

# A gauge glue between spin and charge in cuprates

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joint project with Su Zhao-Bin and Yu Lu  
(happy birthday!)

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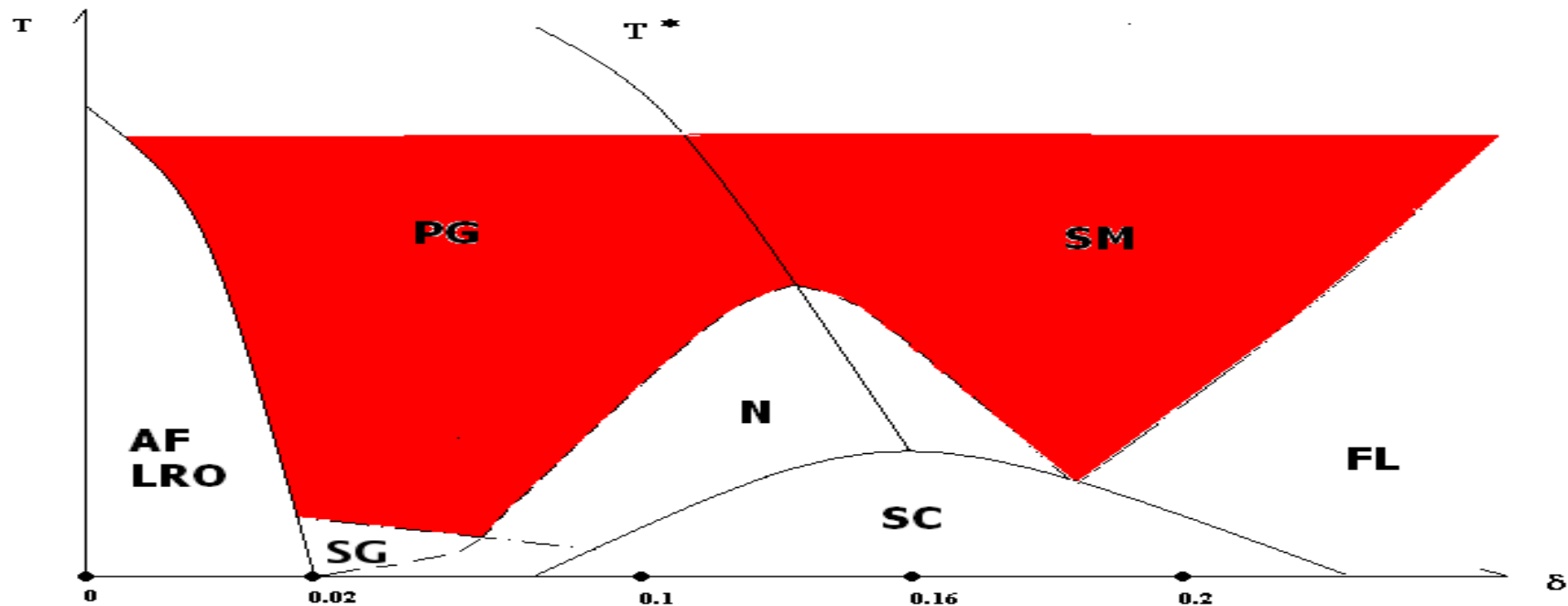
# Aim

Discuss some experimental features of transport and thermodynamic properties of the normal state of hole-doped cuprates suggesting the existence of an emergent "gauge glue" between spin and charge degrees of freedom of the electron.

# Outline

- Phase diagram and model adopted
- Experimental features addressed: metal-insulator crossover of in-plane resistivity and electronic entropy and specific heat
- Where the “gauge glue” come from
- Interpretation of experiments in terms of “gauge glue”

# “Phase diagram” and model

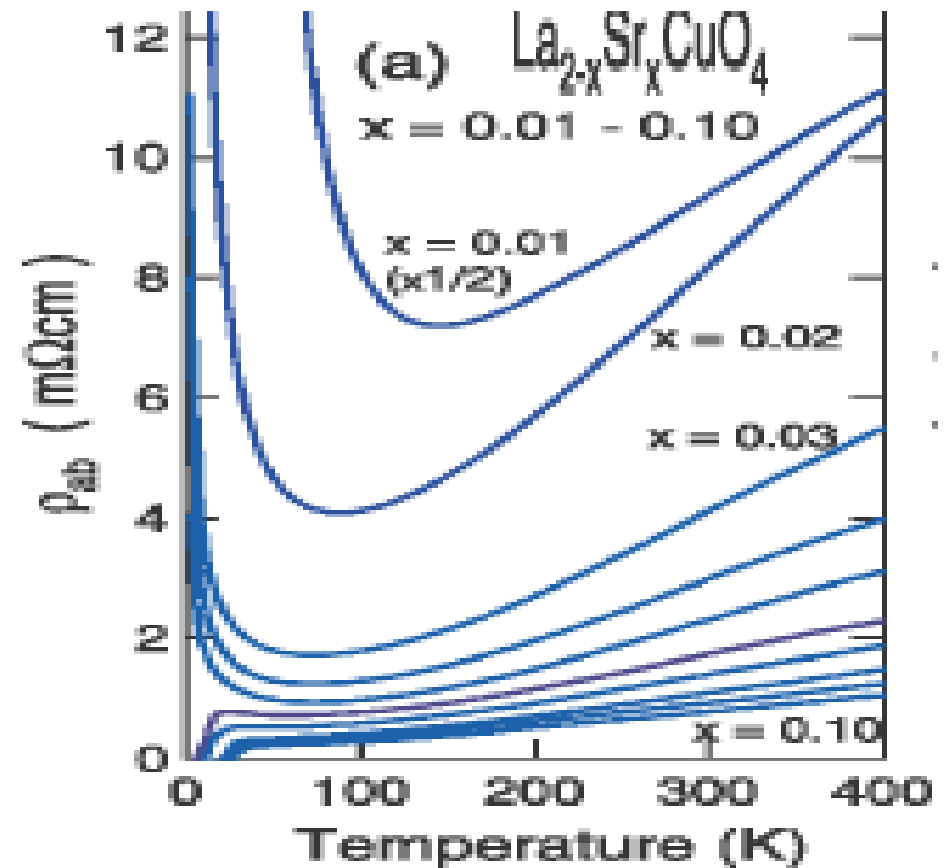
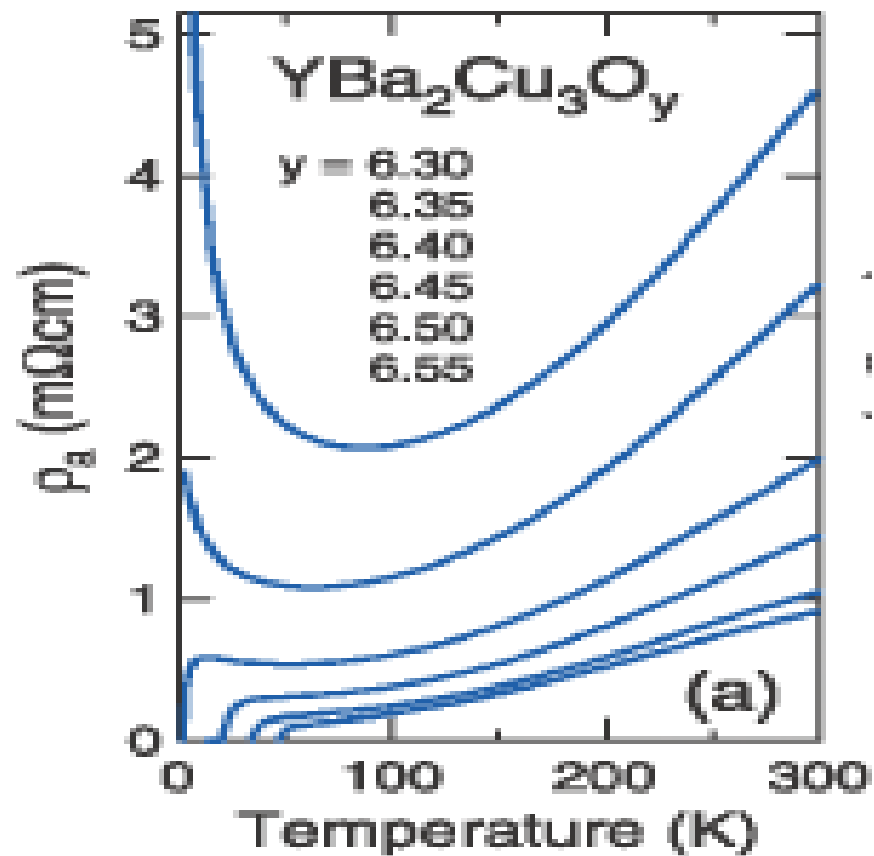


