International Workshop on

# Hong Kong Forum of Physics – Frontiers in Condensed Matter

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# Strings in strongly correlated electron systems

mpipks

# Strings in correlated electron systems

strongly correlated electrons on a frustrated lattice (pyrochlore, checkerboard, kagomé)

but also atoms on optical lattices

solid state physics: frustrated lattices → non-local constrains

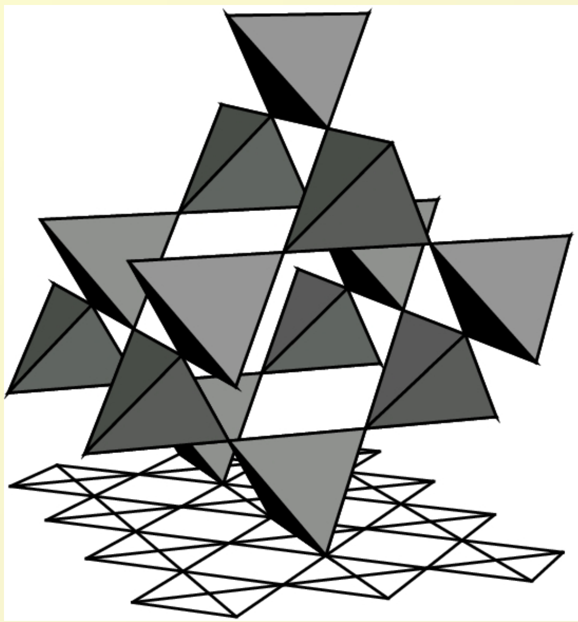
(„tetrahedron rule“) → strings

quantum gravity: strings allow for non-local field theories

not world-lines of point-like particles but

world-sheets of strings

consider a pyrochlore lattice at half-filling with fully spin-polarized electrons (spinless fermions)



pyrochlore → checkerboard

$$H = -t\sum_{\langle ij\rangle}\left(c_i^{+}c_j + \mathrm{h.c.}\right) + V\sum_{\langle ij\rangle} n_i n_j$$

