Nonequilibrium Quantum Criticality in Open Electronic Systems

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Outline

PAST:

Equilibrium magnetic quantum phase transitions
in itinerant magnets; Hertz-Millis-Moriya theory

PRESENT:

General consideration:
nonequilibrium quantum phase transitions

Open system; steady state nonequilibrium state

Development of renormalization group scheme

Study of universality classes

FUTURE:

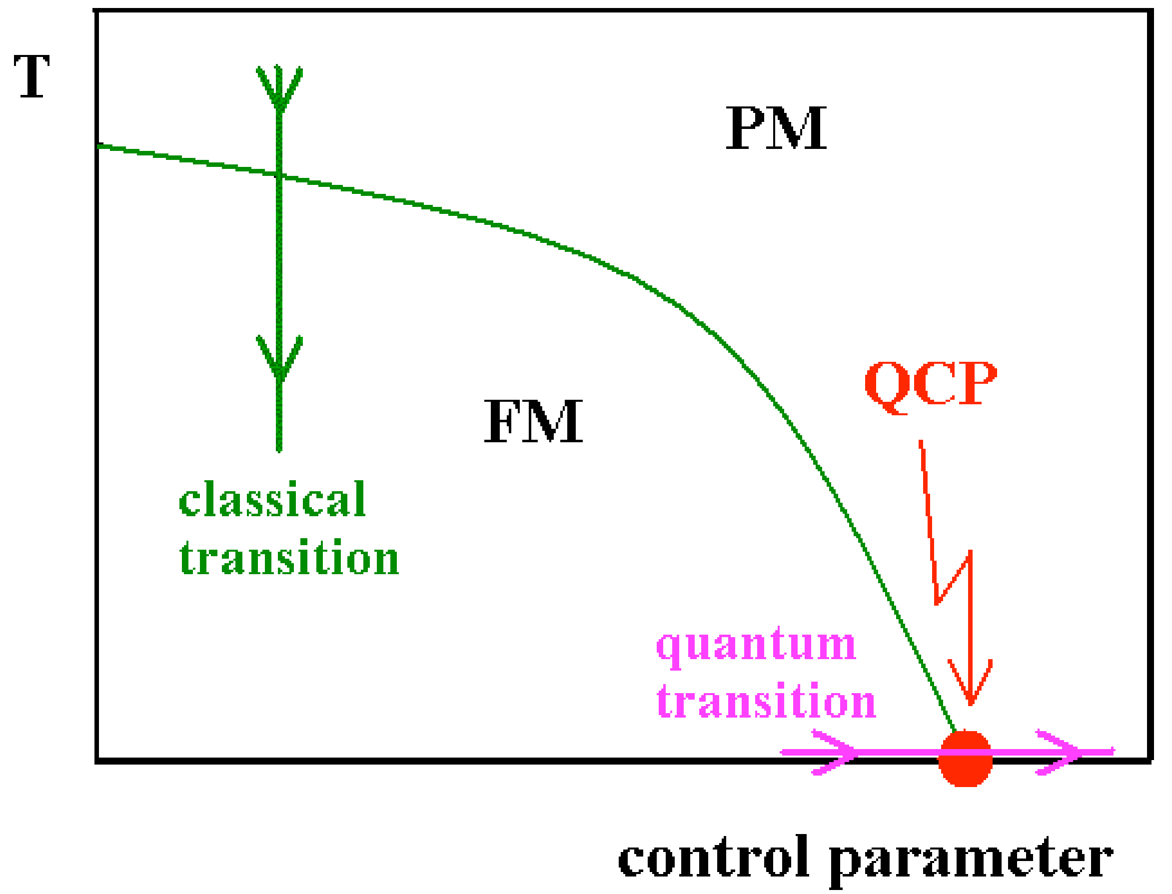
Future directions

Past

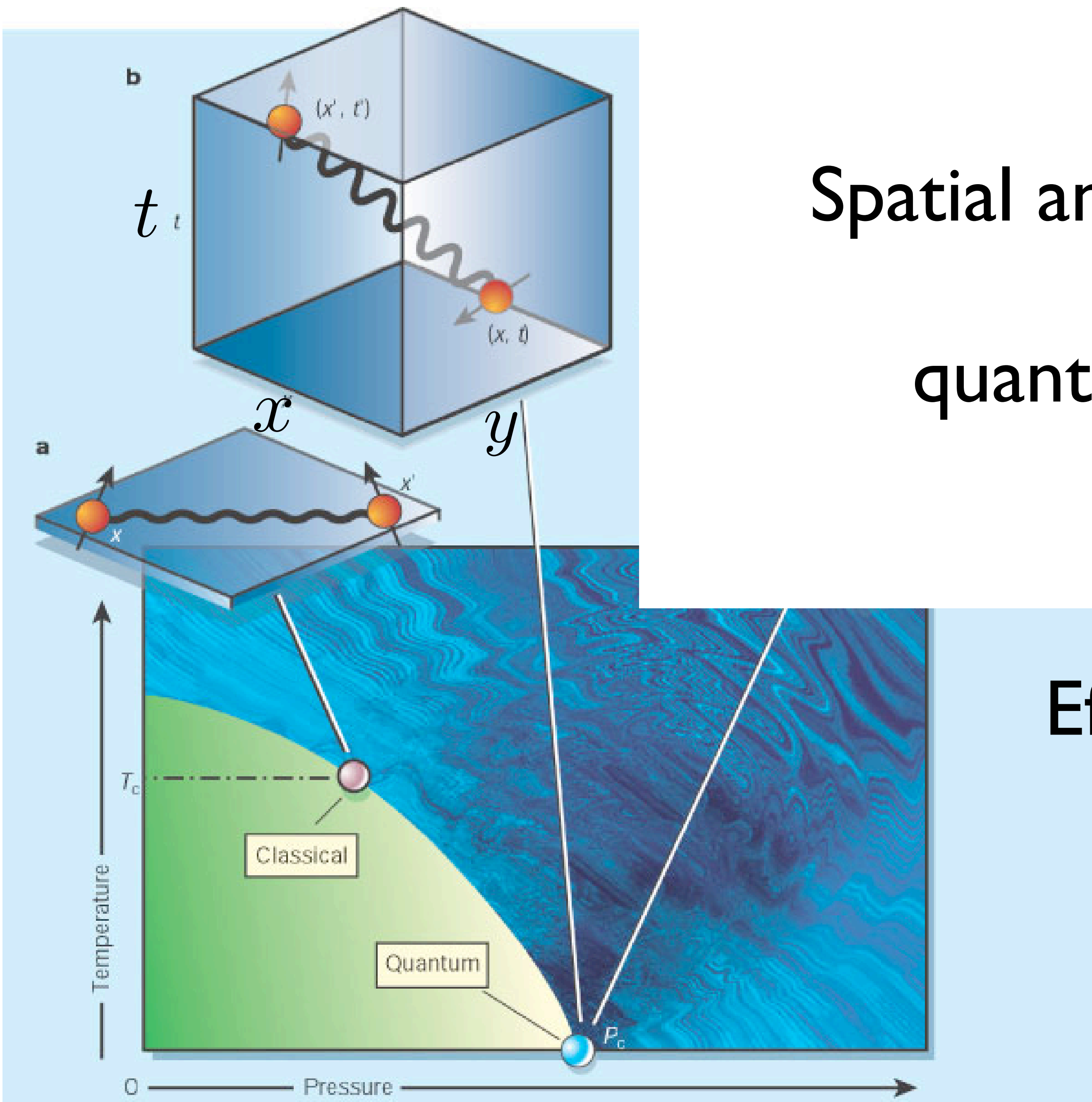
Equilibrium quantum phase transitions

Phase transitions at T=0 tuned by control parameters (e.g. pressure, magnetic fields, chemical doping)

