Fractionalization of charge

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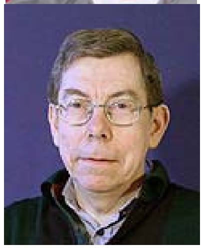


Jon Magne Leinaas

Joel Moore

Outline:

- Fractional quantum Hall effect and polyacetylene.
- Non-abelian quantum Hall state.
- Featureless Mott insulators.

Lee DH, Leinaas JM, PRL 92. 096401 (2004)

Seidel A, Fu H, Lee DH, Leinaas J.M., Moore J.E. PRL 95, 266405,(2005).

Seidel A, Lee DH, PRL 97, 056804 (2006).

Related works

E. H. Rezayi and F. D. M. Haldane, Phys. Rev. B, 50 (1994).

Bergholtz EJ, Karlhede A, PRL (2005)

Bergholtz EJ, Karlhede A, J. STAT. MECH. (2006)

Bergholtz EJ, Kailasvuori J, Wikberg E, T.H. Hansson. PRB 74, 081308 (2006).

N. Read, PRB (2006).

Fractional quantum Hall state and polyacetylene

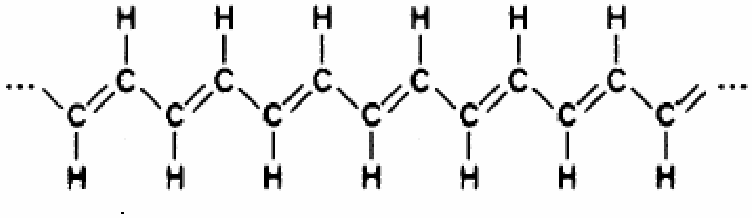
Charge fractionalization

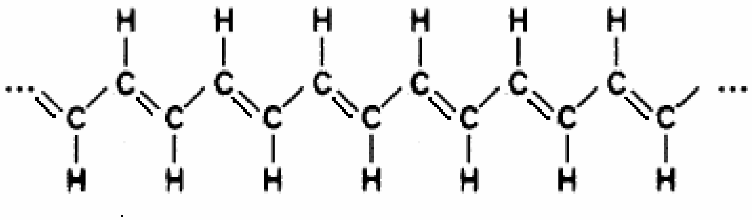
- Solitons in 1D charge density wave
- Quasiparticles in fractional quantum Hall effect

Domain walls in Polyacetylene

R. Jackiw and C. Rebbi, Phys. Rev. D **13**,3398 (1976).

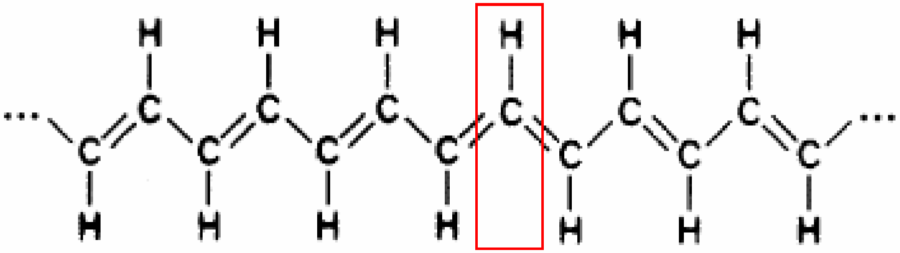
W. P. Su, J. R. Schrieffer, A. J. Heeger, Phys. Rev. B 22, 2099 (1980)





