

PHYS2255 Introductory electricity and magnetism
Laboratory manual 2255-2:
RC circuit

In this laboratory session, CRO is used to investigate various properties of RC circuit. Hence, it is highly recommended that students schedule this laboratory session *after laboratory 2255-1 “Operation of a CRO”*. In any case, make sure you review the operation of CRO, or study it by yourself if you have not done laboratory 2255-1 yet, before attending this laboratory session. Almost all contents in previous session will be applied.

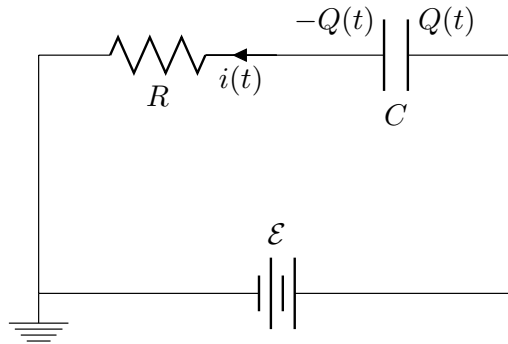


Figure 1: A simple RC circuit. Ground is introduced to define where $V = 0$.

An RC circuit is composed of a resistor R and a capacitor C , sometimes driven by a voltage source which can be either DC or AC. In Figure 1, the presence of capacitor prohibits any current from passing through, so current $i(t)$ is only significant before the capacitor being charged fully. To analyze this circuit, we begin with

$$\mathcal{E} = V_R(t) + V_C(t) = i(t)R + Q(t)/C ,$$

where $Q(t)$ is the amount of charge piled up at the capacitor as a function of time. Now, a key step is to realize

$$i(t) = \frac{dQ(t)}{dt} .$$

This leads us to a linear first-order differential equation

$$\frac{dQ(t)}{dt} + \frac{1}{RC}Q(t) = \frac{1}{R}\mathcal{E} .$$

To solve this equation, multiply both sides by a factor¹ $e^{t/RC}$. Then using the product rule, we get

$$\frac{d}{dt} \left(Q(t)e^{t/RC} \right) = \frac{1}{R}\mathcal{E}e^{t/RC} .$$

Thus,

$$Q(t) = \frac{e^{-t/RC}}{R} \int \mathcal{E}e^{t/RC} dt . \quad (1)$$

Since \mathcal{E} is time-independent, it can be taken out from the integral, resulting in

$$Q(t) = \mathcal{E}C + KCe^{t/RC} \quad \text{or} \quad V_C(t) = \mathcal{E} + Ke^{t/RC} , \quad (2)$$

where K is some constant which can be determined by initial and final conditions.

¹This factor is not some ingenious inspiration that comes out of nothing. Instead, it is a fairly standard procedure to solve differential equations of this kind, and the extra factor we introduce is called “integrating factor”. You should read more about differential equations if you have not learned them before. They will be ubiquitous in your college life as a physics student.

1 Charging and discharging of RC circuit

Consider an RC circuit in Figure 2. When being set to switch 1, the voltage difference over capacitor C builds up and gradually approaching \mathcal{E} , i.e. the RC circuit is in “charging” mode. Now, by turning to switch 2, voltage that was previously built up at capacitor C quickly dissipates, i.e. the circuit is in “discharging” mode.

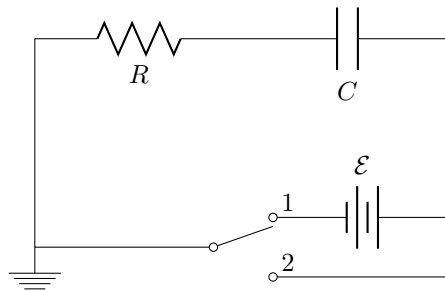


Figure 2: Switch 1 yields charging mode and switch 2 yields discharging mode.

But how fast do these modes occur? More specifically, can we describe the voltage difference V_C over the capacitor as a function of time? The solution to this question lies in Eq. (2).