Change in broken symmetries at a quantum critical point



Equilibrium quantum phase transitions



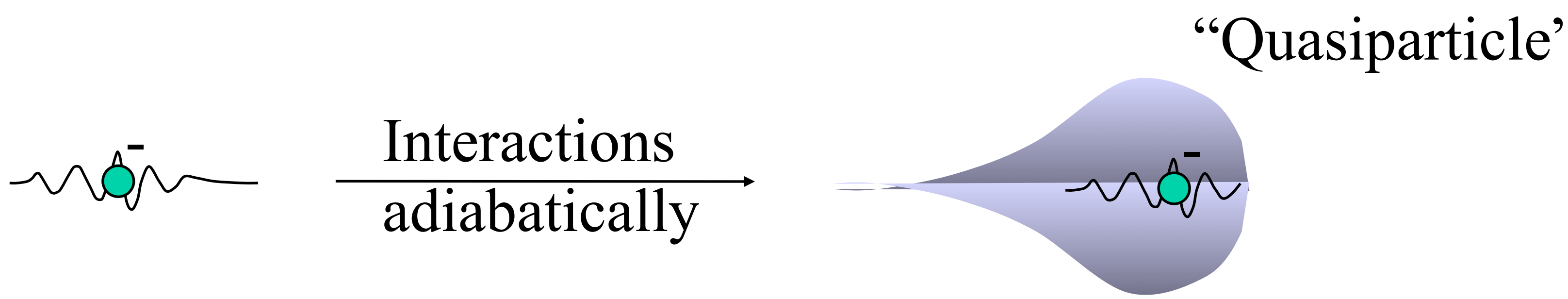
Spatial and Temporal fluctuations are coupled in quantum phase transitions

Effective dimensionality

$$\tau \propto \xi^z$$

$$d_{\mathrm{eff}} = d + z > d$$

Fermi Liquid Theory and Quasiparticle Picture



$$\left|e^-\right\rangle \qquad \left|qp^-\right\rangle$$

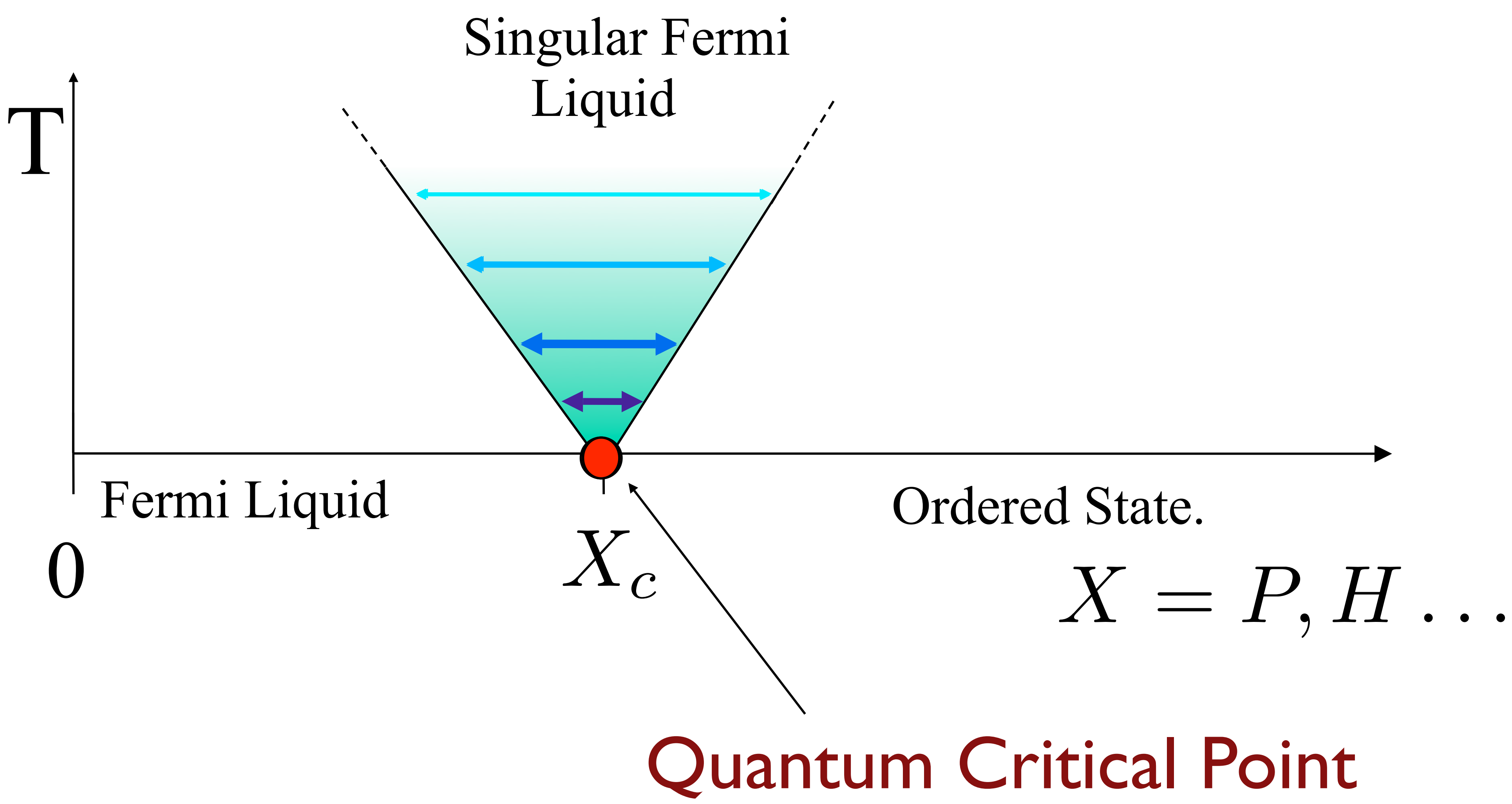
$$\frac{m^*}{m} = \frac{N(0)^*}{N(0)} = 1 + \frac{F_1^s}{3}$$