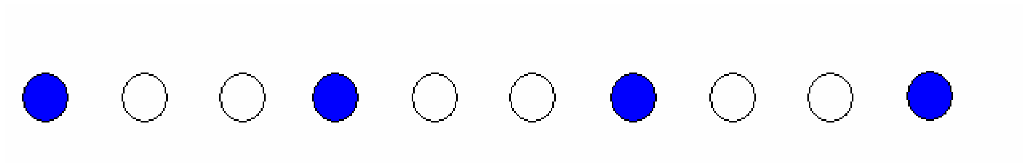
two degenerate ground states

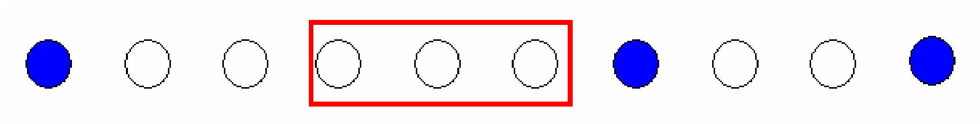
Domain wall



The Su-Schrieffer counting argument

Fractional charged domain wall in 1D charge density wave

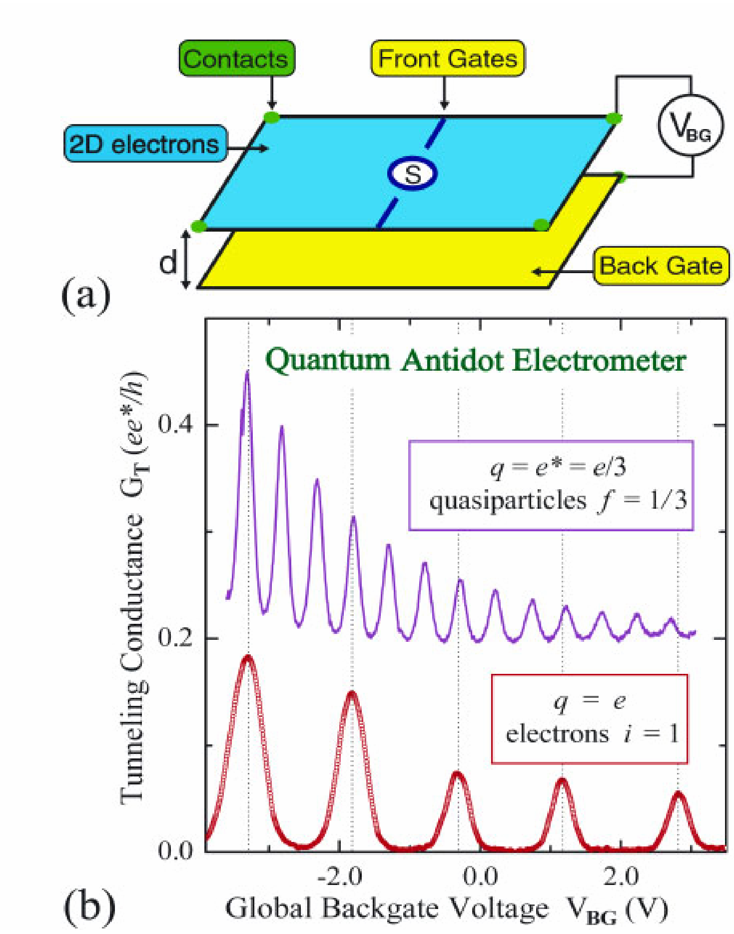




3 domain walls cause $\Delta Q = +e$ → one domain wall causes $\Delta Q = +e/3$.

Fractional-charged quasiparticle in fractional quantum Hall liquid

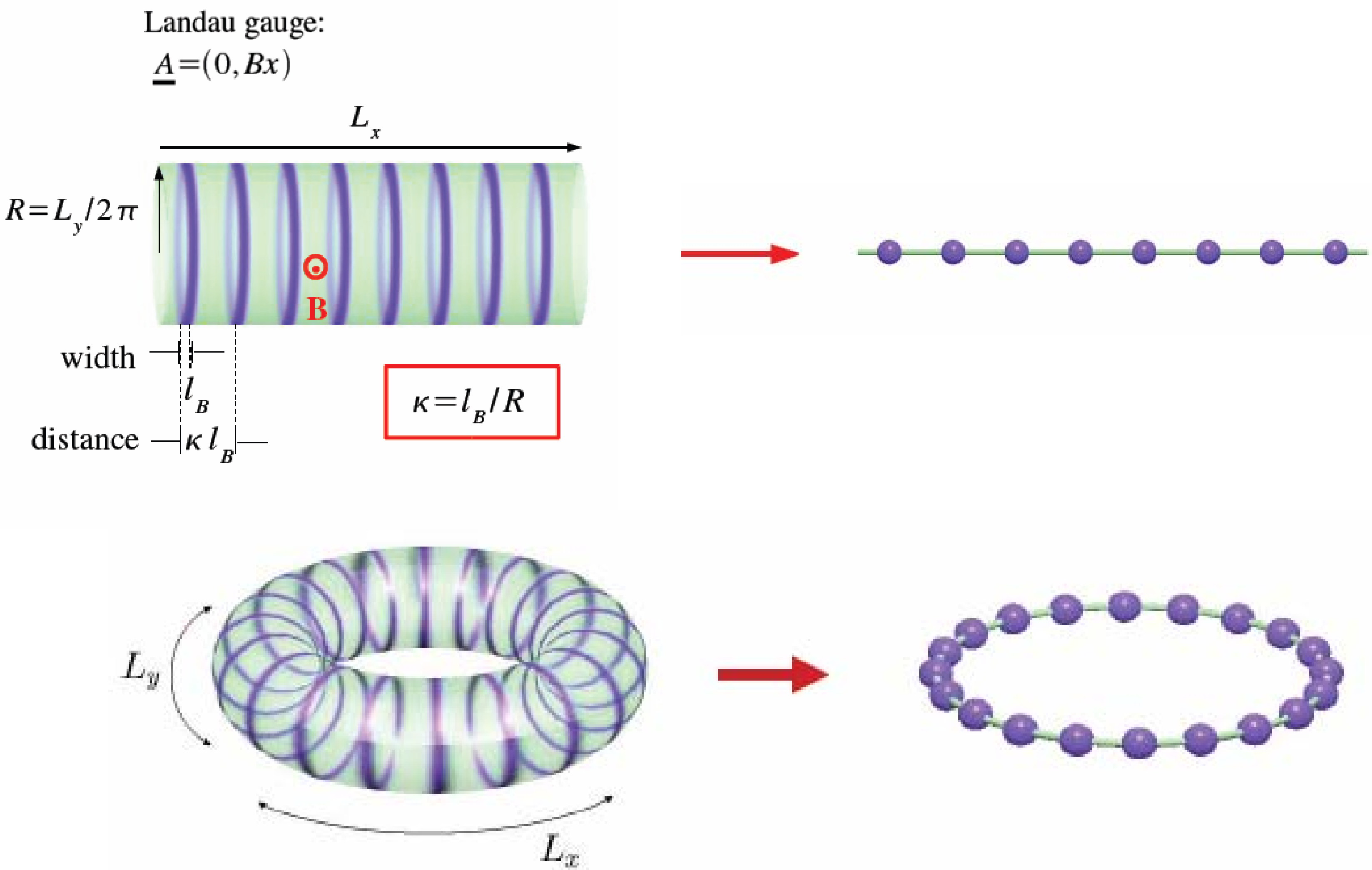
R.B. Laughlin, Phys. Rev. Lett. (1983)