For charging mode, we must have $V_C(0) = 0$ and $V_C(\infty) = \mathcal{E}$, so

$$V_C(t) = \mathcal{E} \cdot \left(1 - e^{-t/RC}\right); \tag{3}$$

for discharging mode, we expect $V_C(\infty) = 0$, hence

$$V_C(t) = V_C(0) \cdot e^{-t/RC}. \tag{4}$$

Task 1: Plot $V_C(t)$

Use your favourite software to generate plots for Eq. (3) and Eq. (4). Vary the parameters \mathcal{E} , R , C and $V_C(0)$. Describe how these variations affect $V_C(t)$. You may want to attach some graphs in your report to help illustration. Also, what would happen if we remove the resistor? It may be convenient to introduce the “time constant” $\tau \equiv RC$ in your analysis.

1.1 Setting up circuit

The charging and discharging processes usually take place in a split second, making them too fast to be observed. A smart experimental trick to overcome this issue is by replacing the entire two-way switch and voltage supply in Figure 2 into an AC generator. Since we would like to monitor the generator directly with a CRO from time to time, a *two-pole switch* is added, resulting in circuit shown in Figure 3.

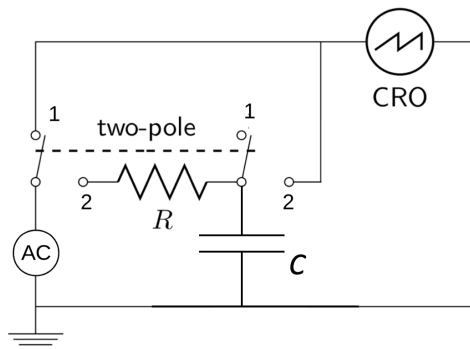


Figure 3: Switch 2 yields a similar topology to circuit in Figure 2 except for the introduction of CRO and the replacement of voltage source; switch 1 allows quick disconnection from RC circuit so that the status of generator can be inspected by the same CRO.

Task 2: Setting up circuit

Set up the circuit shown in Figure 3. We suggest to start with $R = 10\text{ k}\Omega$ and $C = 0.1\text{ }\mu\text{F}$. Remember this circuit was designed to be a better version of the one in Figure 2 in the sense of facilitating the study of charging and discharging modes. Discuss any discrepancies between these two versions of circuits. Also, decide if we should generate sine wave or square wave. Explain your reasoning in detail.

1.2 Measuring time constant τ

Our goal in this part is to measure the time constant τ based on the curves we see from CRO. As always, we should propose a theoretical prediction for our measurement, i.e. $\tau = RC$. However, it is important to realize that the resistance (impedance, indeed) involved in this circuit is not only given by the variable resistor but also the internal resistance of signal generator.

Task 3: Measuring the internal resistance of signal generator

Decide an appropriate output level for the signal generator. You may want to fix it throughout the experiment for consistency. Now, measure the internal resistance of generator using the method you have learned during laboratory 2255-1. Record all the relevant details in your report.

Having the internal resistance R_i measured, the theoretical time constant should now be estimated by using $\tau = R_{\text{eff}}C$, where

$$R_{\text{eff}} = R + R_i .$$

By setting up the circuit in Figure 3 correctly, you should be able to reproduce the following curves:

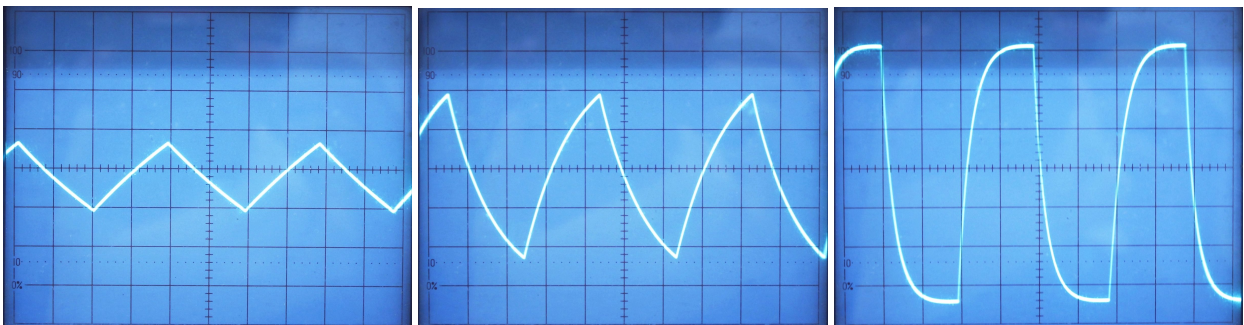


Figure 4: Expected displays from circuit in Figure 3.