strong coupling limit $V \gg t$

tetrahedron rule

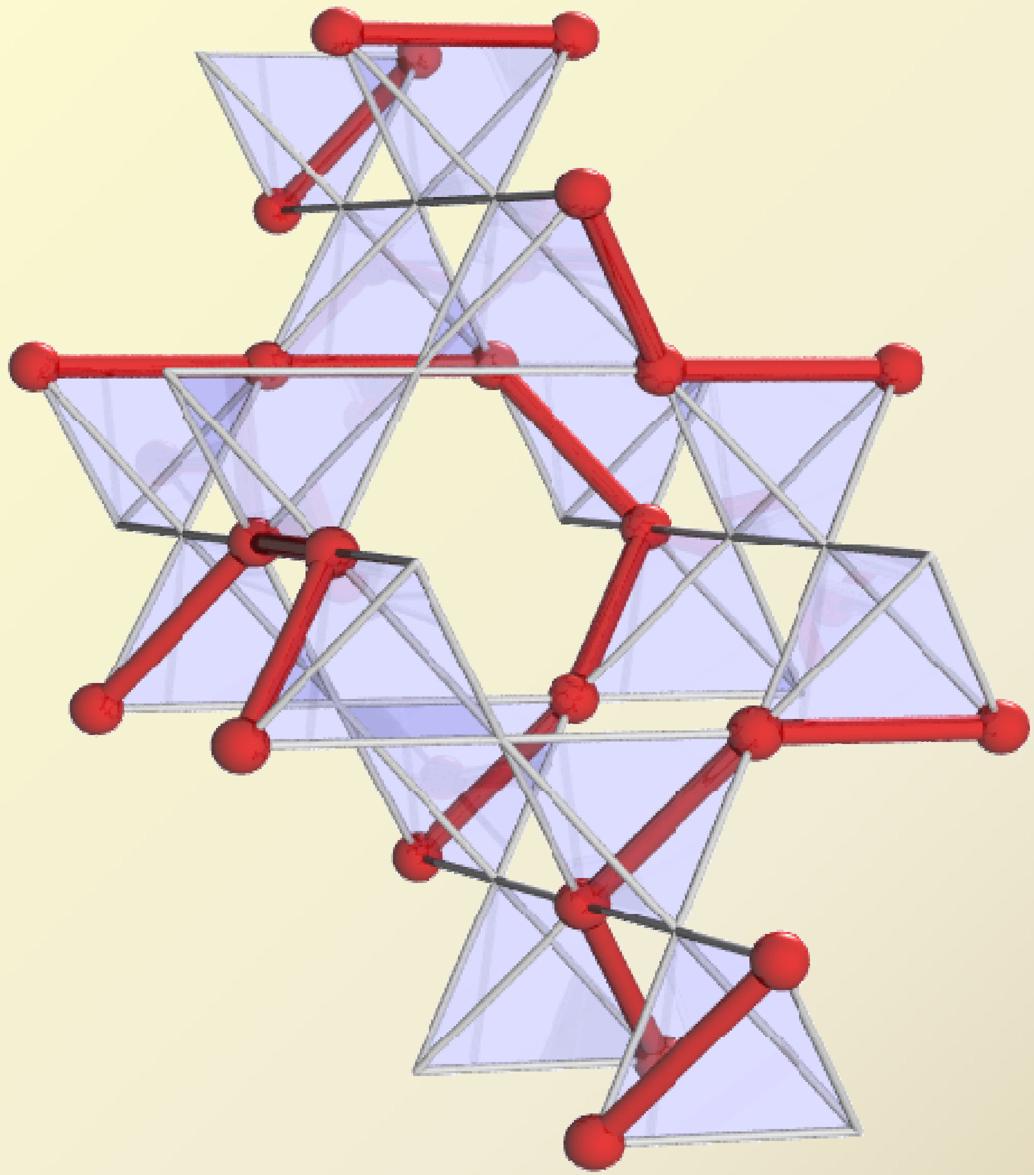
2 empty + 2 occupied sites

**motivation**:

strong electron correlations in

spinels $AB_2O_4$: e.g., $LiV_2O_4$, $Fe_3O_4$

strong non-local constraint!



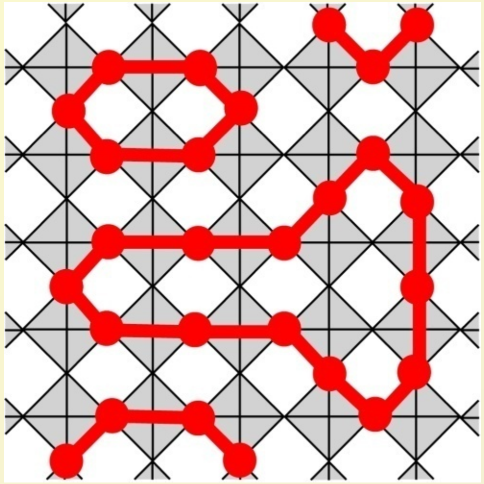
Checkerboard lattice: t = 0

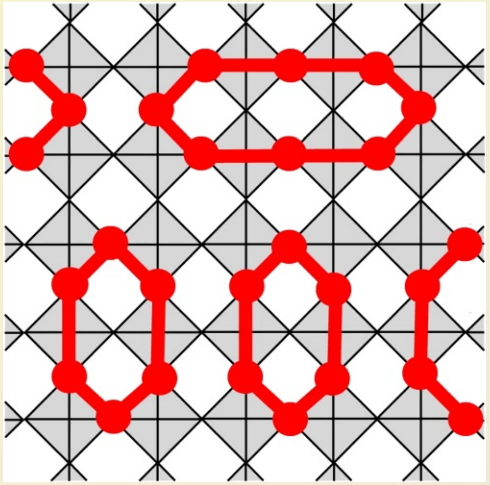
ground-state degeneracy: $N_{deg} = (4/3)^{\frac{3}{4}N}$ N = number of sites

two configurations (examples): → loops

related to ice model

(Pauling)