PG =PseudoGap  
 SM=Strange Metal  
 SG="Spin-Glass"  
 FL=Fermi-Liquid like  
 N=Nernst

- red=region considered here
- MODEL:  $t$ - $J$  in 2D  

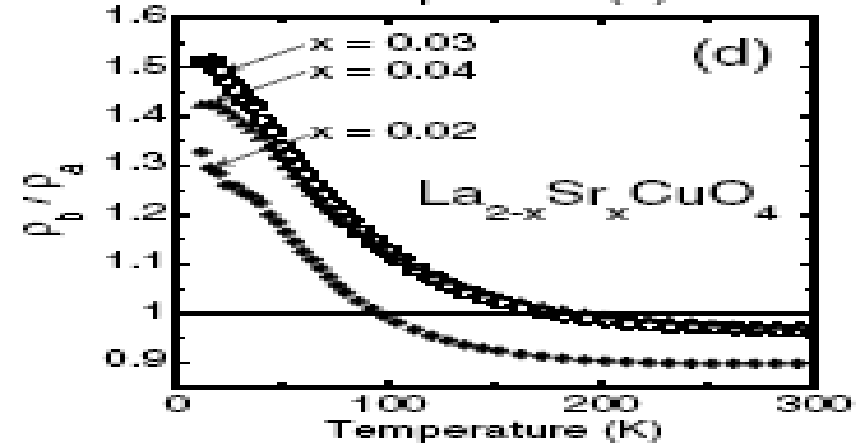
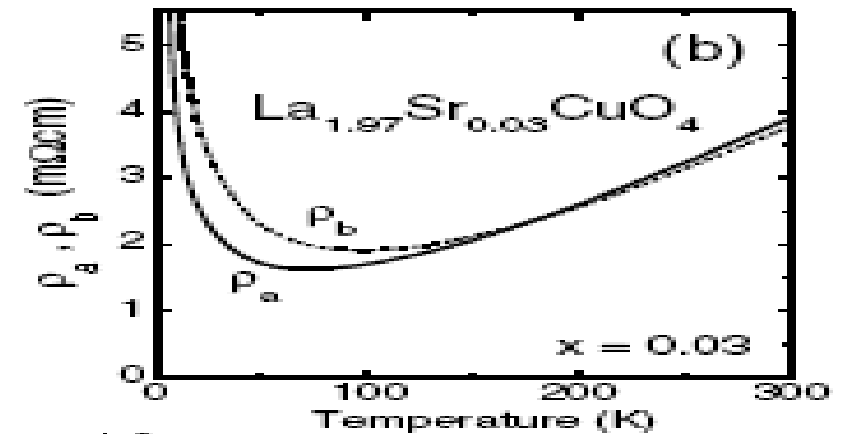
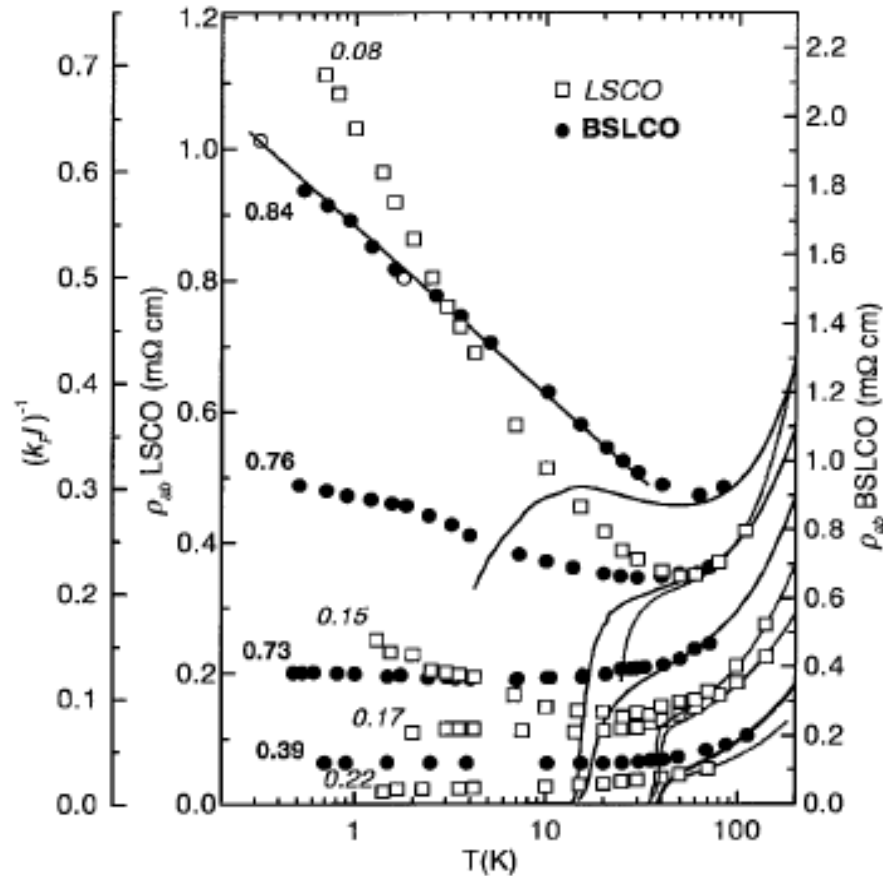
$$H = \sum_{ij} P_G [-t c_i c_j + J S_i S_j + \text{h.c.}] P_G$$
 in square lattice
- $P_G$ =Gutzwiller projection  
 eliminating double occupation

# Metal-insulator crossover (MIC)



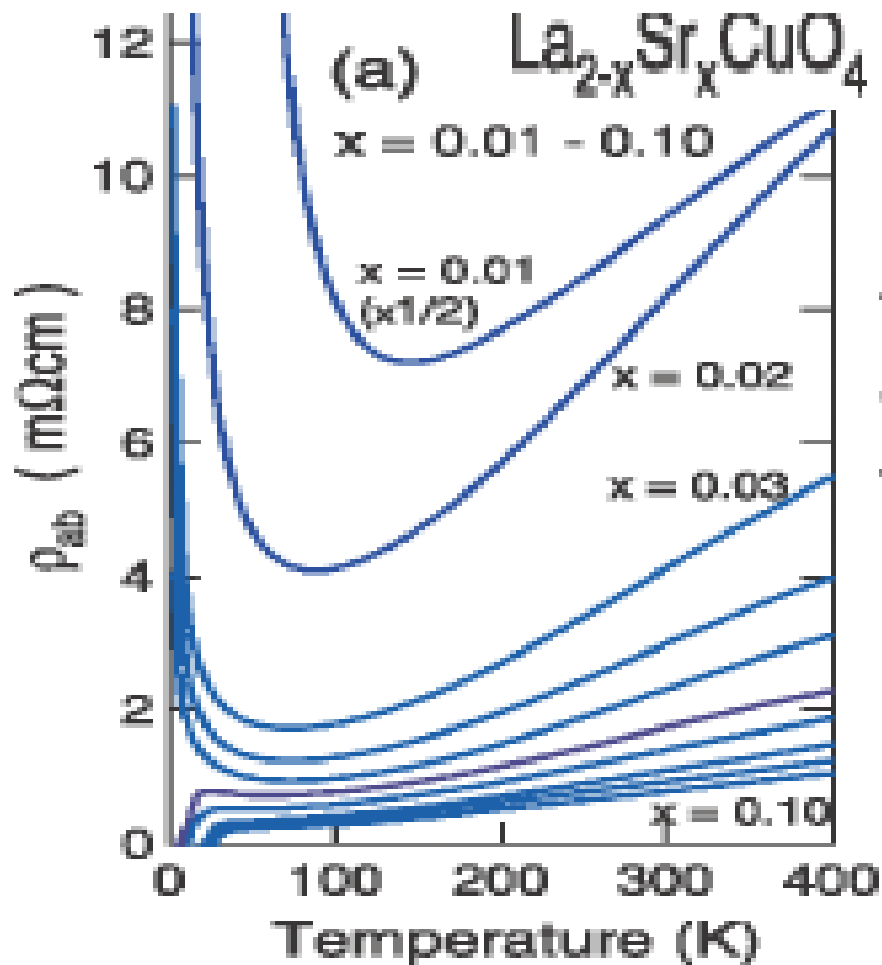
- (Y. Ando et al. PRL 2004)