Task 4: Measure time constant τ

All curves shown in Figure 4 arise from the same circuit and their apparent variations can be attained by changing the frequency of AC source, resistance R and capacitance C . Your job is to figure out the relation between these parameters and the curves. You could explore their relation by trial and error. However, a better practice would be to first give a theoretical prediction, and later verify it from the CRO. You may also want to do some circuit simulation before attending this session (see Appendix A).

Having understood the relation, you should have no problem in deducing the time constant from the displayed curves. Of course, you shall not just make a single measurement. Try different ways to obtain τ . For example, both charging and discharging curves can offer you some hints. In Figure 4, at least two pictures can provide useful information about τ . Write down all the relevant details in your report.

Reminder: When measuring the AC frequency, trust the reading from CRO instead of the one on generator. Remember you have your two-pole switch.

2 Differentiating circuit and integrating circuit

Our goal in this section is to construct a differentiating circuit and an integrating circuit as shown in Figure 5.

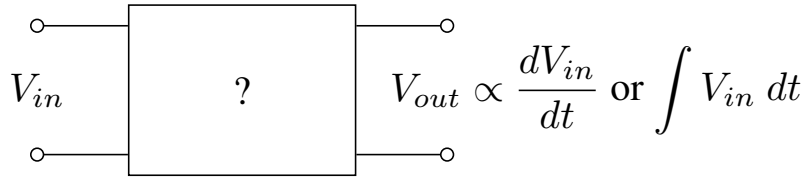


Figure 5: The V_{in} will be supplied by signal generator, while the V_{out} will be read out from CRO.

Task 5: Differentiating circuit and integrating circuit

To construct a differentiating circuit, a key step is to realize that in an RC circuit,

$$V_R(t) \propto \frac{dV_C(t)}{dt}.$$

Having proven this relation in your report, you only need to impose some conditions on R and C so that V_{in} from the generator is well approximated by V_C . A differentiating circuit can then be achieved by outputting V_R . Construction of integrating circuit is done in a similar manner, except you need to swap a few components in the circuit.

Test your proposed circuits during the laboratory session. Do you observe any expected or unexpected deviations from the theory? Describe all relevant details in your report. Whenever illustration becomes confusing by words only, draw a circuit. Presentation matters.

Hint: Task 4 might give you some ideas on how to approximate V_{in} . The conditions you impose may use the much-greater-than sign (\gg) or the much-less-than sign (\ll) on the quantity ωRC .

3 Phase shift in RC circuit

To make the phase shift more evident, we will solely be using sinusoidal waves in this section. Since the voltage source is now time-dependent, we must return to Eq. (1) and derive all the necessary expressions.

Let the voltage supply be $\mathcal{E} = \mathcal{E}_m \sin \omega t$, where $\omega \equiv 2\pi f$. From Eq. (1), using integration by parts, it can be shown that

$$\begin{aligned} Q(t) &= \frac{e^{-t/RC}}{R} \cdot \mathcal{E}_m \int e^{t/RC} \sin \omega t dt \\ &= \frac{e^{-t/RC}}{R} \cdot \mathcal{E}_m \cdot \frac{RC e^{t/RC}}{1 + (\omega RC)^2} (\sin \omega t - \omega RC \cos \omega t). \end{aligned}$$

With $V_C(t) = Q(t)/C$, we arrive at

$$\begin{aligned} V_C(t) &= \frac{\mathcal{E}_m}{1 + (\omega RC)^2} (\sin \omega t - \omega RC \cos \omega t) \\ &= \frac{\mathcal{E}_m}{\sqrt{1 + (\omega RC)^2}} \cdot \sin(\omega t - \phi), \end{aligned} \tag{5}$$

where the phase shift $\phi = \tan^{-1}(\omega RC)$ can be derived by using the identity $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$.²

²We have gone through some rather tedious derivation. You may want to derive all these results by yourself once again as a practice.

Task 6: Phase shift in RC circuit

For simplicity, we fix at $C = 0.01 \mu\text{F}$, $R = 20 \text{ k}\Omega$, and only vary the frequency f . Try to observe the phase shift ϕ from CRO. Record ϕ for several values of f and verify the relation $\phi = \tan^{-1}(\omega RC)$, or perhaps, we are only able to check the linearity

$$\tan \phi \propto \omega .$$

Also, experimentally check if the amplitude of $V_C(t)$ given in Eq. (5) is correct. Present your data using a table with relevant quantities as the columns. When graphical presentation is helpful to illustrate your data, include a plot to your report. Do not forget to apply your knowledge of error analysis.