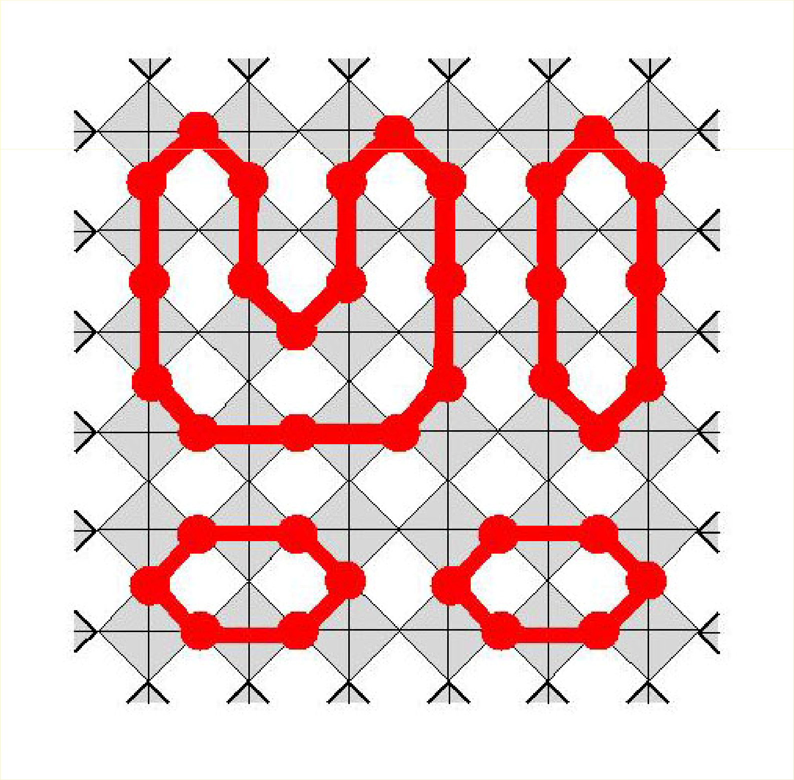
solid lines connect occupied sites

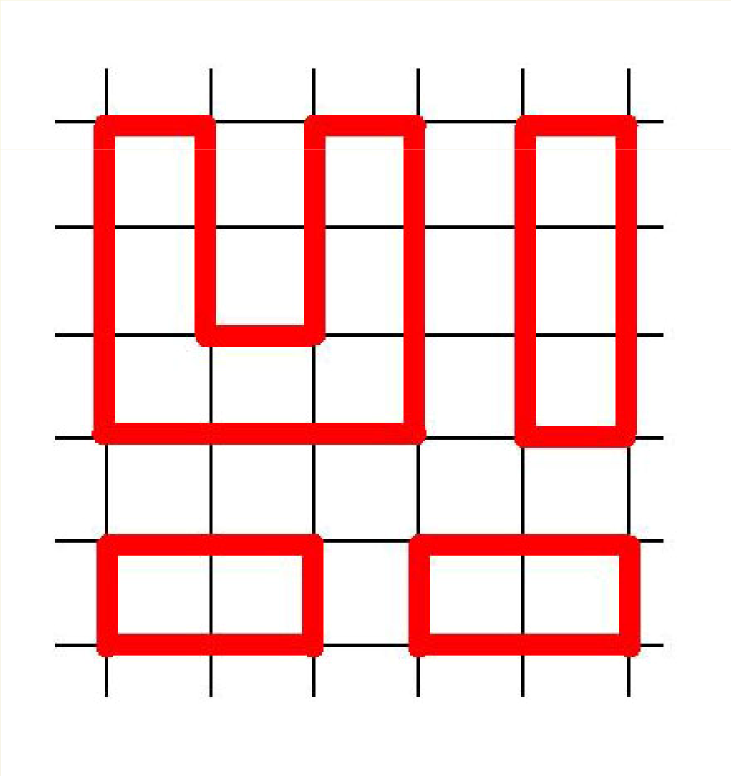
when dynamics is added → simple version of a string theory !

# Medial lattice

Connect all centers of criss-crossed squares by staight lines

➡ particles are sitting on links



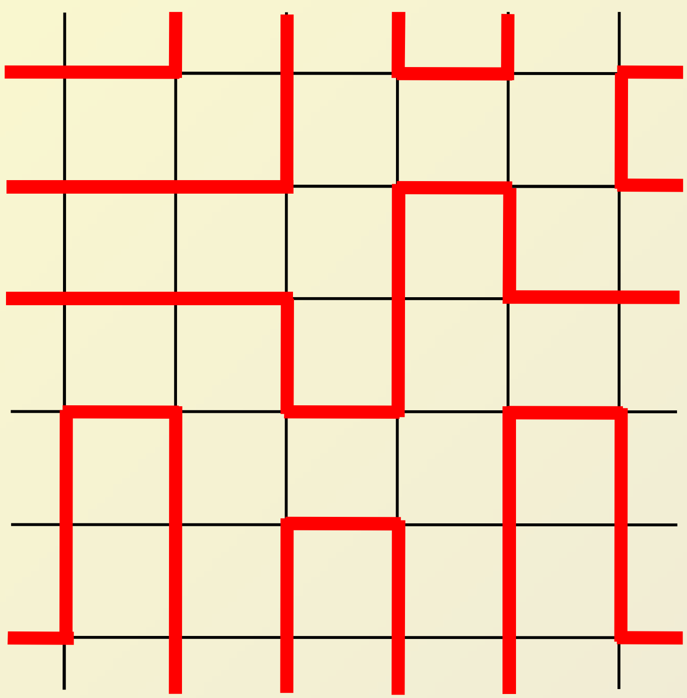


finite hopping $t \neq 0$:

2nd order: constant energy contribution

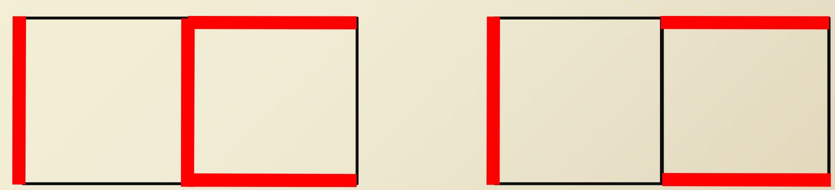
3rd order: effective Hamiltonian $g = \frac{12t^3}{V^2}$ (sign is irrelevant)

$$H_{\text{eff}} = g \sum_{\{\square\square\}} \left( |\square\square\rangle\langle\square\square| - |\square\square\rangle\langle\square\square| + \text{H.c.} \right)$$