Goldman Science (2000)

A connection between charge fractionalization in polyacetylene and fractional quantum Hall effect

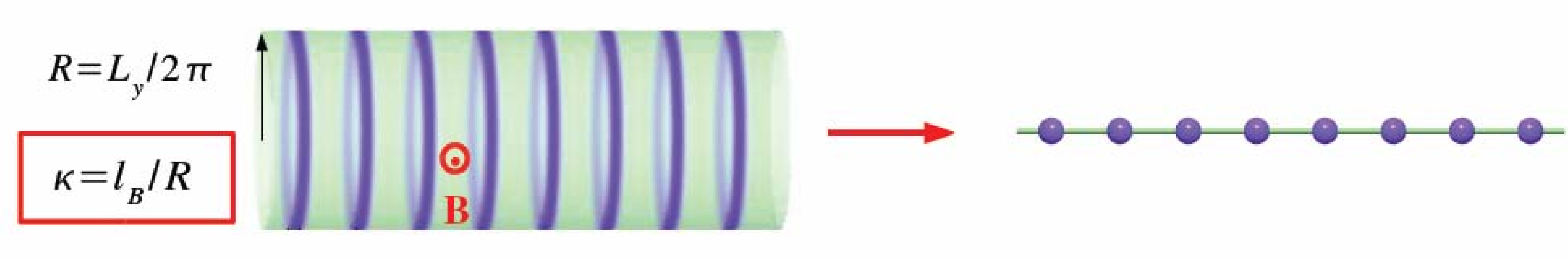
The Lowest Landau level and 1D lattice



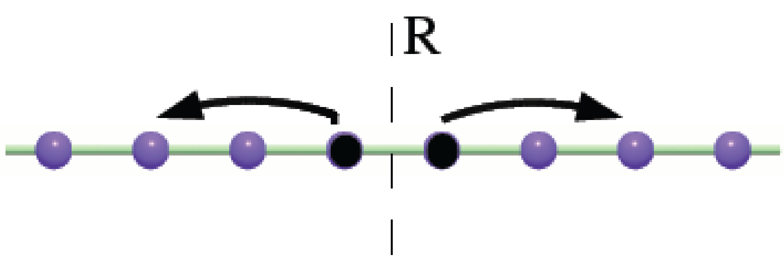
# of Dirac flux quanta → # of sites

$$H = \int d^2r d^2r' \nabla^2 \delta(\mathbf{r} - \mathbf{r}') \psi^+(\mathbf{r}) \psi^+(\mathbf{r}') \psi(\mathbf{r}') \psi(\mathbf{r})$$

Trugman & Kivelson (1985)



$$H = \kappa^3 \sum_{R,x,y} \left(x e^{-\kappa^2 x^2}\right)\left(y e^{-\kappa^2 y^2}\right) C^+_{R+x} C^+_{R-x} C_{R-y} C_{R+y}$$