$$\frac{dE}{dT} = C_V = \gamma T$$

$$\gamma \propto N(\epsilon_F) \propto m^*$$

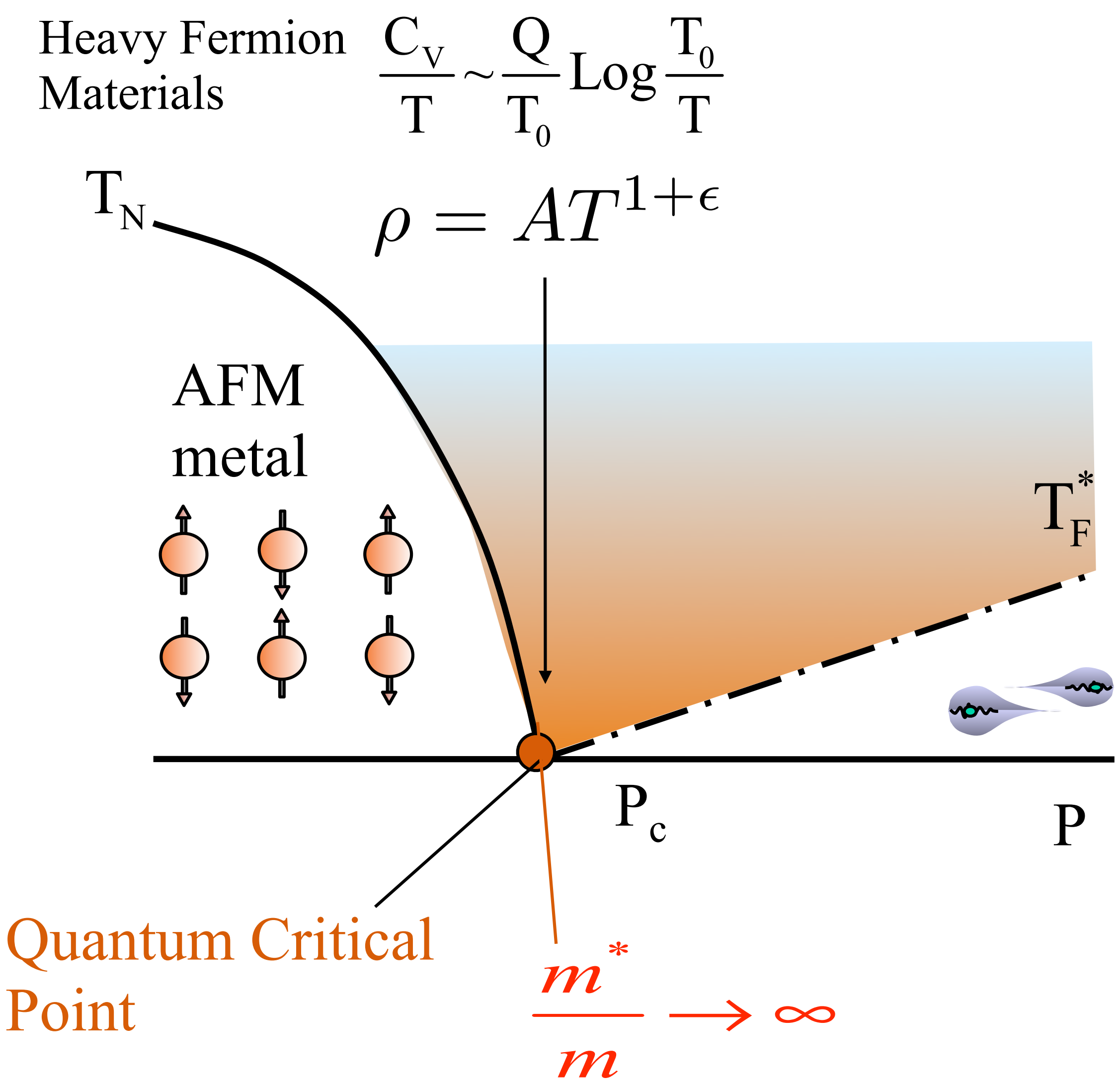
Breakdown of Landau Fermi Liquid Theory

What happens when the interaction becomes too large ?

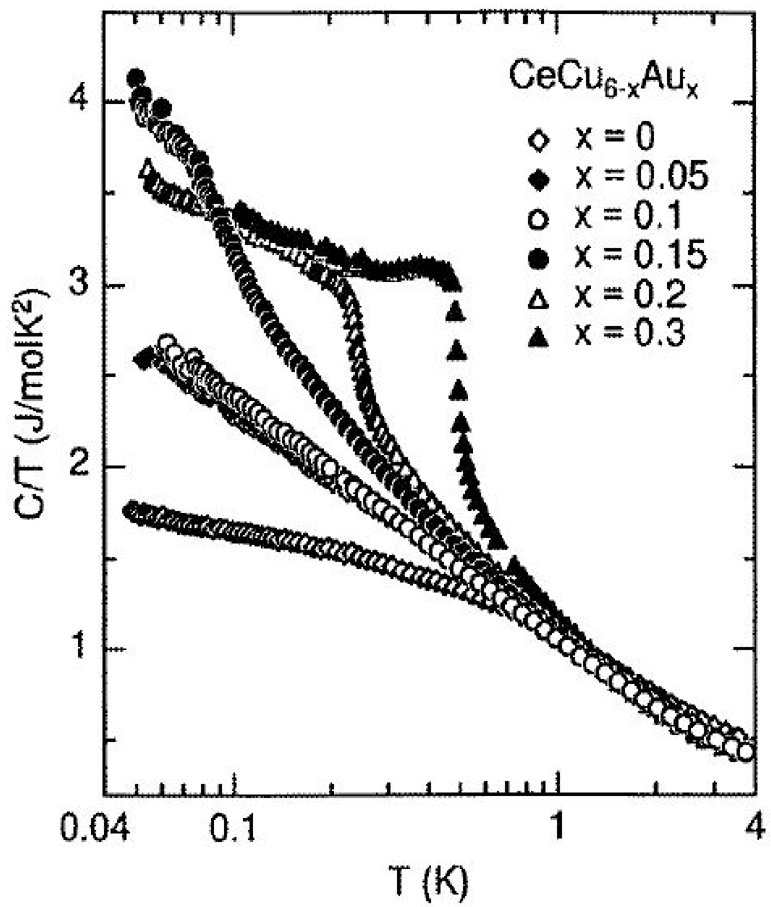


Hertz, 1976

Quantum Criticality: divergent specific heat capacity



H. Von Lohneyson (1996)



Theory of Quantum Criticality in Equilibrium
Itinerant Magnetic Phase Transitions

Effective field theory of the order parameter in d+z dimensions (effective space-time dimensions)

$$\omega \propto q^z \qquad \tau \propto \xi^z$$

$$S_{\text{eff}} = \sum_{\mathbf{q},\omega} \chi^{-1}(\mathbf{q},\omega)|\vec{m}(\mathbf{q},\omega)|^2 + u \int d^d x \; dt \; [\vec{m}(\mathbf{x},t)]^4$$

$$\chi^{-1}(\mathbf{q},\omega) = -i\frac{\omega}{\Gamma(q)} + q^2 + \delta \qquad \delta = 1 - UN(0)$$

Stoner parameter

(c.f. $F = \int d^d x \; \left([\nabla \vec{m}(\mathbf{x})]^2 + \delta[\vec{m}(\mathbf{x})]^2 + u[\vec{m}(\mathbf{x})]^4\right)$ Landau free energy)