lifting of the macroscopic degeneracy

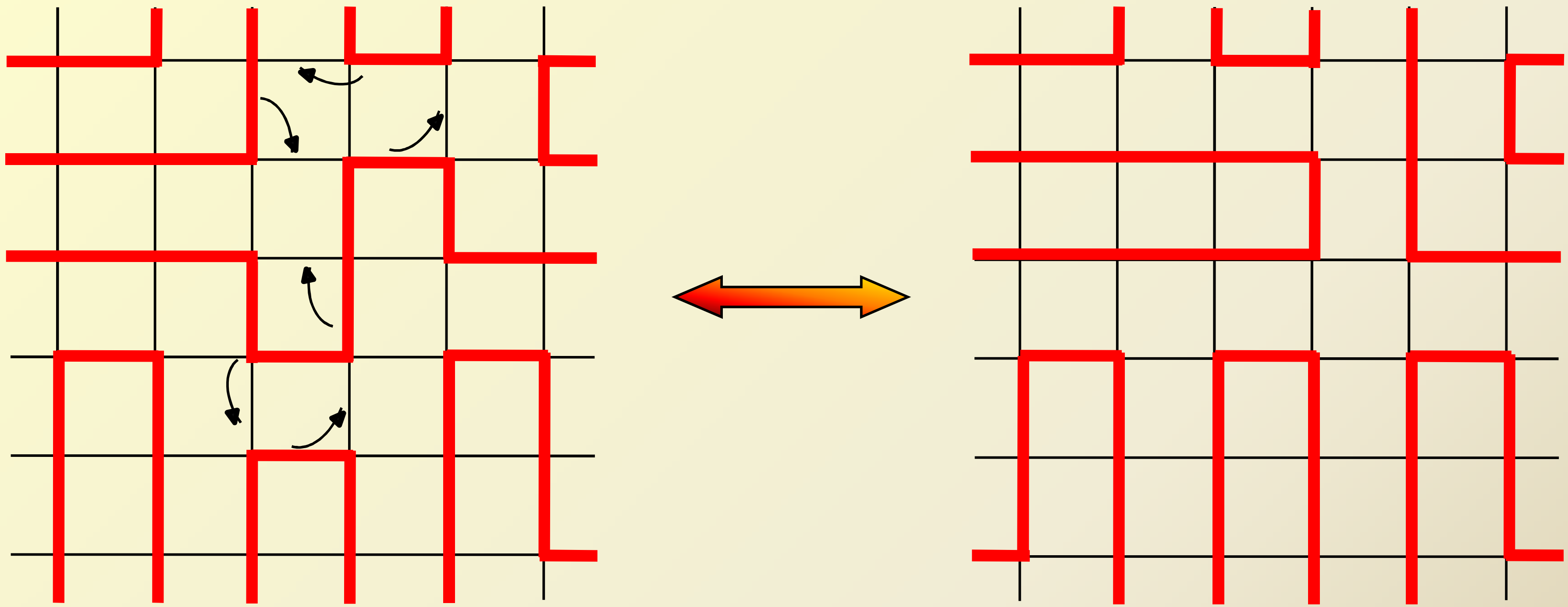
relative sign problem:



B A

**B** processes do not change the topology
**A** processes do it

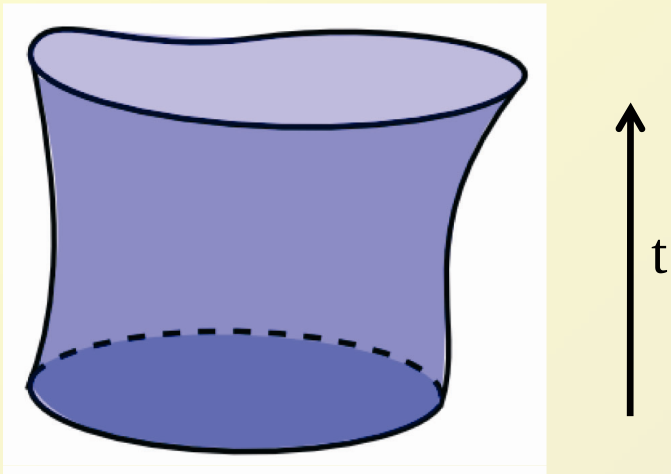
$$H_{eff} = g\sum_{p}\left[|A\rangle\langle\bar{A}| - |B\rangle\langle\bar{B}| + \text{H.c.}\right]$$



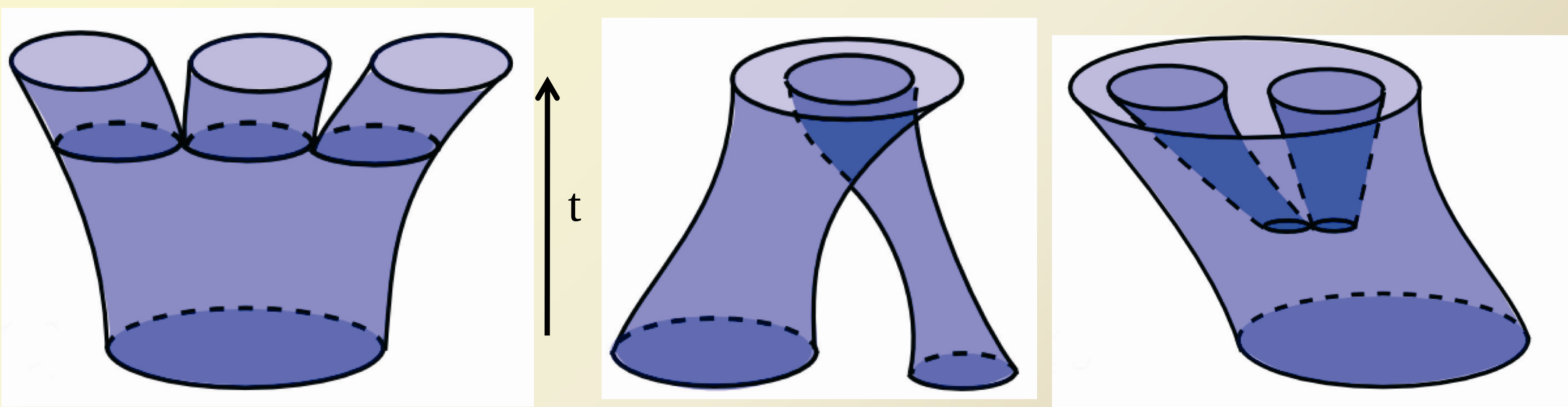
$|B\rangle\langle\bar{B}|$ does not change topology