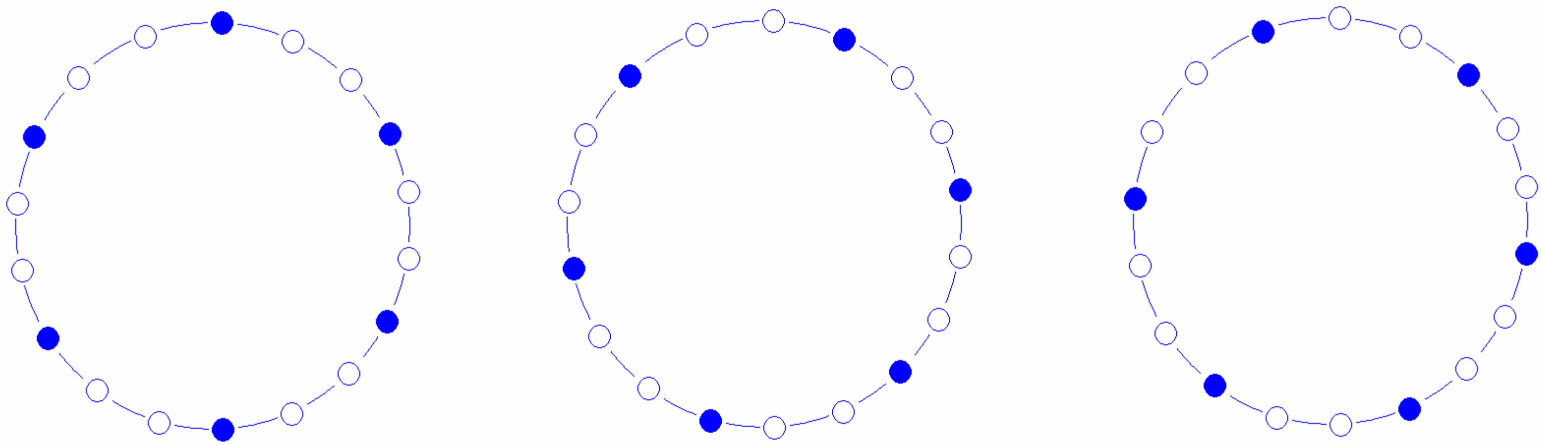


Simultaneously conserves C.M. momentum and position !

$$\boldsymbol{f(x)} = x e^{-\kappa^2 x^2}$$

$$H = \kappa^3 \sum_{R,x,y} \left(x e^{-\kappa^2 x^2} \right) \left(y e^{-\kappa^2 y^2} \right) C^{+}_{R+x} C^{+}_{R-x} C_{R-y} C_{R+y}$$

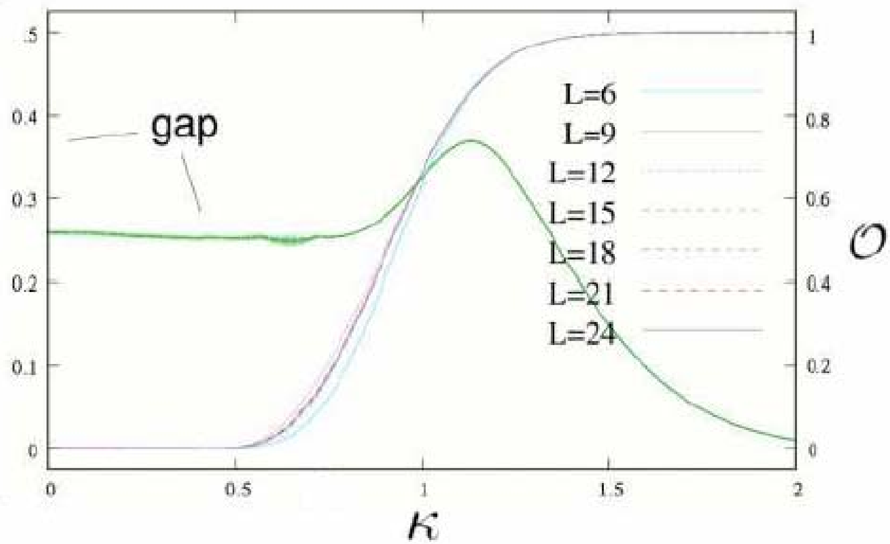
$$\xrightarrow{\kappa >> 1} \quad H = \sum_{i=1}^{L} \left[f(1/2)^2 \; C^{+}_{i+1} C_{i+1} C^{+}_{i} C_i + f(1)^2 \; C^{+}_{i+2} C_{i+2} C^{+}_{i} C_i \right]$$



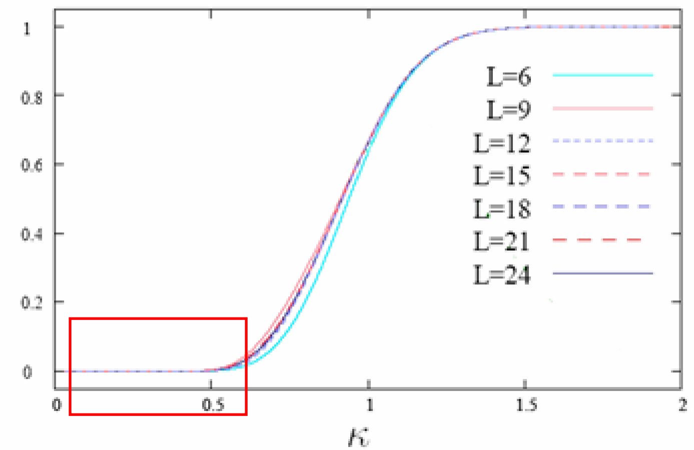
Three degenerate ground states

What happens as we reduce κ ?