# MIC not due to disorder localization

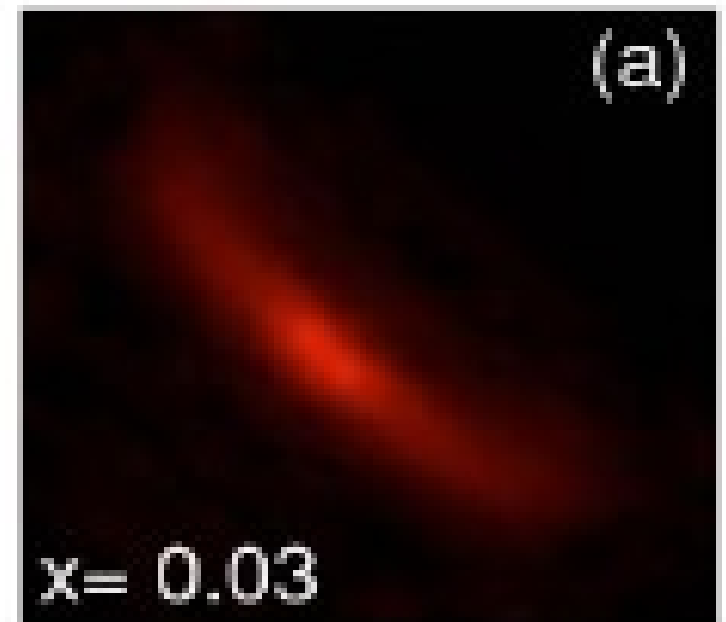


- S. Ono et al PRL 2000 : MIC well above (and below) Ioffe–Regel limit ( $k_F \ell \sim 1$ ) expected for disorder loc.
- Y. Ando et al PRL 2002: different MIC temperatures for a, b directions: incompatible with standard localization, anisotropy too small for 1D structures

# Insulating $\rho$ and Fermi “surface”

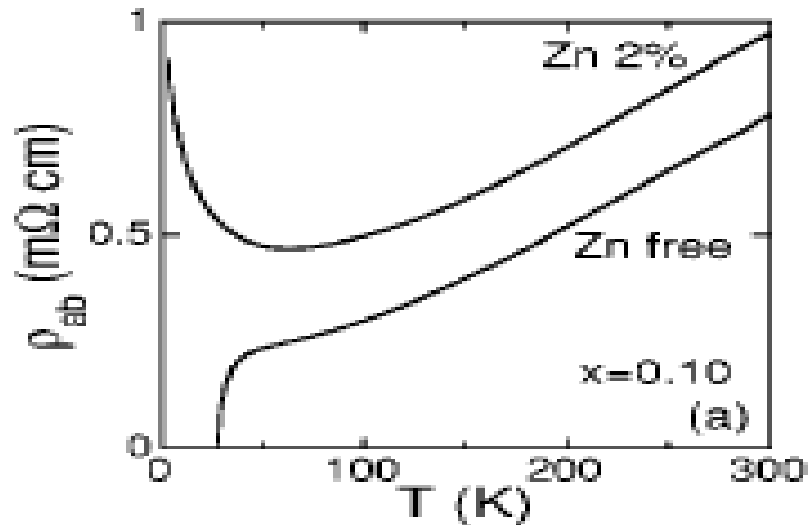


- If MIC is not due to disorder–localization (plausible however at lower  $T$ ), how to reconcile gapless FS “arcs” with insulating resistivity?

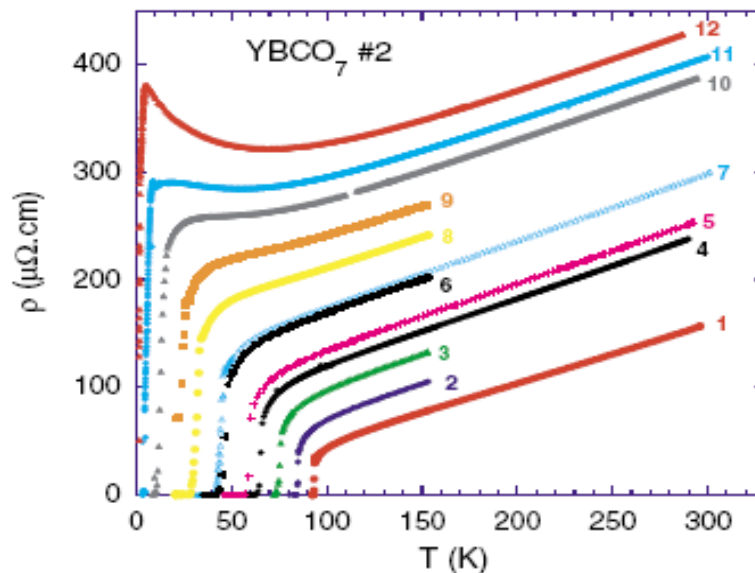


- LSCO ARPES T.Yoshida et al PRB 2006

# Dependence on impurities



- LSCO S. Komiya et al. PRB 2004

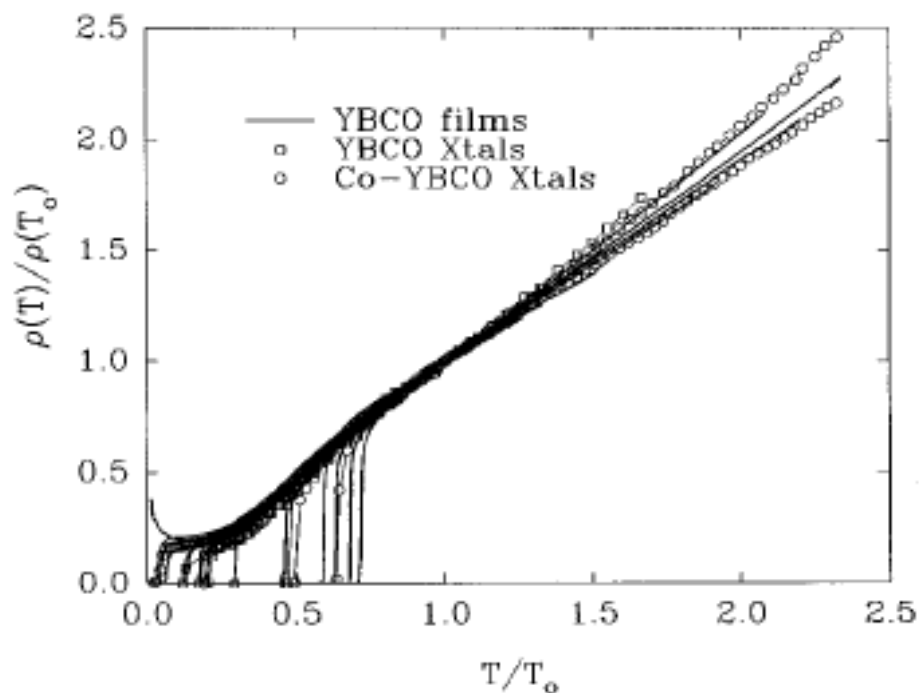


- For magnetic impurities two effects:
- Upward shift with slope unaffected for  $T \gg T_{MIC}$   $\rightarrow$  Matthiessen rule satisfied  $\rightarrow$  FL-like
- Near  $T_{MIC}$  shape affected  $\rightarrow$  Matthiessen rule violated