$|A\rangle\langle\bar{A}|$ does change it

# Continuum representation of loop dynamics due to $H_{eff}$



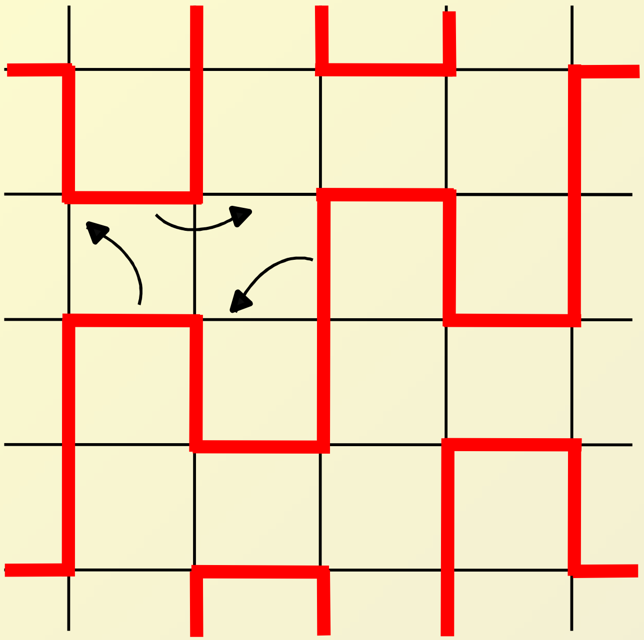

time evolution due to B processes




due to A processes

squiggle configuration

10-fold degeneracy

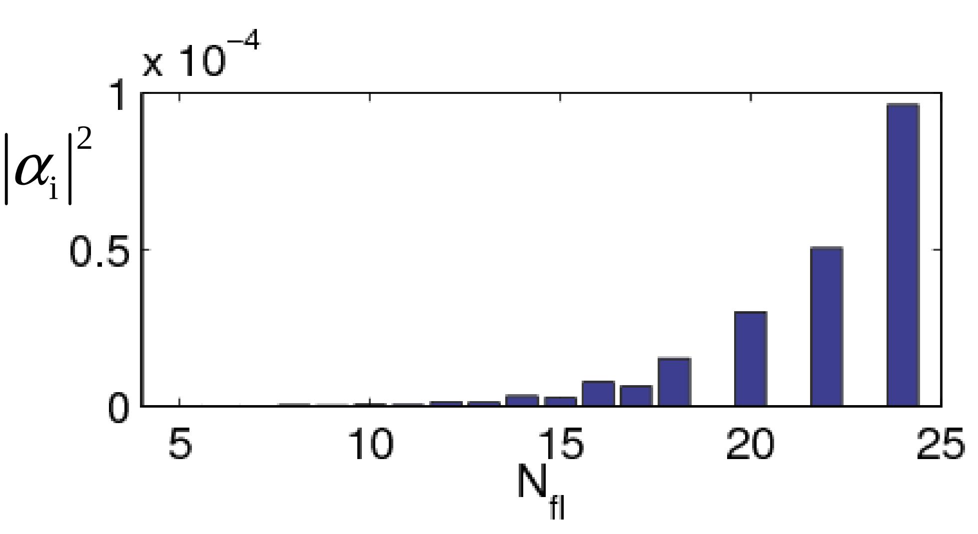


ordered ground state with quantum numbers

$$\kappa = (0,0) \quad , \qquad \mathbf{F} = (3,3,2,2)/10$$

72 links cluster

$$|\psi_0\rangle = \sum_i \alpha_i |c_i\rangle$$

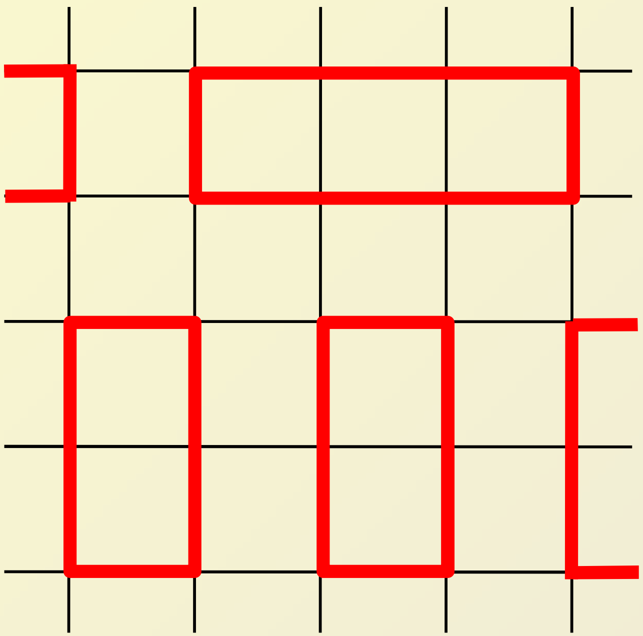



„order by disorder"

energy added to the system ⟹ vibrating squiggle + fluctuations and opening of new „channels“ with different quantum numbers