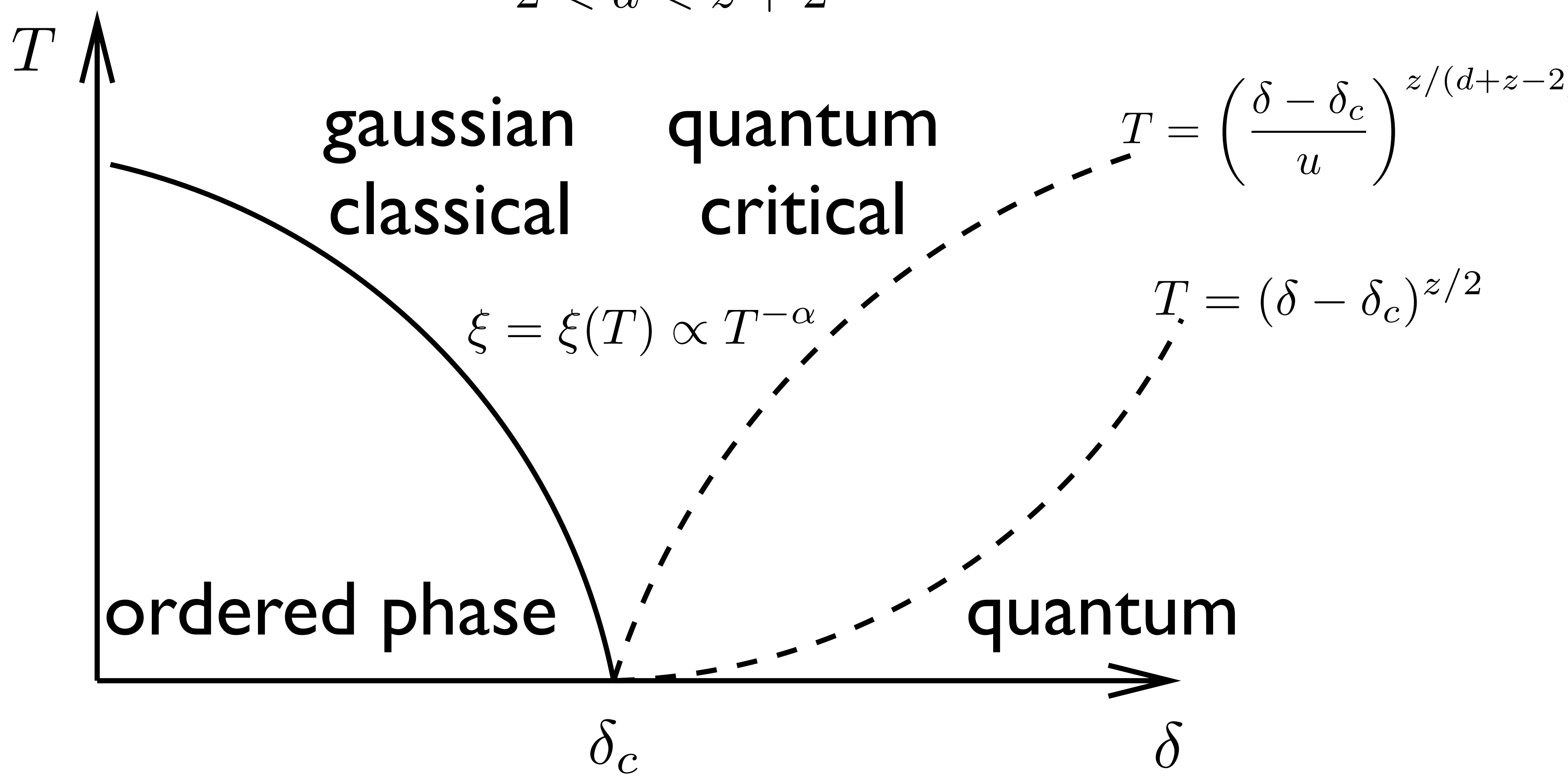
$\Gamma(q) \propto q, q^2$ **Clean/Dirty Ferromagnet (z=3,4)**

$\Gamma(q) \propto \text{constant}$ **Open System (z=2)**

- Renormalization Group Analysis;
  quartic term is irrelevant ($d_{\text{eff}} > 4$); relevant ($d_{\text{eff}} < 4$)

- T is a relevant perturbation $\dfrac{dT(b)}{d\ln b} = zT(b)$
  Thermal fluctuations decouple the dynamics and statics - classical transitions in d-dimensions

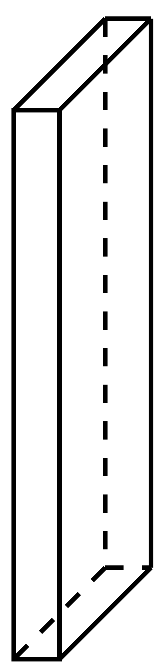
$2 < d < z+2$

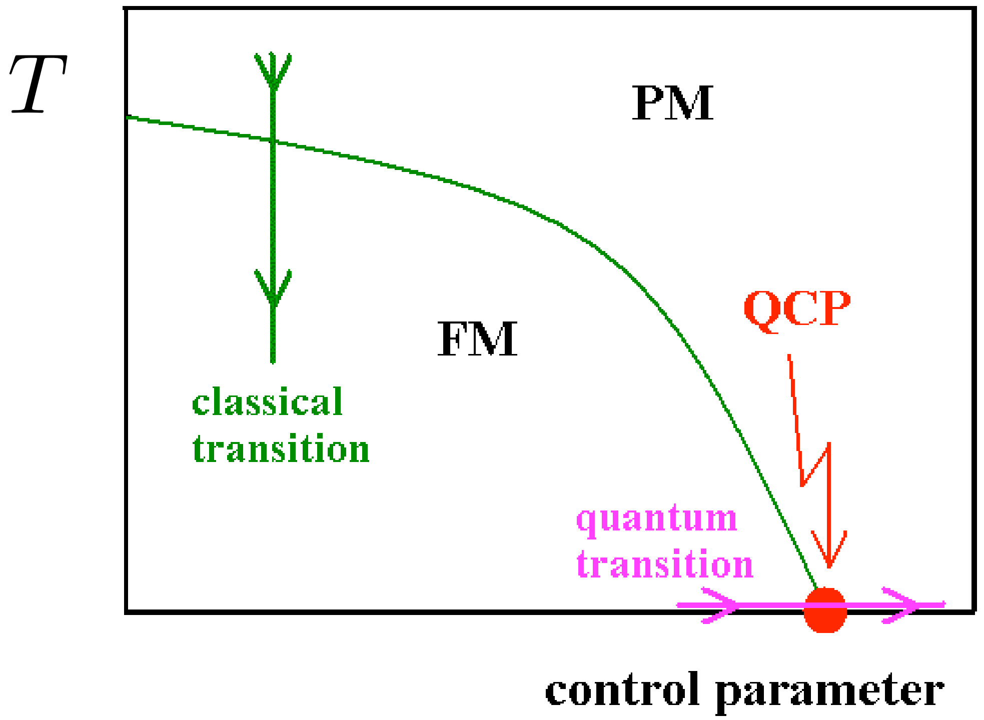


Present

General Consideration

The fields which drive the system out of equilibrium typically increase its energy and destroy phase coherence; this may be analogous to temperature ?
similarity between non-equilibrium transitions and thermal transitions ?