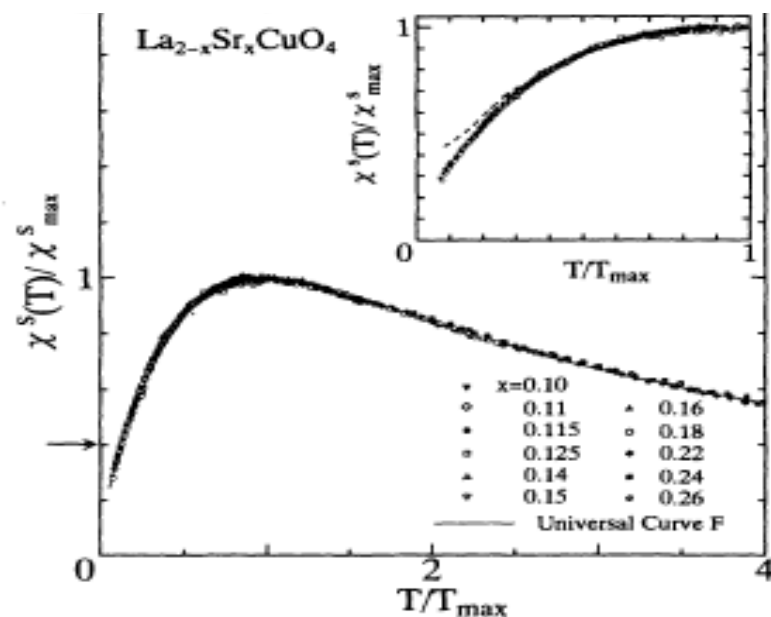
YBCO F. Rullier-Albenque et al PRL 2003



# Universality

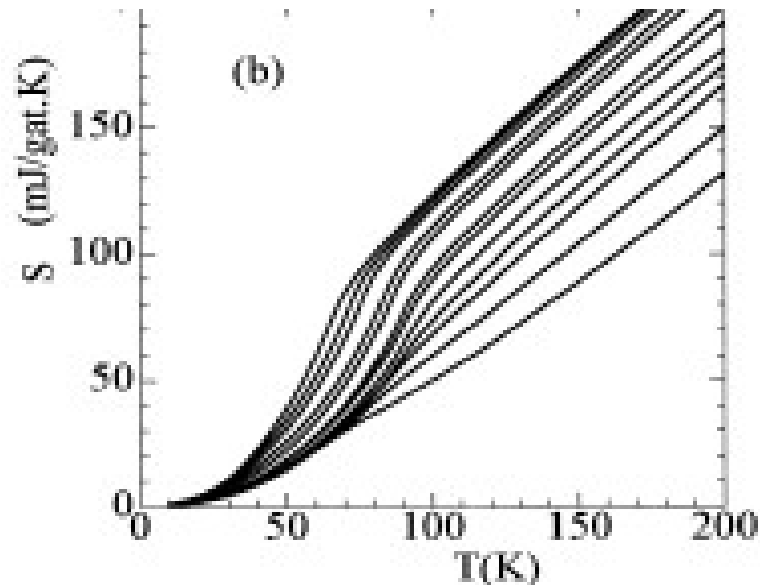


- Universality of resistivity with impurity term removed, in terms of  $T/T^*$  (B. Wuyts et al PRB 1996); where comes from?
- Similar universality for other quantities; in uniform magnetic susceptibility related to spin dof (Heisenberg model at moderate T)

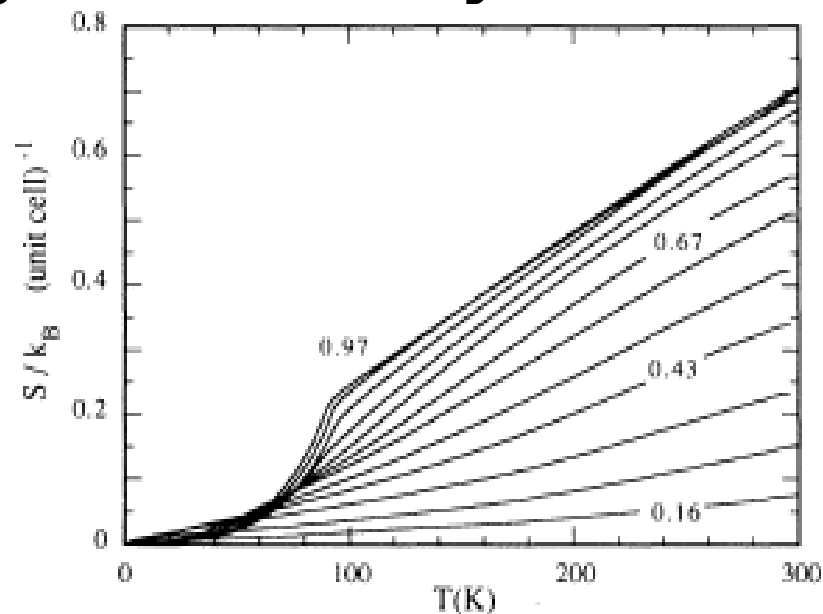


T. Nakano et al PRB 1994

# Electronic entropy



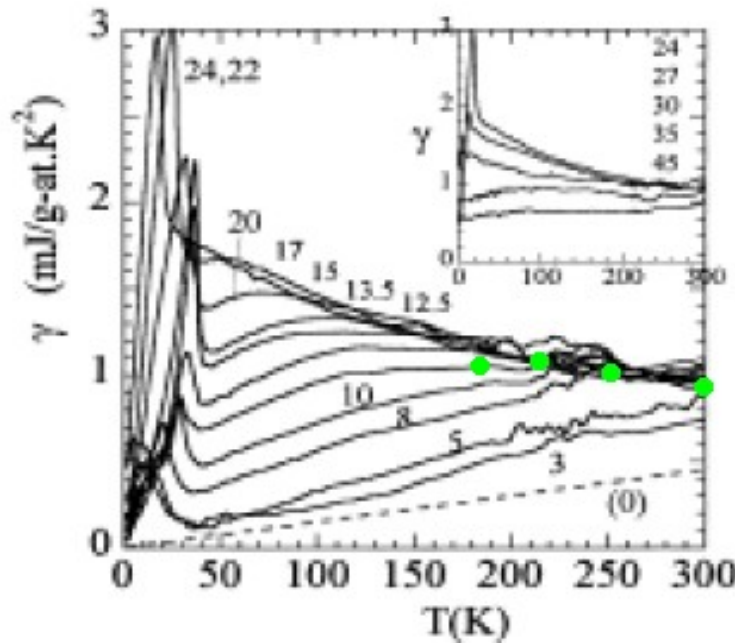
- Y-Pb doped Bi2212:  
JW Loram et al JPCS 2001



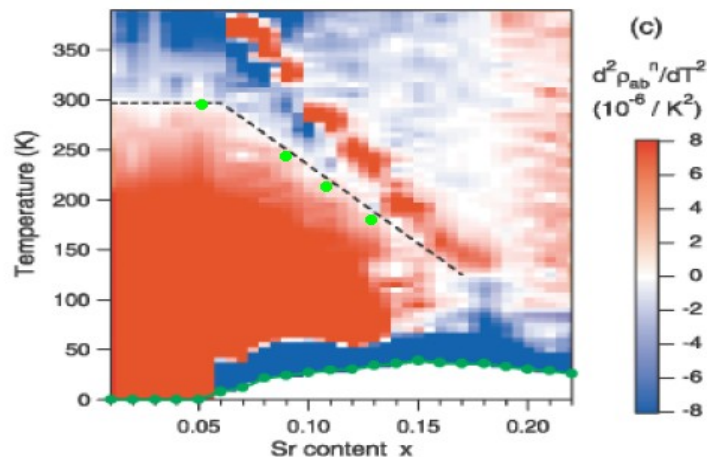
- Linear in  $T$  at moderate temperatures  $\rightarrow$  Fermi liquid-like
- Anomalous feature: negative intercept of extrapolation to  $T=0$  in PG  $\rightarrow$  Negative contribution?
- Slope increasing at low doping then saturating at higher dopings

YBCO : JW Loram et al PRL 1993

# Electronic specific heat coefficient



- Decreasing  $T$  in NS:
- First increase up to  $T^*$  (identified by inflection point of resistivity)
- Then decrease (as for undoped  $\rightarrow$  spin waves) with roughly the same slope at different dopings
- For non super-conducting samples there is an upturn at lowest  $T$

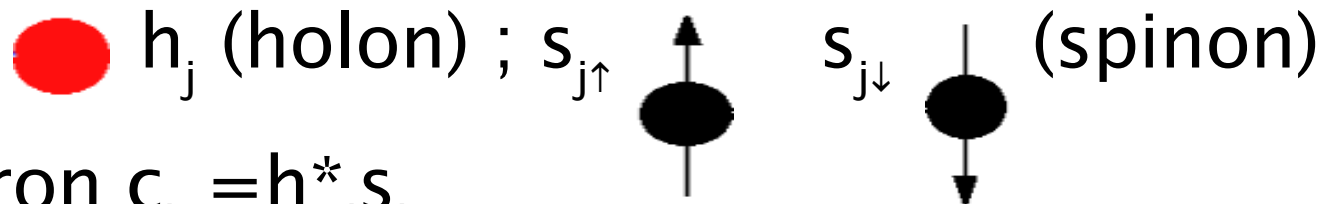


(Y. Ando et al PRL 2004)

# Origin of the gauge “glue”

- We propose an interpretation of the discussed experimental features in terms of an emergent “gauge glue” between spin and charge dof. But where this “glue” comes from?