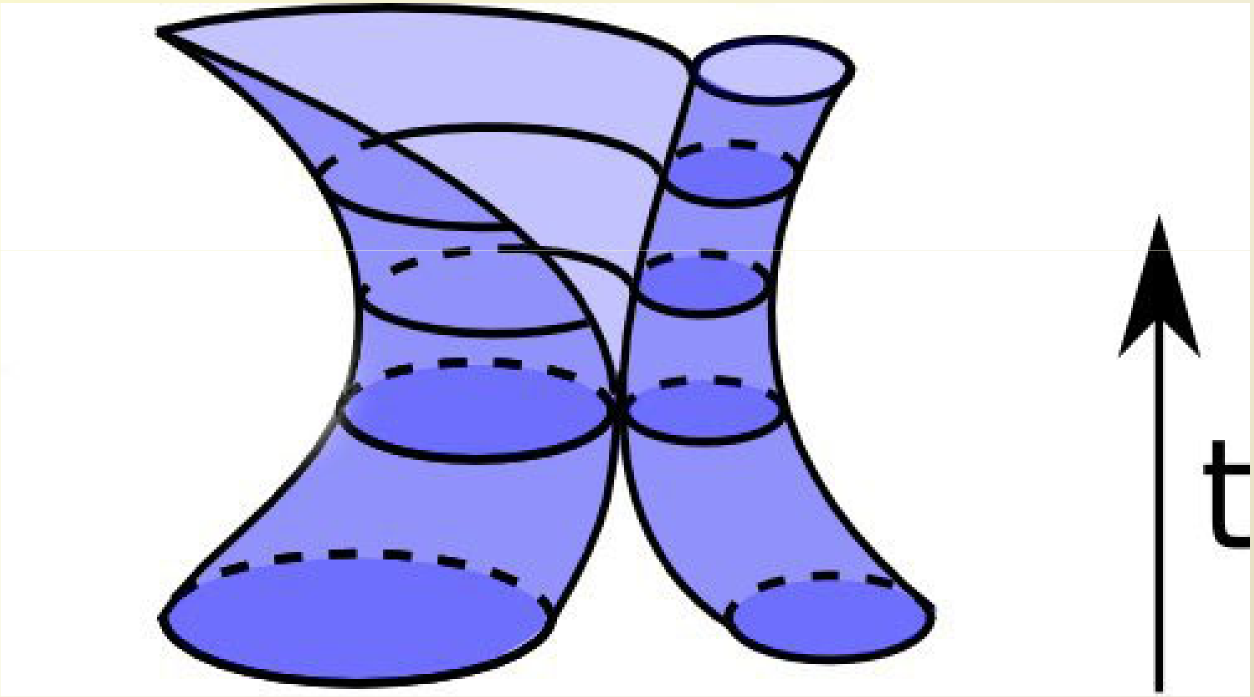
energy $E > V$ added to system ⟹ strings disrupt

closed strings become open strings



fractional charges $+e/2$, $-e/2$

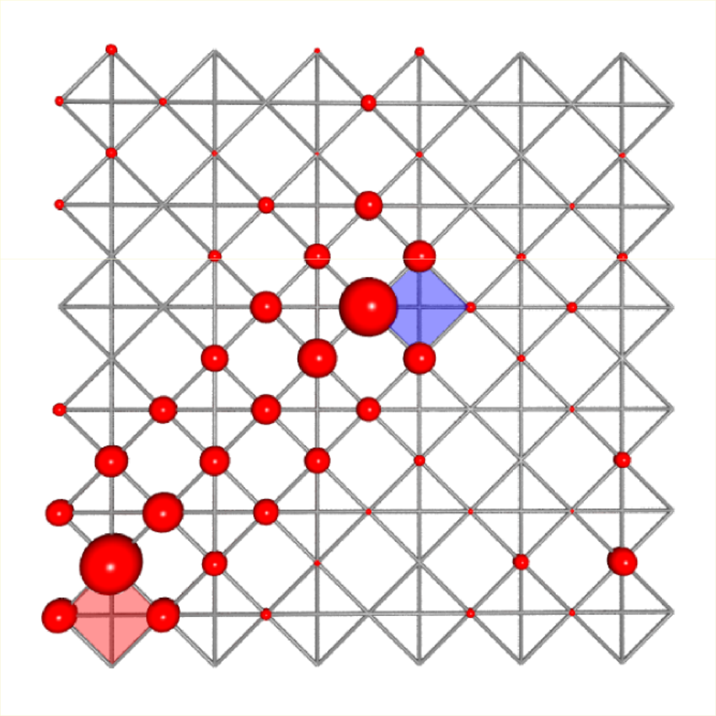
**breaking of a loop** $\Longrightarrow$ **creation of a pair** $+\frac{e}{2},-\frac{e}{2}$