Having investigated the phase shift of $V_C(t)$, we should be wondering the phase shift of $V_R(t)$ as well. Make a prediction based your theory. If you do not know how, perform an experiment to gather more hints. The answer is not complicated.

Hint: Since we would have to compare $\mathcal{E}(t)$ and $V_C(t)$ on CRO very often, you may want to establish a connection from the generator to CH 2 of CRO. So far we have only been using CH 1, but a CRO is usually capable of showing more than signals simultaneously. To allow multi-display, remember to set the VERT MODE to DUAL.

----- END OF LABORATORY SESSION -----

Discussion

A few questions have been listed below for you to ponder:

- We have been assuming that both resistor and capacitor provided are well calibrated. In fact, they could be the main source of error. Could you come up with some methods to check their calibration, or even better, to actually calibrate them?
- Electrical grounds have been drawn to all the circuits in the manual. But how important are they in this series of experiments?
- Since a CRO accepts dual channels, do you think a two-pole switch still has its value here?

Discuss them in your report under the “Discussion” section. Feel free to include more constructive comments.

Suggested link

- Desmos - online graphing calculator: <https://www.desmos.com/calculator>

Appendix A NL5 Circuit Simulator

In modern days, one of the important research skills for physicists is doing simulation. With a personal computer and the internet, performing simulations for many small to mid-scale circuits have become rather uncomplicated. In this appendix, we will be introducing a circuit simulator called “NL5”, which can be downloaded from

<http://nl5.sidelinesoft.com/>

for free. To run a simulation, it is sufficient to download just the .exe file, even though most likely as a user, you would also want to consider checking out its official manual.

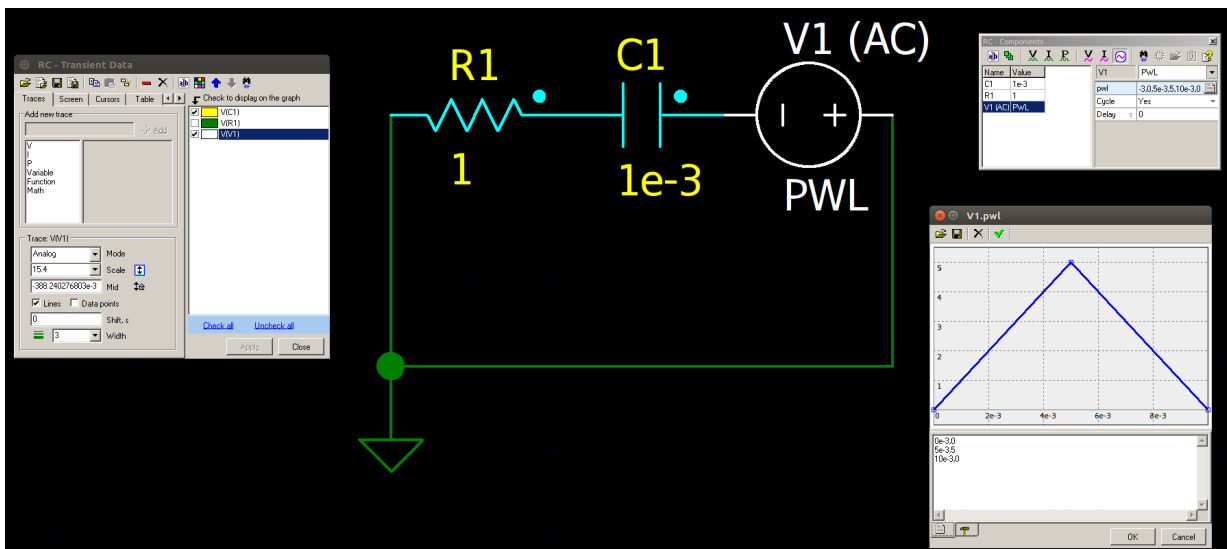


Figure 6: This is a screenshot of the NL5. NL5 allows users to specify a cycle of AC voltage supply by selecting PWL (piece-wise linear), e.g. a triangular wave can then be generated by providing three pairs of coordinates on plane (bottom right window). A ground always has to be defined before running a simulation on NL5.