- $t$ - $J$  : NO-DOUBLE OCCUPATION  $\rightarrow$  Fixed site  $j$   
alternatives: hole; spin up, spin down



- Electron  $c_{j\alpha} = h_j^* s_{j\alpha}$
- Redundant dof (Anderson, Baskaran, Kivelson, ..Lee, Nagaosa...):

$$h_j \rightarrow h_j e^{i\Lambda_j}, s_{j\alpha} \rightarrow s_{j\alpha} e^{i\Lambda_j} \quad e^{i\Lambda_j} = \text{U(1) group element}$$

- EMERGENT LOCAL U(1) GAUGE SYMMETRY(unphysical)

# The gauge field

- It is convenient to introduce emergent U(1) gauge field  $A_\mu$  (not independent, no new dof:  
 $A_\mu \sim s_\alpha^* \partial_\mu s_\alpha + \dots$ ) coupled to  $h$  and  $s$  to explicitly realize local U(1) gauge symmetry
- Gauge fixing  $A_\mu \rightarrow$  elimination of unphysical gauge dof
- $A_\mu$  gauge fixed  $\sim A_\perp$  (vector transverse),  $A_0$  (scalar)
- $A_0$  is purely constraint field: enforces “Gauss law of holon+spinon density conservation”
- The field  $A_\mu$  will provide our “gauge glue” between charge and spin dof of the electron

# Statistics for holons and spinons

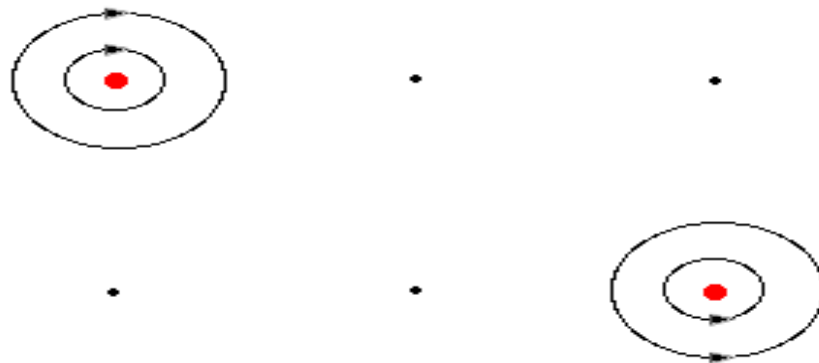
- Slave boson:  $h$  boson,  $s_\alpha$  fermion  $\rightarrow c_\alpha = h^* s_\alpha$   
fermion
- But in 2D more possibilities: one can add fluxes (like in LRO for FQHE) to  $h$  and  $s$  (Chern–Simons)

$$c = \underbrace{e^{-i\Phi_h}}_{\text{U(1) charge dof}} h^* \underbrace{e^{i\Phi_s}}_{\text{SU(2) spin dof}} s$$

with statistical compensation  $e^{-i\Phi_h} e^{i\Phi_s} \sim 1$  ;  
if  $h^* s$  fermion  $\rightarrow c$  fermion

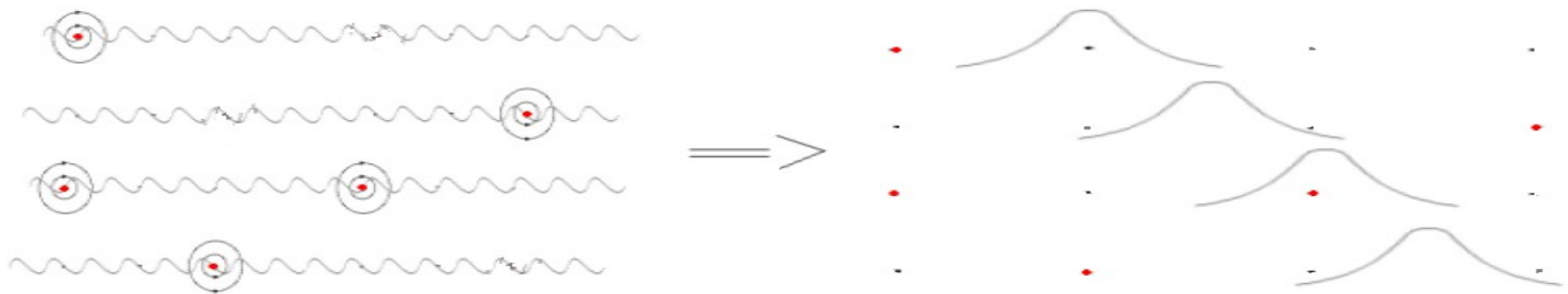
# Optimize spin flux

- Our strategy: choose  $h$  spinless fermion (hence  $s$  boson)  $\rightarrow$  no-double occupation automatically satisfied by Pauli principle; then find optimal “charge” and “spin” fluxes  $\Phi_h$ ,  $\Phi_s$  in Mean Field compatible with statistics
- Effect of optimal spin flux  $\Phi_s$  : to attach a spin vortex to the holon positions (empty sites) with opposite chirality in the two Neel sublattices



# Optimal spin flux $\rightarrow$ Short Range AF

- Gapless spinons ( $\sim$  spin waves) of Heisenberg AF model ( $\rightarrow$  undoped) travelling through gas of spin vortices centered on empty sites (induced by  $\Phi_s$ ) in MF acquire a gap  $m$  ( $\langle \Phi_s^2 \rangle s^* s = m s^* s$ )



- Doping: LongRangeAF  $\rightarrow$  ShortRangeAF (via magnon formation by spinon binding)
- $m \sim J (\delta + |\ln \delta|)^{1/2}$   
density of empty sites      long range tail of vortices
- Neutron experiments in LSCO (B. Keimer et al 1992) AF correlation length  $\xi_{AF} \sim 1/\sqrt{\delta}$



# Optimize charge flux

- Fixed the spin flux, in MF for  $s$   
 $\rightarrow$  MF Hamiltonian for  $h$ :  
 $H \sim \sum_{ij} -t h_i^* h_j e^{i \Phi_h} =$  spinless fermions hopping in 2D square lattice in presence of a “magnetic” flux  $\Phi_h$
- Lieb's theorem: at  $\delta=0$  flux optimizing energy is  $\pi$  per plaquette (peculiar consequence of Pauli principle in 2D bipartite lattices)
- Diamagnetism: for  $\delta$  sufficiently large and finite  $T$  optimal flux per plaquette is 0
- Numerical simulations (using Harper's equation) $\rightarrow$   
for  $T$  sufficiently large only  $\pi$  and 0 flux are optimal

# Optimal charge flux $\rightarrow$ SM-PG crossover

- Conjecture:  $0 \rightarrow \pi$  flux transition in holon MF leads via t-J model to the StrangeMetal  $\rightarrow$  PseudoGap crossover in phase diagram of cuprates ( $T^*$  line viewed as the region of formation of the flux lattice  $\sim$  slave boson flux phase Lee-Nagaosa)
- Dispersion relations for holons ( $\Phi_h \rightarrow$  Haldane-Wu semion statistics : allowed double occupation in k)  
SM  $E \sim -t (\cos k_x + \cos k_y)$   $k_F \sim (1 - \delta)$   $\sim$  band FS  
PG  $E \sim -t (\cos^2 k_x + \cos^2 k_y)^{1/2}$   $k_F \sim \delta$  small FS pockets near  $(\pm\pi/2, \pm\pi/2)$
- Hofstadter phenomenon of Fermi surface reduction for holons (not yet of electrons!)