# Confinement of fractional charges

change in kinetic energy in the presence of two fixed charges e/2 and –e/2

site *i*: $$\varepsilon_{\mathrm{i}} = -\frac{1}{6}\sum_{\hexagon / i\varepsilon \hexagon} \left\langle \bar{\psi}_0\left(0,\mathbf{r}\right)\middle| \mathrm{H}_{\mathrm{eff}} \middle| \bar{\psi}_0\left(0,\mathbf{r}\right)\right\rangle$$



constant confining force

reason: vacuum fluctuations are reduced in the vicinity of the string

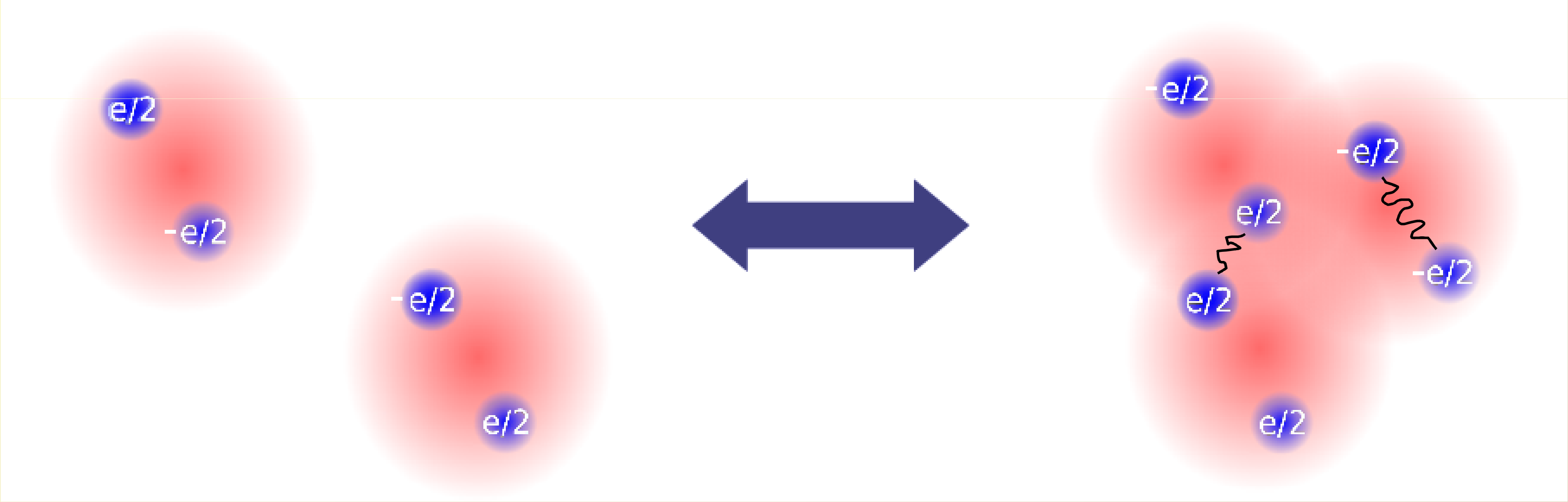
numerics: $\Delta\varepsilon_{\mathrm{kin}} \simeq 0.2\,\mathrm{g}\cdot\mathrm{r}$ r = units of a

$$\mathrm{r} > \mathrm{r_c} \simeq 0.4\left(\mathrm{V}/\mathrm{t}\right)^3 \longrightarrow \left(\frac{\mathrm{e}}{2}, -\frac{\mathrm{e}}{2}\right)$$ pair production

since $\Delta\varepsilon_{\mathrm{kin}} > \mathrm{V}$

- with increasing number of created pairs → formation of particle and antiparticle pairs e/2, e/2 and –e/2, -e/2

- since $g = \frac{12t^3}{V^2} \ll t$ → huge (extended) quasiparticles e.g., d ~ 100a




→ eventually transition to e/2, -e/2 plasma

## extension to pyrochlore lattice:

fractional charges $\pm\frac{e}{2}$ which are deconfined

## What have we learned?

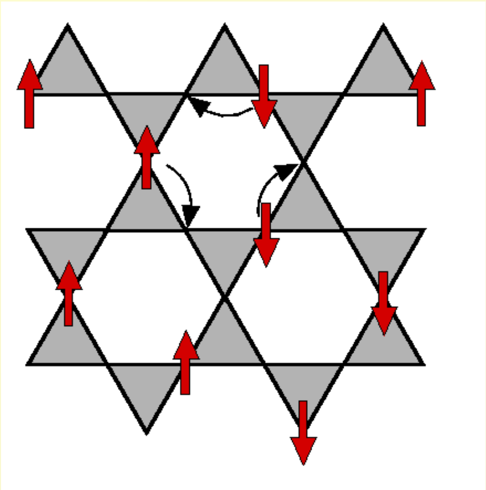
- frustrated lattices lead to fractional charges at special fillings even in 3D

- in 3D they are deconfined while in 2D they are confined with a weak constant restoring force (like quarks)