General Consideration

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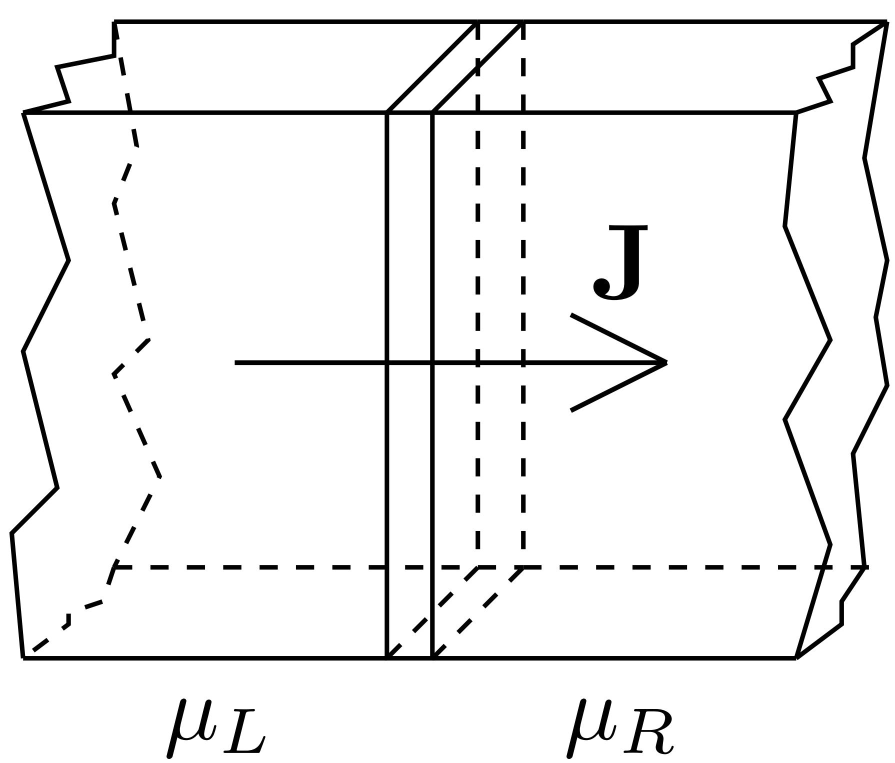
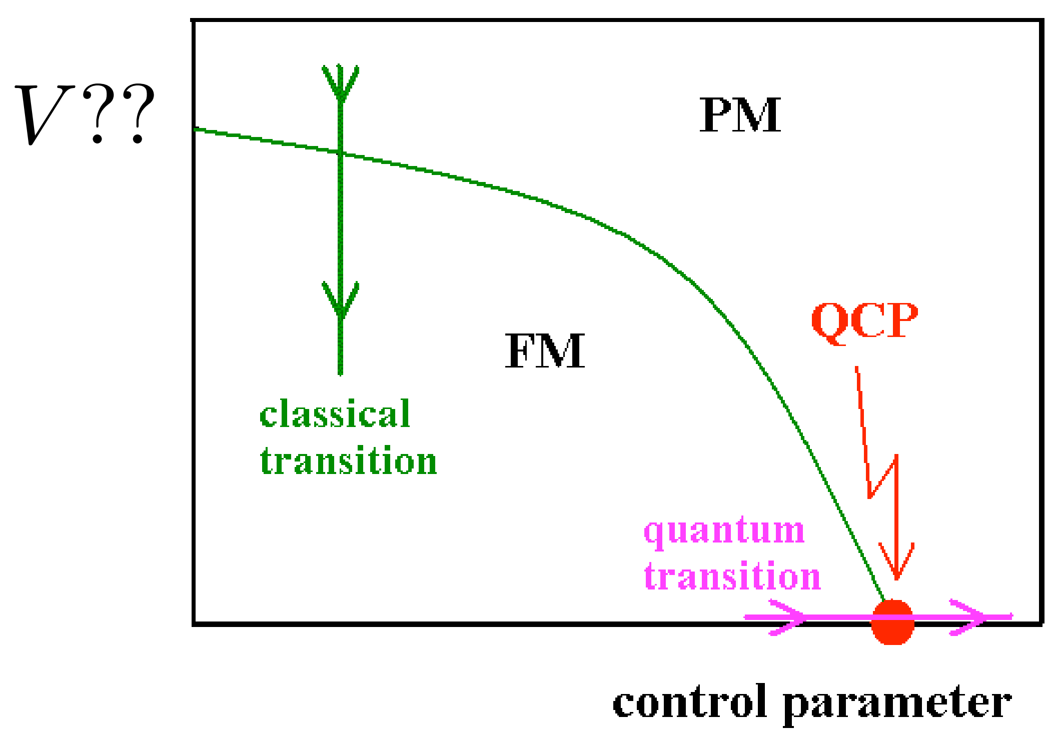
$$V = \mu_L - \mu_R$$





General Consideration

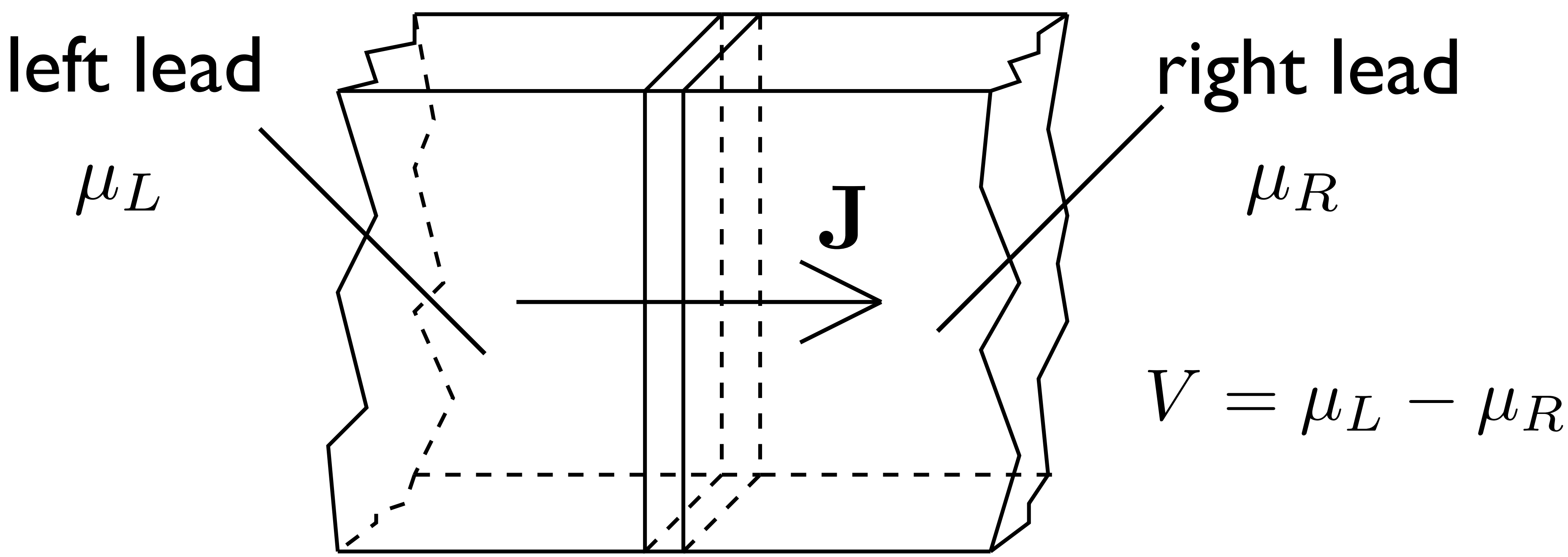
Departures from equilibrium may also break basic symmetries (e.g. time reversal invariance, spatial symmetries such as rotation and inversion)