Will there be a phase transition?



$$\mathcal{O} = \frac{1}{N}\sum_{j=1}^{L} e^{i\frac{2\pi}{3}j} < C_j^{+} C_j >$$



Seidel *et al*, PRL (2005)

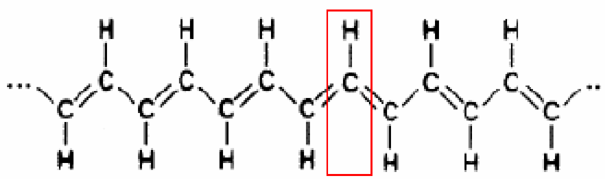
$\mathrm{Exp}(-0.6/\kappa^2)$

Small and large κ limit are adiabatically connected !!!

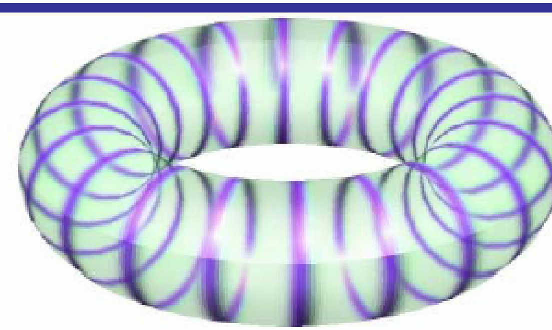
Topological degeneracy ⟷ symmetry breaking degeneracy

Niu & Wen (1990)

Laughlin quasiparticle ⟷ Domain wall



Fractional charged domain wall in polyacetylene



Fractional charged quasiparticles in quantum Hall liquid

Unified

Seidel et al (2005)

Besides the Chern-Simons theory, the 2D → 1D mapping offers another simple way to understand the fractional quantum Hall state.

Non-abelian quantum Hall states

Non-abelian quantum Hall state

Pfaffian facts

ground state wavefunction (disc geometry):

$$\Psi = Pf\left[\frac{1}{z_i - z_j}\right] \prod_{(ij)} (z_i - z_j)^q \exp[-\sum_k |z_k|^2/4]$$

filling factor: $\nu=1/q$

Moore and Read, Nucl. Phys. B (1991).

Quasi particle charge: $Q=1/(2q)$

degeneracy of 2n quasi-hole state: 2^{n-1}

ground state degeneracy on torus: 3q

Now, specialize to $\nu=q=1$ (bosons):

pseudo potential Hamiltonian: $$H = \sum_{(ijk)} \delta(z_i - z_j)\delta(z_i - z_k)$$

lattice version: $$H = \sum_R Q_R^+ Q_R$$

$$Q_R = \sum_{m+n+p=3R \bmod N} f(R-m, R-n, R-p)\, c_m c_n c_p$$

Seidel and Lee, PRL 2006
Bergholtz et al, PRB 2006.

The thin torus limit for ν=1 Pfaffian

Again, take $\kappa \to \infty$ limit :

$$H \approx \kappa^2 \sum_n [(c_n^\dagger)^3 (c_n)^3 + \exp(-\kappa^2/3)(c_n^\dagger)^2 c_{n\pm 1}^\dagger (c_n)^2 c_{n\pm 1}]$$

=> No more than two particles may occupy two adjacent sites!

$$H \xrightarrow{\kappa >> 1} \kappa^2 \sum_n [(c_n^\dagger)^3 (c_n)^3 + \exp(-\kappa^2/3)(c_n^\dagger)^2 c_{n\pm1}^\dagger (c_n)^2 c_{n\pm1}]$$

