2. Since the momentum will be transfered completely into the pendulum, we may then deduce the velocity of the metal ball right after being launched.

Figure 2: After being launched, the metal ball will be trapped within the ballistic pendulum. This pendulum-ball system will then swing upward to some angle.

Naively, we would be begin by writing down the conservation of momentum. But this omits the rotational inertia of the ballistic pendulum. Instead, we should consider the conservation of angular momentum.

Right after the metal ball leaves projectile launcher, it carries an angular momentum of $L_b = m_b R_b v_0$ with respect to the pendulum pivot, where m_b is the mass of metal ball and R_b measures the distance form the pivot to the metal ball. Conservation of angular momentum then says

$$L_{cm} = L_b = m_b R_b v_0 \ ,$$

where L_{cm} is the angular momentum of the pendulum-ball system. Denote E as the total energy, then we have

$$E = \frac{L_{cm}^2}{2I} \implies v_0 = \frac{1}{m_b R_b}\sqrt{2IE} \ ,$$

where I is the moment of inertia of the pendulum-ball system. Suppose we assume that there is no energy loss *after* the ball is trapped into the pendulum, then the initial velocity right after launch is given by

$$v_0 = \frac{1}{m_b R_b}\sqrt{2I \cdot (m_b + m_p)\, g R_{cm} \left(1 - \cos\theta\right)} \ , \tag{1}$$

where m_p is the mass of pendulum (including the pivot) and R_cm measures the distance from the center of mass of the pendulum-ball system to the pivot.

It is important to realize that we are not saying the collision between metal ball and pendulum has no energy loss. Contrary to that, the collision is indeed inelastic, which means the only conserved quantity is (angular) momentum. In the literature, we refer to this mechanical setup as the *ballistic pendulum*.

Task 2: Determining initial velocity by ballistic pendulum

All parameters in Eq. (1) can be directly measured except for I, the moment of inertia of the pendulum-ball system. In the previous laboratory session (*2250-1 Rotations*), we have acquired a way to measure moment of inertia. Here, another method which better suits pendulum will be introduced and summarized as following:

$$I = \frac{MgR_{cm}}{\omega^2} ,$$

where ω is the angular frequency.

Hint: It is hard to directly measure ω. Rewrite it into some quantity that is more measurable. Notice that $\omega \neq \dot{\theta}$ - the former one describes the frequency of oscillation, while the later one is simply the angular velocity of pendulum that changes depending on position.

2 Projectile angle and horizontal range

It is easy to show that the horizontal range x of projectile can be given by

$$x = v_0 t \cos\theta ,$$

where v_0 is the initial speed of projectile, t is the time of flight and θ is the angle of inclination above horizon.

Task 3: Projectile angle and horizontal range

This task requires you to measure the horizontal ranges as functions of projectile angle θ in two scenarios:

1. Shooting on a level surface
2. Shooting off a table

For both cases, begin your measurements from $\theta = 10^\circ$ to $\theta = 80^\circ$ with 10° increment. Three trials are required for every angle θ, and the average of these three trials will be adoped for your plot. Interpolate/Best-fit your data points to determine the angle that yields the maximum horizontal range. Also, compare your data (not just the maximum) against the theoretical prediction that assumes no air resistance.

Hint: The angle that yields maximum range is not always 45°. This is because the time t contains a θ-dependency when shooting off at some height, making the solution to $dx/d\theta = 0$ becomes less trivial. You may use numerical method here.

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Discussion

A few questions have been listed below for you to ponder:

- Discuss the advantages and disadvantages between the two ways described in Task 1 and Task 2, of measuring initial velocity.
- So far we have assumed that no air resistance is present. However, how reasonable is such assumption in this particular experiment? Present your arguments with quantitative support.

Discuss them in your report under the "Discussion" section. Feel free to include more constructive comments.