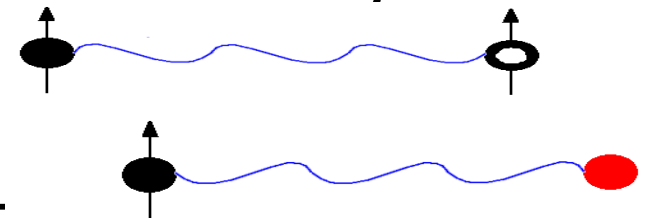
# The “gauge glue” at work

- Inserting gauge fluctuations ( $A_\mu$ ) necessary because physical objects are gauge invariant  $\rightarrow$  Effects from eikonal resummation:
- Finite holon FS  $\rightarrow A_\perp$  transverse is a gapless diffusive mode dominated by Reizer singularity with momentum scale  $Q \sim (T k_F^2)^{1/3}$
- Coupling  $A$  to spinons provides an effective dissipative term correction to the mass :
 

$m \rightarrow (m^2 + i c T k_F)^{1/2}$	$= M + i \Gamma$	PG
$\rightarrow m + i c T k_F Q / m^2$	$= m + i \Gamma$	SM
- $A$  acts at low energy as a “gauge glue” binding
 

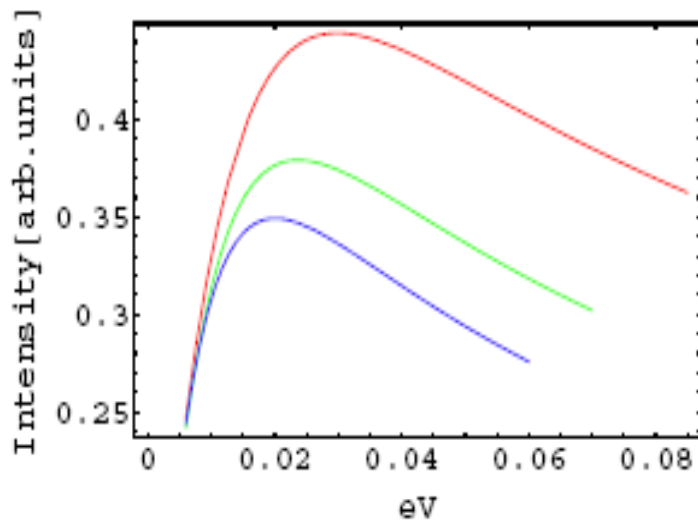
spinon–antispinon	$\rightarrow$ magnon
spinon–holon	$\rightarrow$ electron

  
 resonances with inverse life–time  $\Gamma$

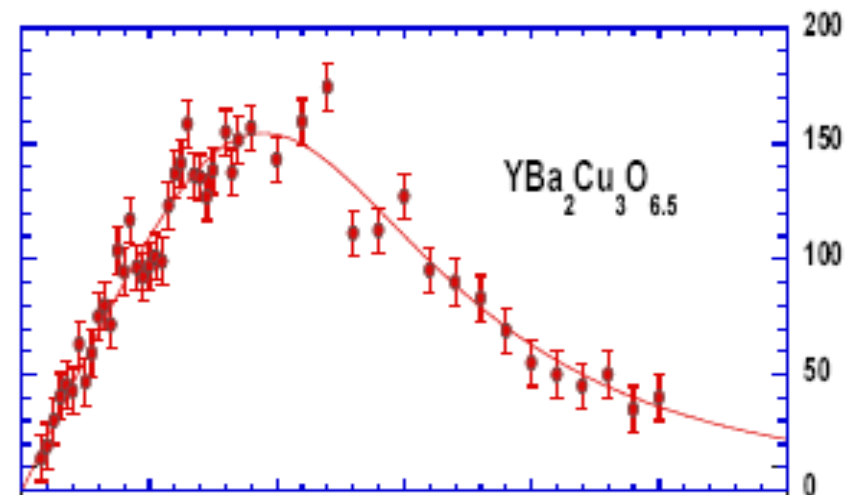


# Effect of gauge glue on magnon: ShortRangeAF but no spin gap

- Dissipative term introduced by  $A_{\perp} \rightarrow$  magnon can move by thermal diffusion at low  $T \rightarrow$  spin fluctuations are not gapped (as m suggests)
- Check: dynamical spin susceptibility at AF point



Normal State, 100 K,  $Q=(\pi,\pi)$

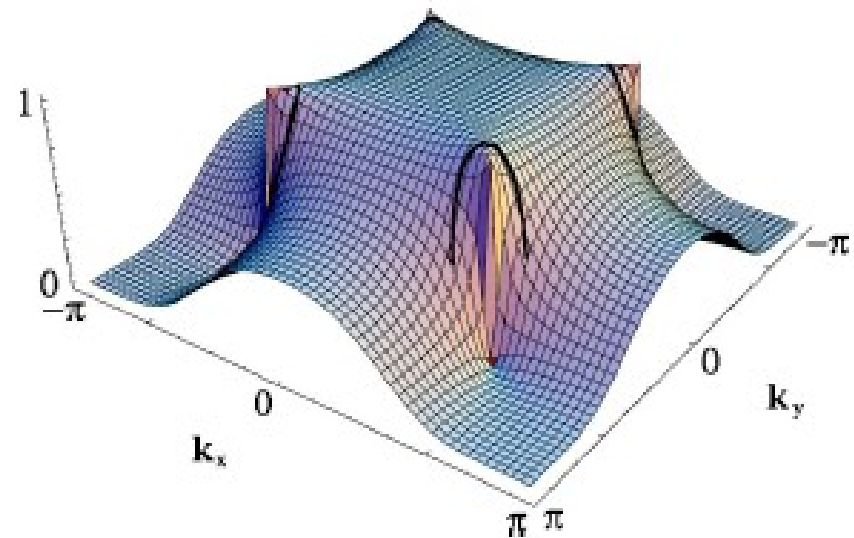


SRAF  $\rightarrow$  diffusion crossover (P.Bourges 1999)  
(from top  $\delta = 0.04, 0.07, 0.1$ )

# Effect of gauge glue on electron: electron Fermi Surface

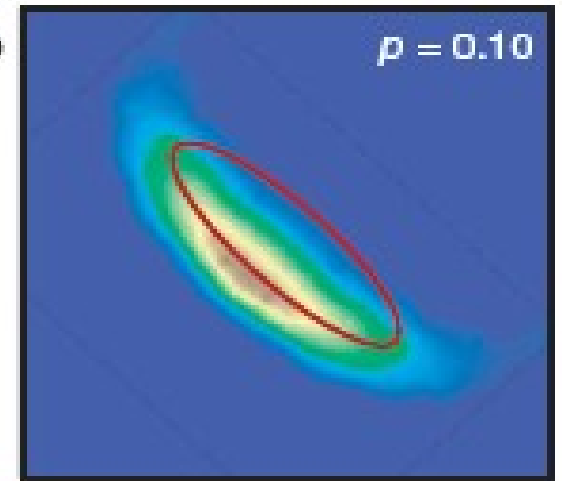
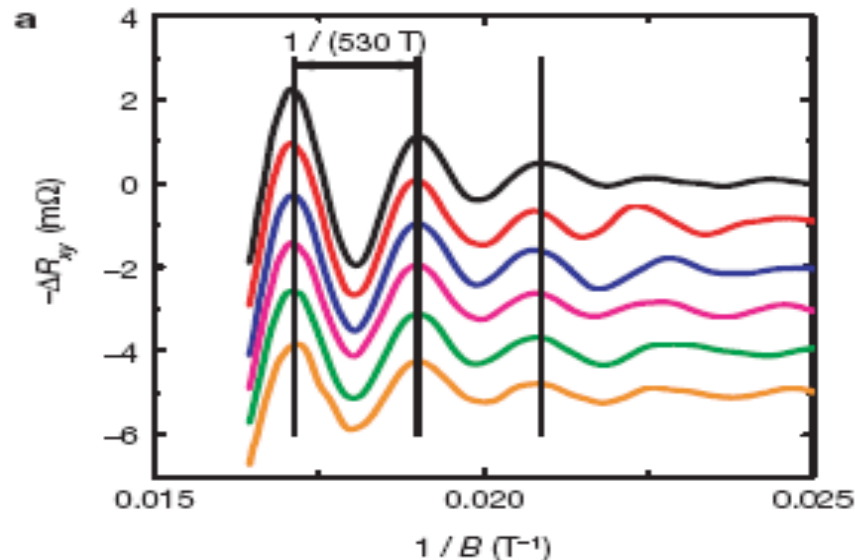
- Momenta near holon FS: binding  $\rightarrow$  electron FS
- SM : holon band FS  $\rightarrow$  electron band FS
- PG : small holon pocket FS  $\rightarrow$  small electron pocket FS near  $(\pm\pi/2, \pm\pi/2)$ , but reconstruction of electron from holon-spinon  $\rightarrow$  reduction of spectral weight outside MBZ (mimicks, but not exactly, Fermi arcs)