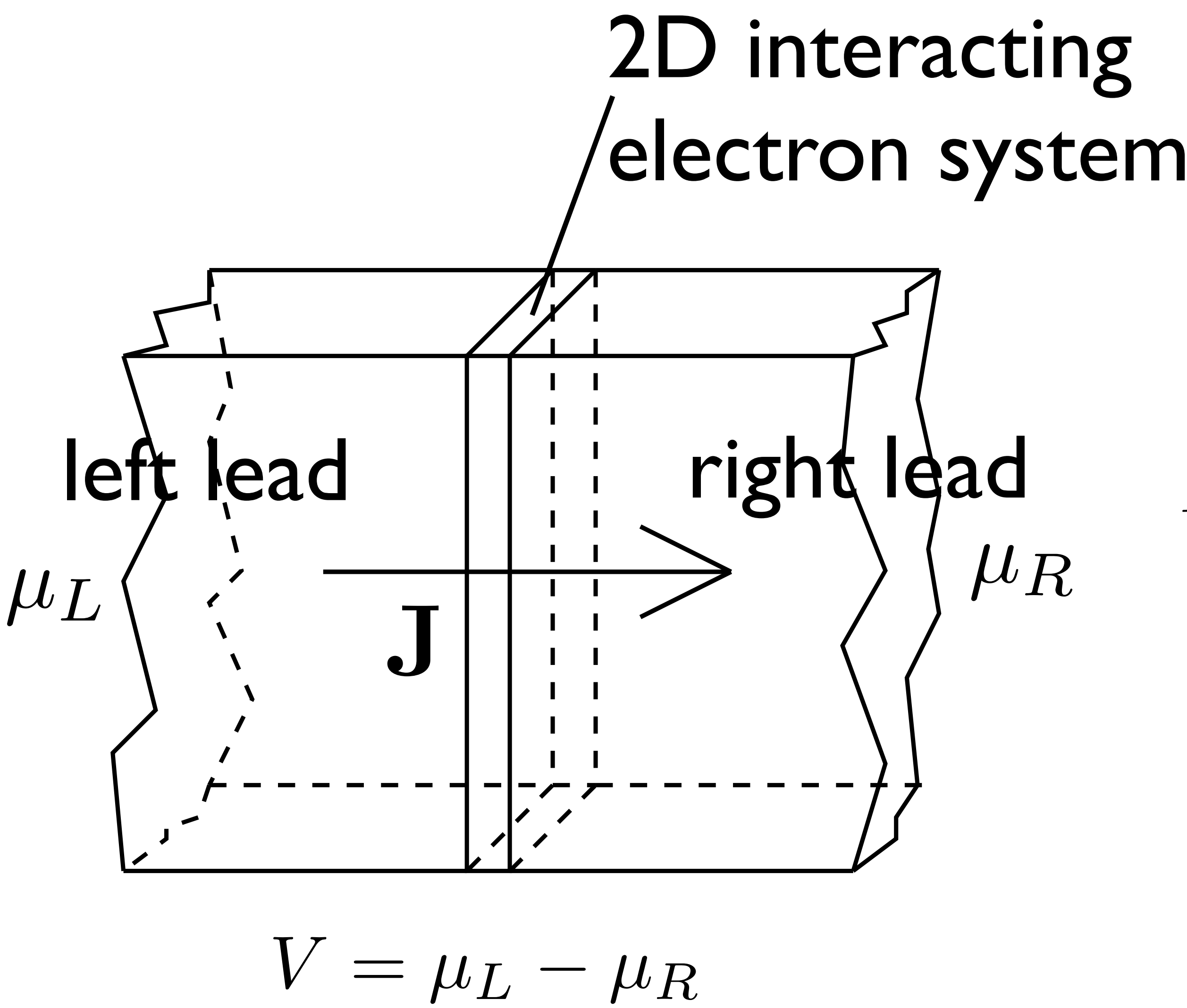
These new effects may change the critical behavior.

What problems do we want to solve ?

Theory of nonequilibrium quantum criticality in itinerant electron systems

Open systems coupled to reservoirs - non-conserved order parameter; nonequilibrium by differences between reservoirs - time-independent drive and steady state





$$
\begin{aligned}
H &= H_{layer} + H_{mix} + H_{leads} \\
H_{layer} &= \sum_{i,\delta,\sigma} t_\delta c^\dagger_{i+\delta,\sigma} c_{i,\sigma} + H_{int}
\end{aligned}
$$

$$H_{mix} = \sum_{i,k,\sigma,b=L,R} \left(t_{k,b} c^\dagger_{i,\sigma} a_{i,k,\sigma,b} + h.c. \right)$$

2D Itinerant electron system coupled to two 3D leads

Let us consider the ISING limit (longitudinal magnetization)

Goals

Difficulty in the formalism: Hertz-Millis-Moriya theory
is based on a quantum generalization of the
Landau-Ginzberg free energy.
But free energy is an equilibrium concept.

Find a way to express nonequilibrium problems
in a Feynman path integral form;
sum over histories on the Keldysh time contour

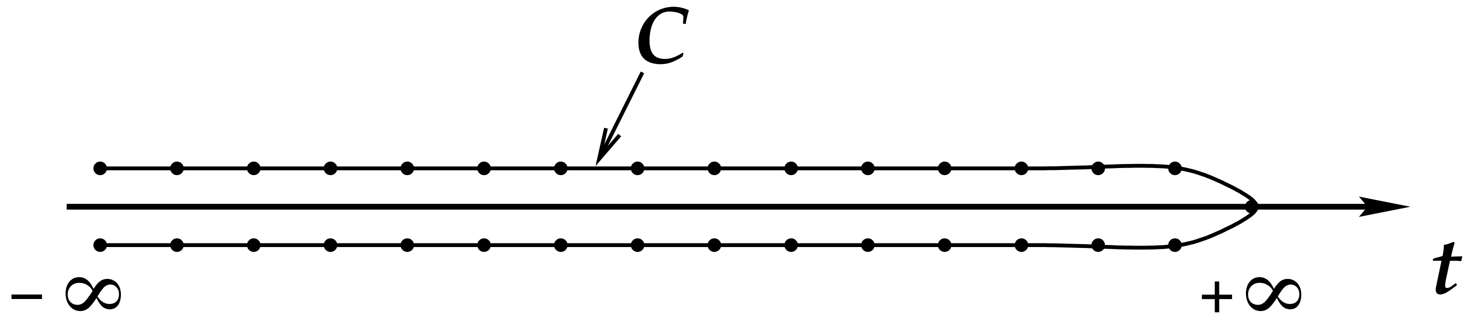
Determine the nature of quantum phase transition;
determine dynamic and static universality classes;
generalize renormalization group scheme
to nonequilibrium systems

Keldysh Path Integral Formalism

Equilibrium field theory is based on the crucial assumption that the asymptotic state in the distant past (initial state) and distant future are identical

Out of equilibrium this assumption is invalid and we have no knowledge of the asymptotic state in the distant future