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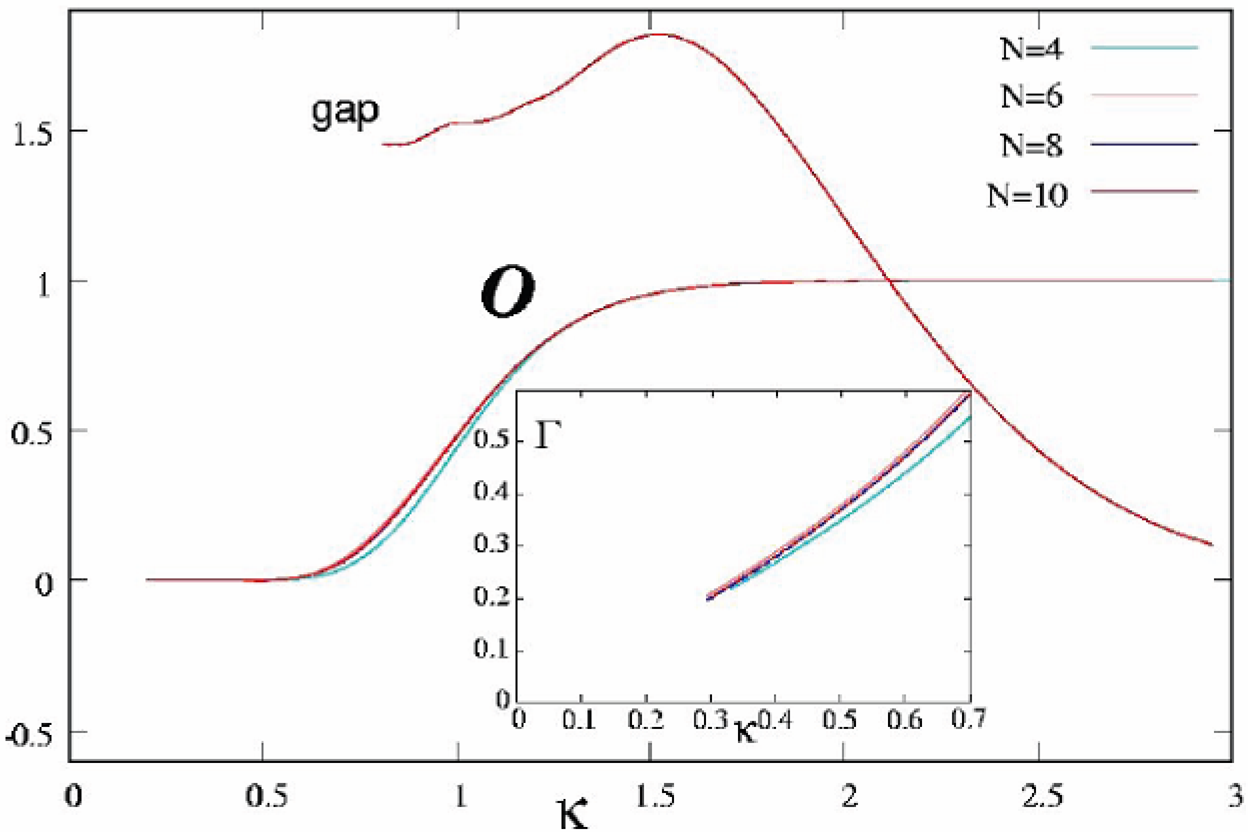
Bergholtz *et al* PRB (2006)

Seidel and Lee, PRL (2006)



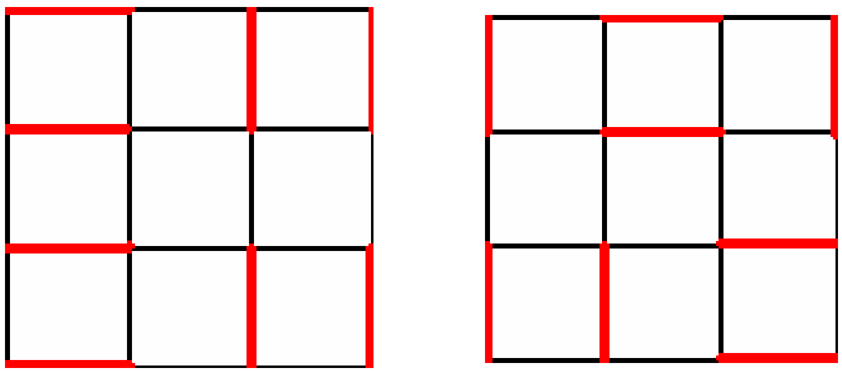
$$O = \exp\left(-1/\Gamma(\kappa)^2\right)$$

Featureless Mott insulators

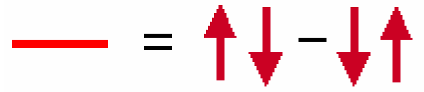
Anderson's spin liquid proposal: the parent compound of high T_c conductor is a "*spin liquid*".