Spectral weight in PG  
PA Marchetti et al PRB 2004



# Small Fermi Surface in PG?

- Reduction of spectral weight recovers full lattice translational symmetry for the physical electron even in PG. Recently explained in terms of Projective Gauge Symmetry (XG Wen 2002, R Kaul et al 2007)
- Is this small FS in PG the one of quantum oscillations in YBCO (N. Doiron-Leyraud et al 2007) and Y124 (AF. Bangura et al , E A Yelland et al 2007) ?



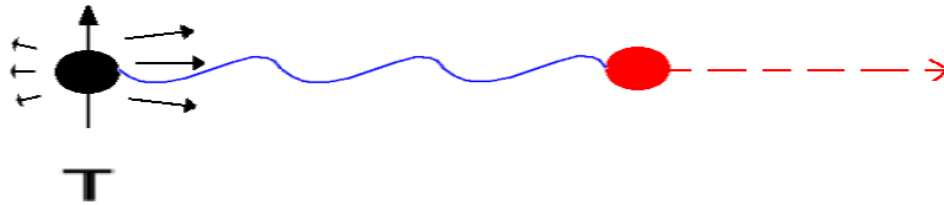
N Doiron-Leyraud et al Nature 2007

# Gauge interpretation of MIC

- Ingredients: spinon is massive but can thermally diffuse, holon has FS, physical electron is gauge invariant and there is a gauge string between spinon and holon
- The electromagnetic field is coupled to the physical electron  $\rightarrow$  to move the electron one should move the spinon and the (gapless) holon, but due to its gap the spinon can move only by thermal diffusion  $\rightarrow$  solves the puzzle of FS coexisting with insulating resistivity
- Calculation of resistivity via Kubo formula and Ioffe-Larkin rule ( $\leftarrow$  gauge invariance)

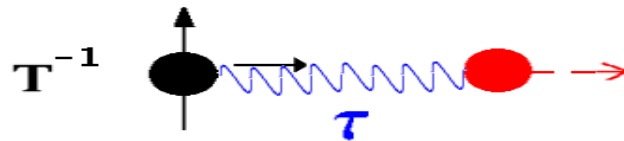
# MIC: the basic gauge mechanism

- $T < \text{MIC}$  SRAF dominates, basic time scale is set by spinon diffusion  $\sim T$  conductivity  $\sigma \propto T$



Possible evidence for a SRAF origin of MIC (muon spin rotation JE Sonier et al PRB 2007)

- $T > \text{MIC}$  basic time scale is scattering time against gauge fluctuations  $\sim$  Drude-like  $\sigma \sim \tau \sim T^{-\alpha} \propto T^{-1}$  as  $T \propto$

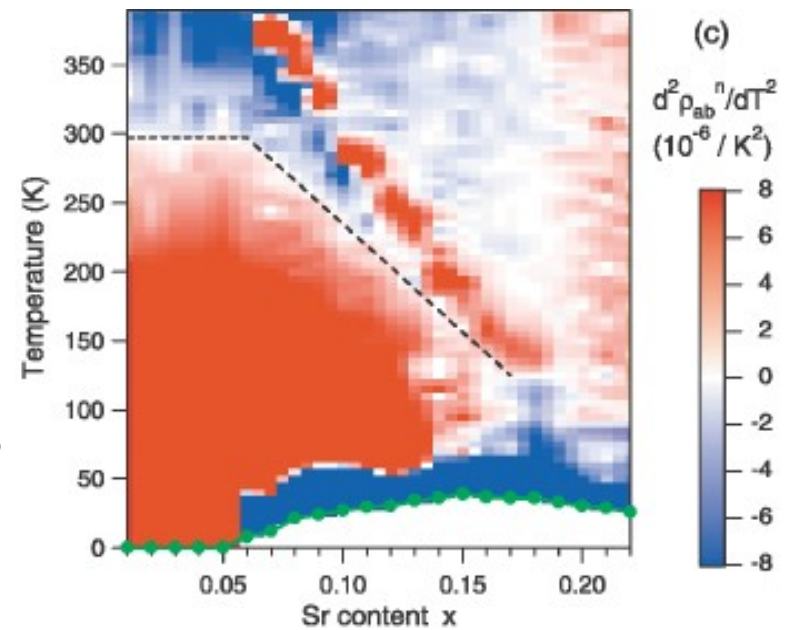


- Limit of eikonal treatment: at  $T \lesssim 50-20\text{K}$  gauge interaction too weak  $\rightarrow$  SG?



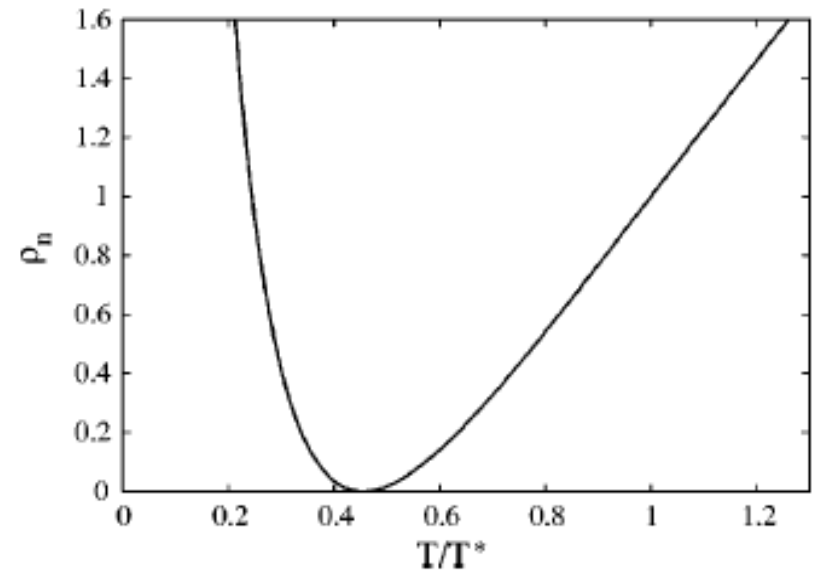
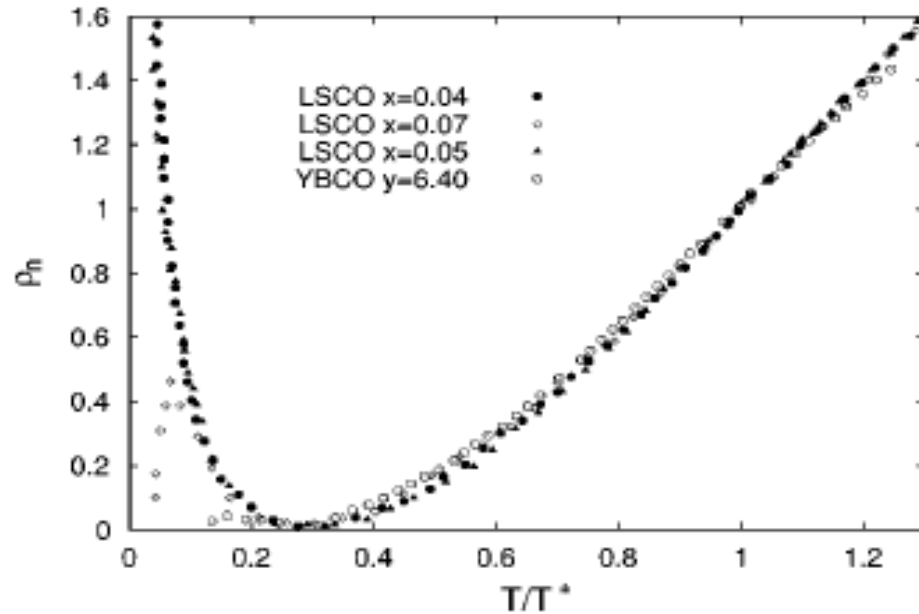
# Gauge MIC : features