- we deal with a string theory which can be managed

- not explained: theory can be cast into the form of a gauge theory

# kagomé lattice



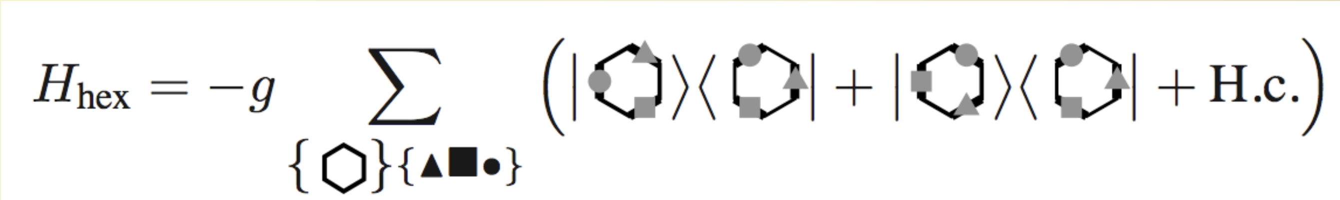
$$H = -t\sum_{\langle ij\rangle\sigma}\left(c^{+}_{i\sigma}c_{j\sigma} + \text{h.c.}\right) + U\sum_{i} n_{i\uparrow}n_{i\downarrow} + V\sum_{\langle ij\rangle} n_i n_j$$

1/6 filling, i.e., one electron per triangle

$U \rightarrow \infty \quad |t| \ll V$ strong correlations

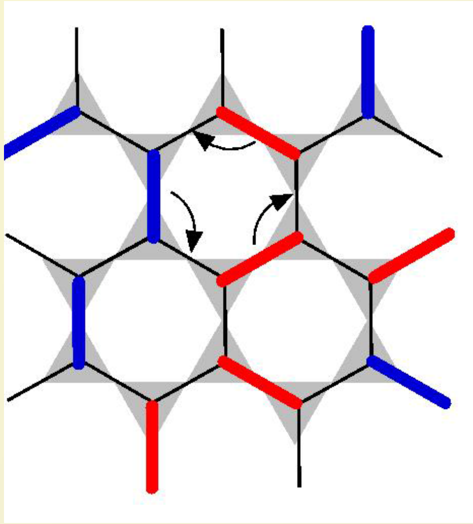
(1) $t = 0$ → ground state is macroscop. degenerate

(2) $t \neq 0$ →




$$g = \frac{6t^3}{V^2}$$

no fermionic sign problem!



*dual lattice*

Perron-Frobenius Theorem:

consider $n \times n$ matrix with positive entries ⟹ largest eigenvalue is positive and nondegenerate and the corresponding eigenvector is nodeless

consider $\exp\left(-\tau H_{hex}\right)$ ; $\tau > 0$

choose $g > 0$ ⟹ all matrix elements of $H_{ex}$ are nonpositive

⟹ theorem is applicable ⟹ fully polarized ground state

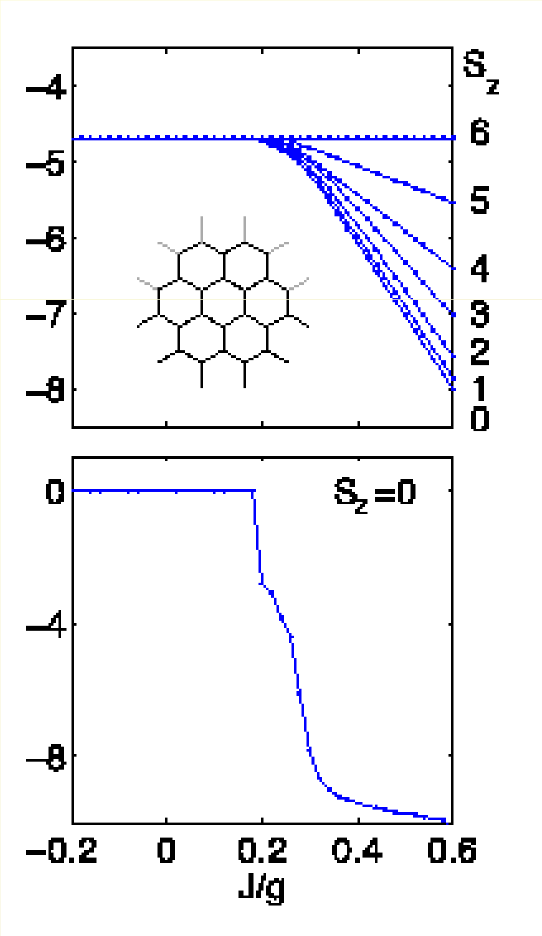
# Robustness of ferromagnetic ground state

add n.n.n interaction

$$H_{spin} = J \sum_{\langle\langle i,j \rangle\rangle} \left( \mathbf{S}_i \mathbf{S}_j - \frac{1}{4} n_i n_j \right)$$

$\frac{E}{g}$

$\langle 0 | H_{spin} | 0 \rangle$




24 sites honeycomb lattice
(period. boundary cond.)

similar results for filling factor 1/3

and pyrochlore lattice with 1/8 filling