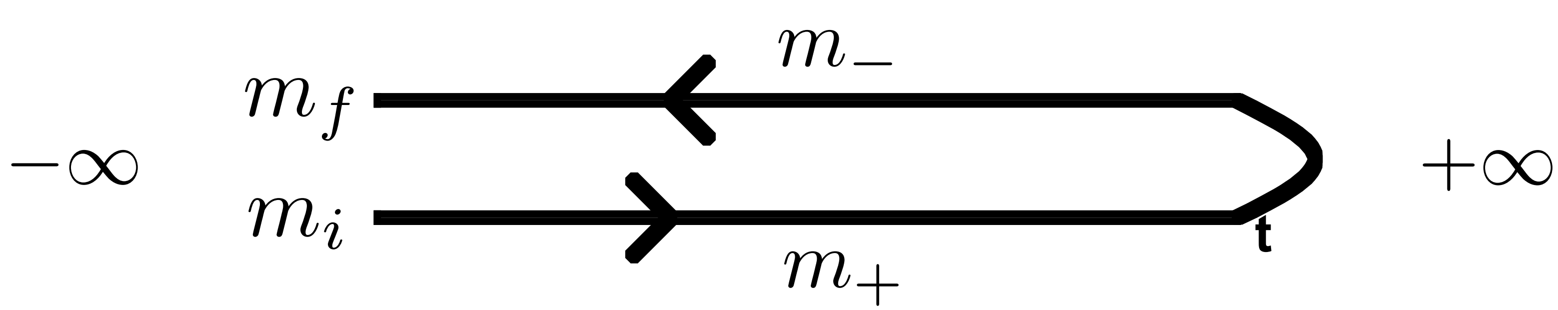
Keldysh approach; let the quantum system evolve forward in time then rewind its evolution back; then no knowledge of the distant future is necessary



Keldysh Path Integral Formalism

Density matrix $\hat{\rho}(t) = e^{-i\hat{H}(t-t_{init})}\hat{\rho}(t_{init})e^{i\hat{H}(t-t_{init})}$.

Keldysh generating functional $Z_K = Tr[\rho(t)]$



Electrons may be integrated out; Keldysh path integral in terms of order parameter fields on the forward and backward paths

$$Z_K \sim \sum_{\text{all config. of } m_+, m_-} e^{S_K[\text{each config. of } m_+, m_-]}$$

Effective Field Theory

Integrate out electronic degree of freedom and obtain an effective field theory of order parameter

$$S_K = S^{(2)} + S^{(4)} + \ldots \qquad m_{cl} = \frac{1}{2}(m_- + m_+) \quad m_q = \frac{1}{2}(m_- - m_+)$$

$$S^{(2)} = -i \int dt\ dt' \int d^d r\ d^d r' \left(m_{cl}(t,r),\ m_q(t,r)\right) \begin{pmatrix} 0 & \left[\chi^{-1}\right]^A \\ \left[\chi^{-1}\right]^R & \left[\chi^{-1}\right]^K \end{pmatrix} \begin{pmatrix} m_{cl}(t',r') \\ m_q(t',r') \end{pmatrix}$$

$$[\chi^{-1}]^R(\mathbf{q},\Omega) = [\chi^R(\mathbf{q},\Omega)]^{-1} = \delta - i\frac{\Omega}{\gamma} + \xi_0^2 q^2$$

$$[\chi^{-1}]^K(\mathbf{q},\omega) = \frac{\mathrm{Max}(\omega, T, V)}{\gamma'}$$

$[\chi^{-1}]^K$ acts as a "mass" for quantum fluctuations

Effective Field Theory

$[\chi^{-1}]^K$ acts as a "mass" for quantum fluctuations

At T=0 and at equilibrium (V=0)

$$[\chi^{-1}]^K(\omega \to 0) = 0$$ strong quantum fluctuations

At finite T and at equilibrium (V=0)

$$[\chi^{-1}]^K(\omega \to 0) = \frac{T}{\gamma'} \neq 0$$ quantum fluctuations suppressed by T

At T=0 and in non-equilibrium (finite V)

$$[\chi^{-1}]^K(\omega \to 0) = \frac{V}{\gamma'} \neq 0$$ quantum fluctuations suppressed by V

Quartic Interactions

$$S^{(4)} = -i \int (d\{k\}) \sum_{i=1\ldots4} u_i m_q^i m_{cl}^{4-i}$$

u_i are interaction functions depending on the momenta and frequencies of all the fields

Renormalization Group

Integrate out fluctuations with $\Lambda/b < q < \Lambda$

Rescaling $q \to q'/b, (\Omega, T, V) \to (\Omega', T', V')/b^z, m_{cl,q} \to m_{cl,q} b^{1+(d+z)/2}$

Solutions of the Scaling Equations

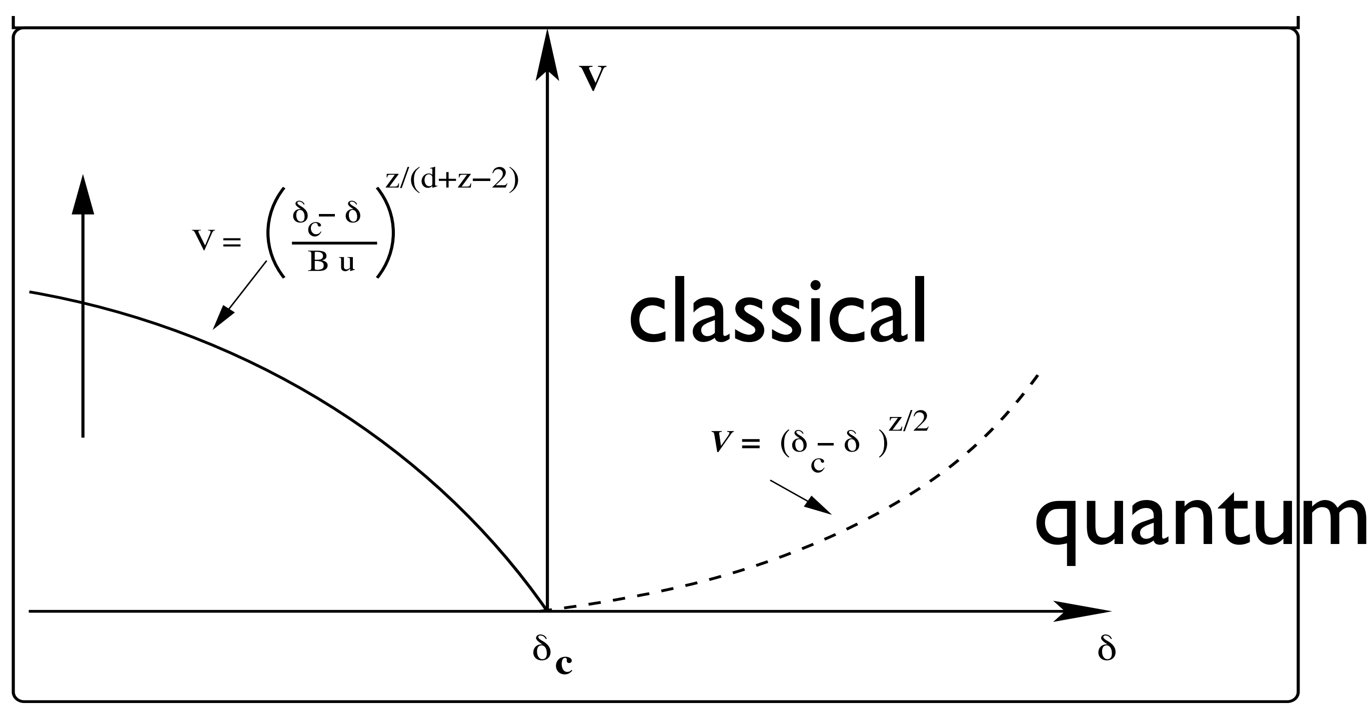
The quartic interaction is irrelevant/relevant for d+z>4 or <4