Anderson, Science 1987

Resonating singlet patterns

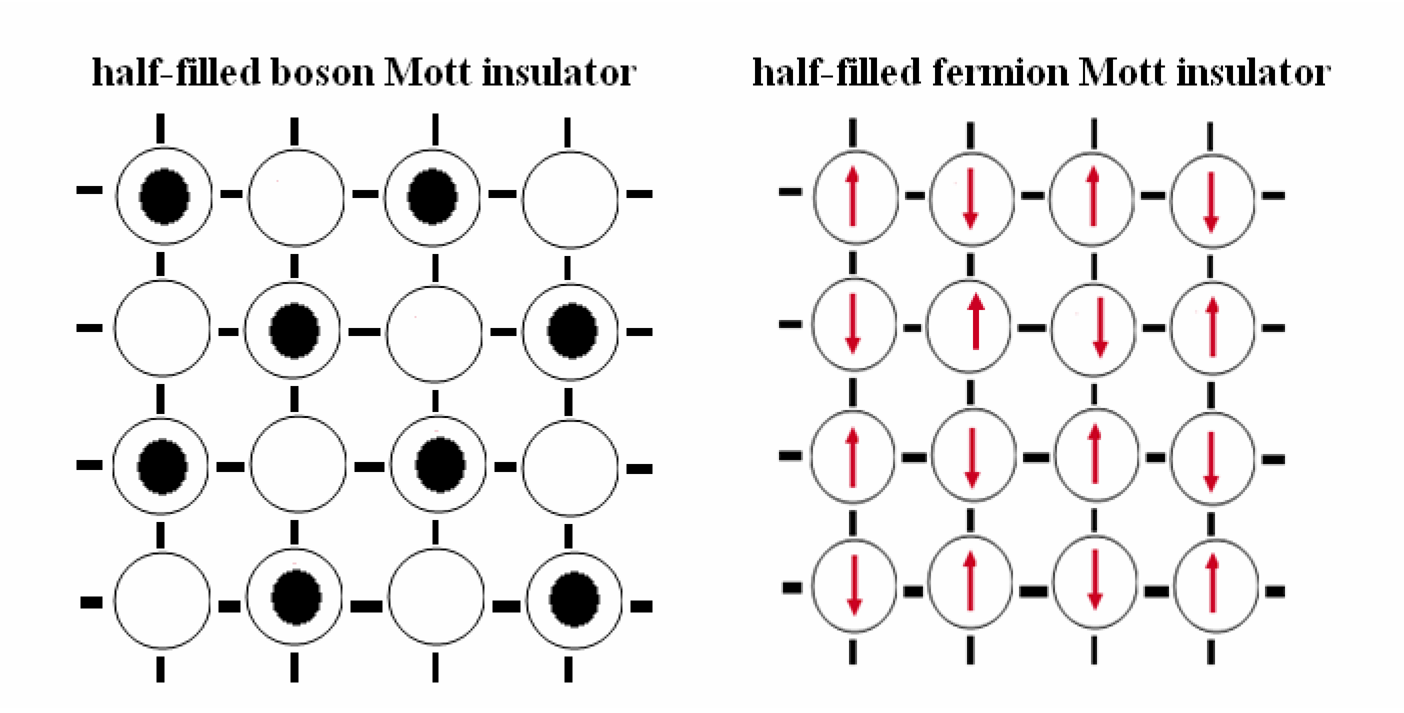


\+ . . .



Spin liquid is a *featureless* Mott insulator ! *This is a state with no symmetry breaking.*

A curious fact: Most Mott insulators break symmetry, except for integer filling factors.

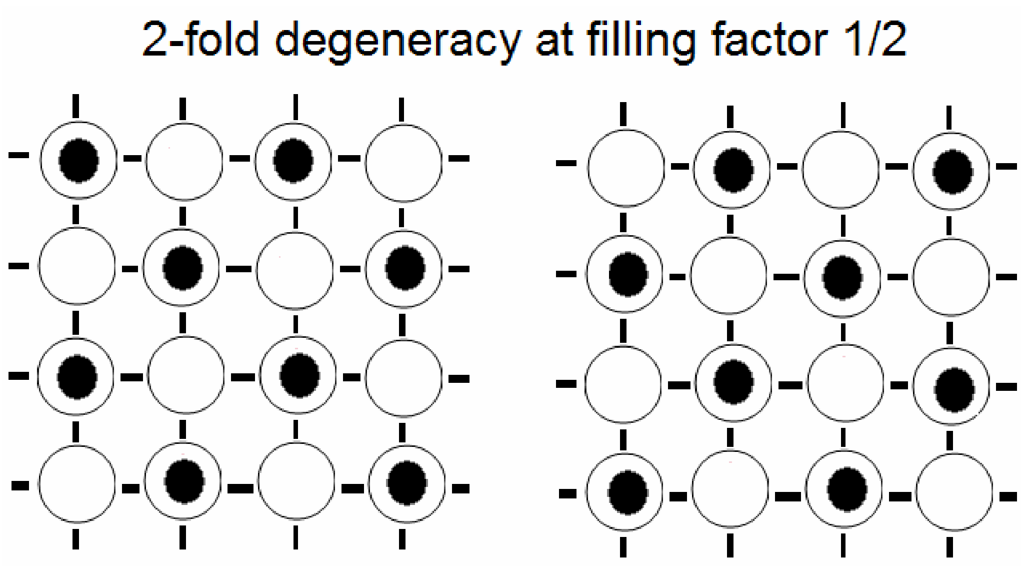


Oshikawa's degeneracy

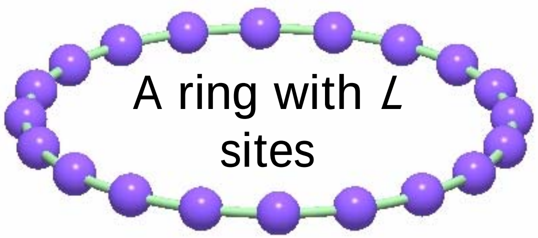
If a Mott insulator exists at filling factor = *p/q*, ⇨ the ground state must be at least *q*-fold degenerate.

Oshikawa, PRL (2000)

Usually the required degeneracy is achieved through symmetry breaking.



Theorem: Simultaneous conservation of the C.M. position and momentum implies Oshikawa's degeneracy.



$$U = \exp(2\pi i/L \sum_{j=1}^{L} j\ C_j^\dagger C_j) = \text{the c.m. position modulo L}$$

$$T = \text{translation by one lattice constant}$$

$$[H,U] = [H,T] = 0 \qquad\qquad U\ T = e^{i2\pi\frac{p}{q}} T\ U$$

$$|e^{i\phi}, E>$$

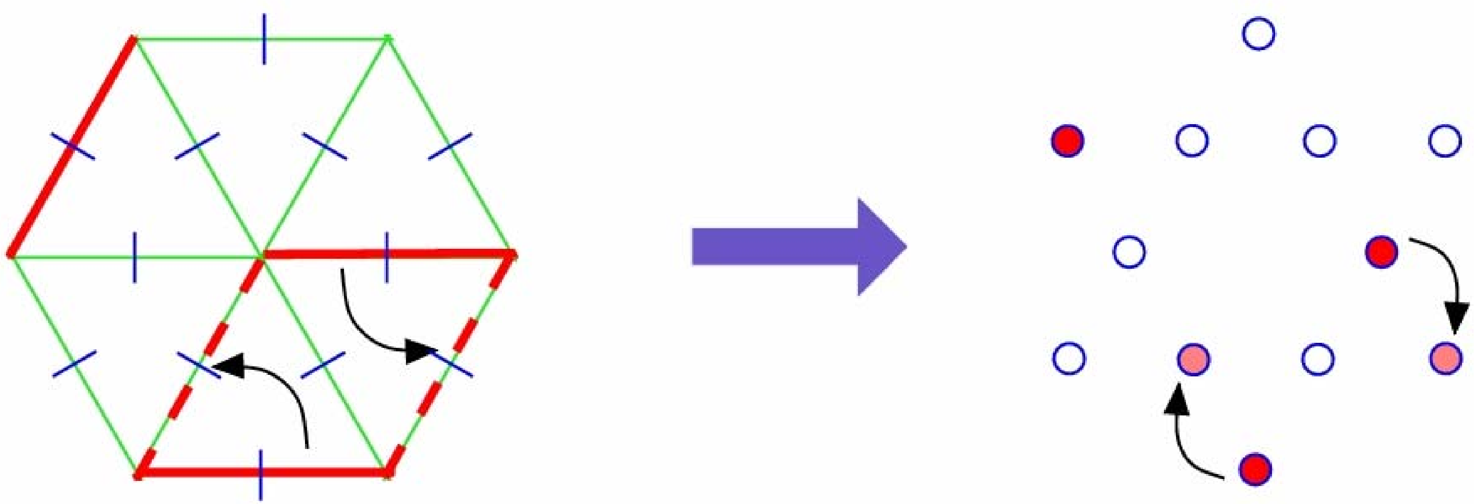
$$T|e^{i\phi}, E> \ \propto\ |e^{i2\pi p/q} e^{i\phi}, E>$$

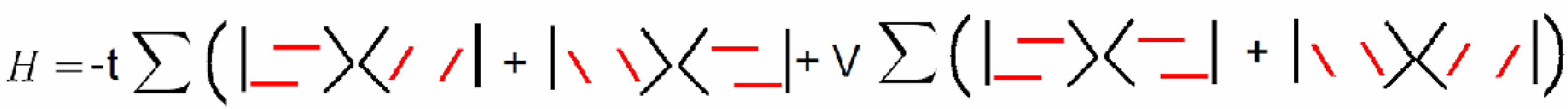
$$\underbrace{|e^{i\phi}, E>\ ,\quad T|e^{i\phi}, E>\ ,\quad TT|e^{i\phi}, E>\quad \ldots}_{q}$$

Seidel *et al* (2005)

Quantum Dimer Moldel

Moessner and Sondhi, PRL(2001)





C.M. conservation

Summary

1. Unify the charge fractionalization in 1D CDW and fractional quantum Hall effect.
2. Derive a 1D model for the pfaffian quantum Hall state.
3. C.M. conservations and featureless Mott insulator.