d=z=2 case is marginal; details of the crossover complicated

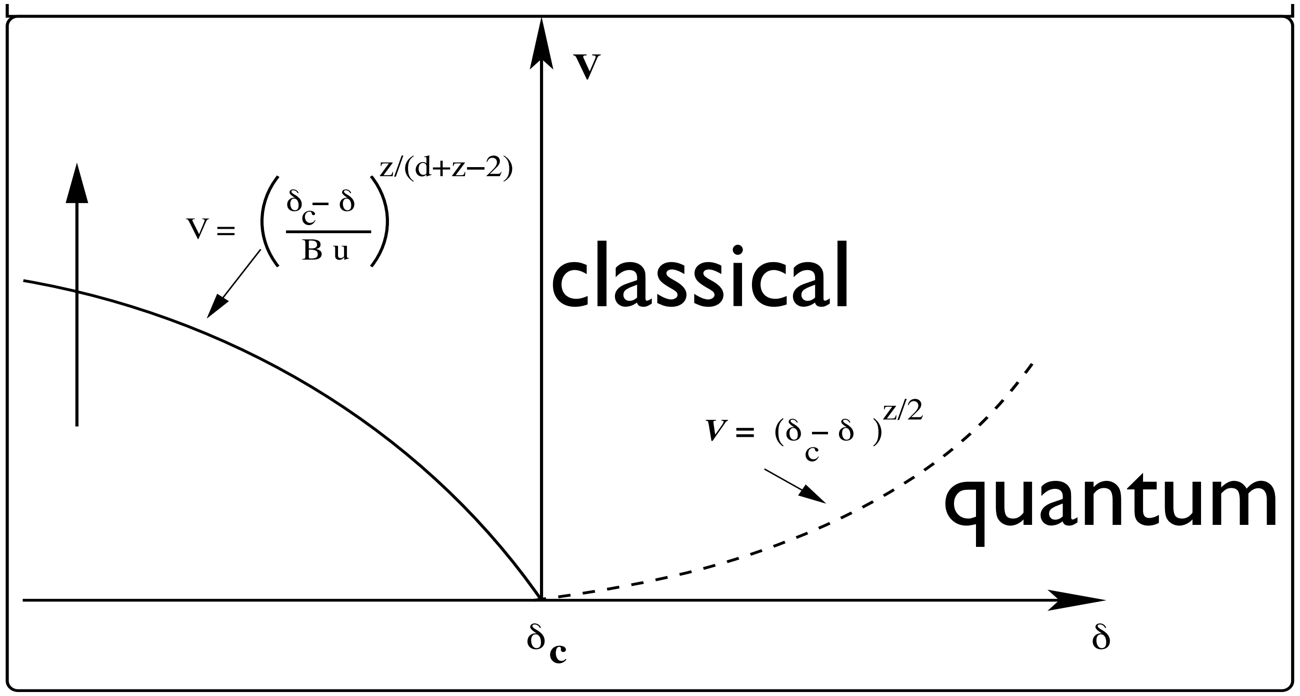
At the end of scaling $\delta(b^*) \sim 1$

$V(b^*) \ll 1$ quantum regime $V < r = \delta - \delta_c$

$V(b^*) \gg 1$ classical regime $V > r$



Phase Diagram at d=2



Dynamical critical exponent z=2

V is a relevant perturbation

Generalized Fluctuation-Dissipation Theorem

At T=0 and in the classical regime, V > r

$$\chi^K(\Omega) = \frac{2T_{\text{eff}}}{\Omega}[\chi^R(\Omega) - \chi^A(\Omega)] \qquad T_{\text{eff}} \sim \frac{\Gamma_L \Gamma_R}{\Gamma} V$$

(c.f. $\chi^K(\Omega) = \coth\left(\frac{\Omega}{2T}\right)[\chi^R(\Omega) - \chi^A(\Omega)]$ in equilibrium at finite T)

fluctuation dissipation

Magnetization Dynamics

Quantum Langevin Equation $$-\frac{1}{\gamma_r}\frac{\partial m_{cl}}{\partial t} = (\delta_r - \xi_0^2\nabla^2 + v_{1,r}m_{cl}^2)m_{cl} + \xi$$

The noise is determined by the Keldysh response function

$$-i\langle\xi(\mathbf{q},\Omega)\xi(\mathbf{q}',\Omega')\rangle = \frac{1}{2}[\chi^{-1}]^K(\mathbf{q},\Omega)\delta(\mathbf{q}+\mathbf{q}')\delta(\Omega+\Omega')$$

In the classical regime, V > r $\quad [\chi^{-1}]^K \propto V$

$\xi(t)$ becomes gaussian white noise

$$\langle\xi(x,t)\xi(x',t')\rangle = \delta(x-x')\delta(t-t')\frac{2V}{\gamma_{rLR}}$$

This is the same as the Model A dynamics;
the voltage driven transition is in the same universality class as the usual thermal Ising transition;
voltage acts like temperature

Heisenberg Magnet

The physics of the disordered and quantum-classical crossover regimes is weakly dependent on the spin symmetry

Differences appear in the 'renormalized classical' regime corresponding to adding a weak non-equilibrium drive to an ordered state

The Langevin Equation in the ordered phase (near QCP)

$$\frac{a_{xx}}{\Gamma}\frac{\partial \vec{m}}{\partial t} + \hat{z} \times \frac{a_{xy}\Delta}{\Gamma^2}\frac{\partial \vec{m}}{\partial t} - \left(b_{xx} - \frac{b_{xy}\Delta V}{\Gamma^2}\hat{z}\times\right)\xi_0^2\nabla^2\vec{m} = \vec{\xi} \qquad (V/\Gamma, \Delta/\Gamma \ll 1)$$

spin precession as in the Landau-Lifshitz-Gilbert Eq.

spin-torque effect when $\Gamma_L(\varepsilon_1)\Gamma_R(\varepsilon_2) - \Gamma_L(\varepsilon_2)\Gamma_R(\varepsilon_1) \neq 0$

Future

Future Directions

Investigation of other dynamical universality classes

RG in the full quantum problems

$\Rightarrow$ Classical limit

$\Rightarrow$ Dynamical universality classes (model A to J)

Systems where the drive couples linearly to the order parameter (e.g. superfluid-insulator transition)

Generalization to other geometries

Driven Bose condensates

Summary

studied the steady-state nonequilibrium magnetic quantum critical phenomena in open systems

generalized renormalization group scheme to nonequilibrium systems

voltage playing a role of effective temperature in the Ising limit and close to the quantum critical point in the Heisenberg case;
dynamical phase transitions in the Heisenberg case, in the classical regime, far away from the quantum critical point