- Coexistence with gapless FS: Even if  $\rho$  is insulating for  $T < \text{MIC}$  holon still has FS  $\rightarrow$  electron has FS “arcs” (due to spectral weight reduction)
- Impurities:  $\rho$  is dominated by spinons  $\rightarrow$  most sensitive to magnetic impurities; non-magnetic ones act on Fermi liquid holons, thus obeying Matthiessen rule
- PG: inflection point of  $\rho$  due to square root in  $\Gamma$   
 $\sim \text{Im} (m^2 + i c T k_F)^{1/2}$
- SM: no square root in  $\Gamma \sim T^{4/3}$   
 $\rightarrow$  no inflection point ;  
 $m < \Gamma \rightarrow$  Drude-like behaviour  $\sigma \sim \tau \sim T^{-1} \rightarrow$  resistivity linear in  $T$



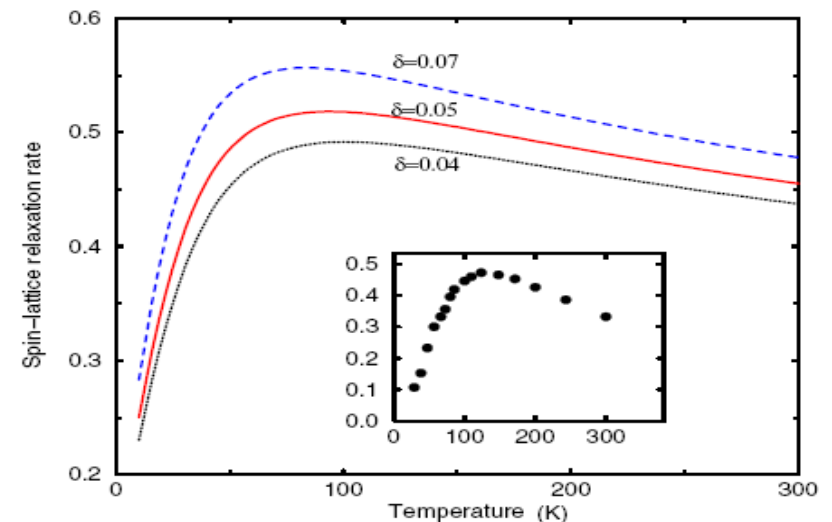
# Gauge MIC: universality

- Dominating spinon ( $\rightarrow$  spin) contribution  
 $\rho \sim c_1(\delta) \rho_n(T/T^*) + c_2(\delta)$ ,  $T^* = \text{inflection point} \sim m^2 / k_F$ ,  
 $\rho_n$  universal function



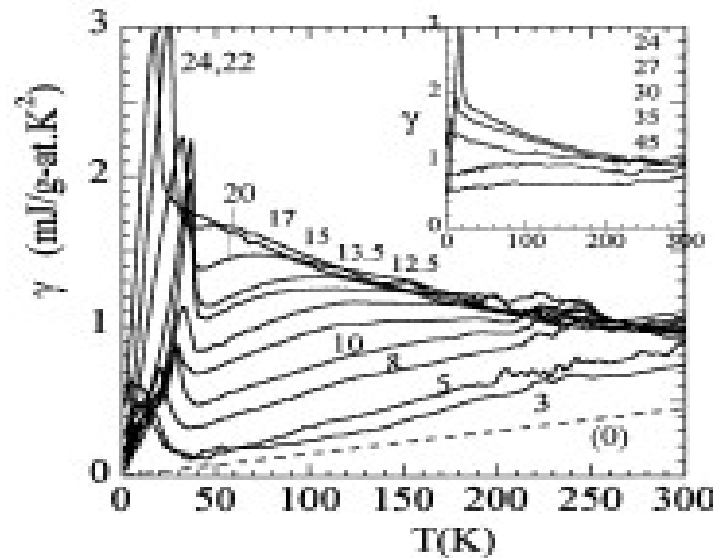
- Crossover SRAF  $\rightarrow$  diffusion,  
 same e.g. in  $^{63}(1/T_1 T)$

Marchetti et al JPCM 2000 ,  
 PRB 2004



# Gauge interpretation of specific heat

- Contributions: massive spinons (irrelevant at low T), Fermi liquid holons with  $k_F \sim \delta$  in PG,  $\sim 1 - \delta$  in SM, transverse gauge acting as gapless diffusive mode (Reizer), scalar gauge acting as “constraint field”



→

upturn

due to  $A_{\perp}$ ,  $\gamma \sim T^{-1/3}$

PG:  $k_F \sim \delta \rightarrow$  increase

SM:

$k_F \sim 1 - \delta \rightarrow$  saturation

decrease in T

due to  $A_{\perp}$ ,  $\gamma \sim T^{-1/3}$

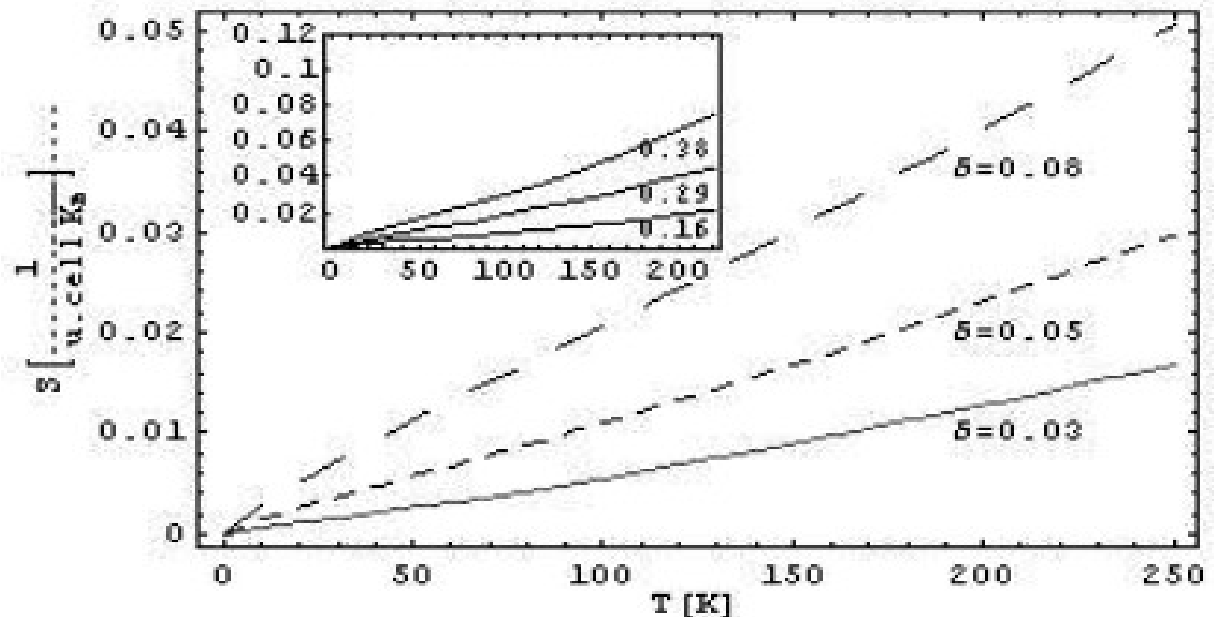
- $A_{\perp}$  enhances  $\gamma$  at low T in PG and then in SM

# Gauge interpretation of entropy

- $A_0$  acts constraining  $\rightarrow$  reduces dof  $\rightarrow$  negative contribution to entropy (also Hlubina et al 1992 in slave boson)

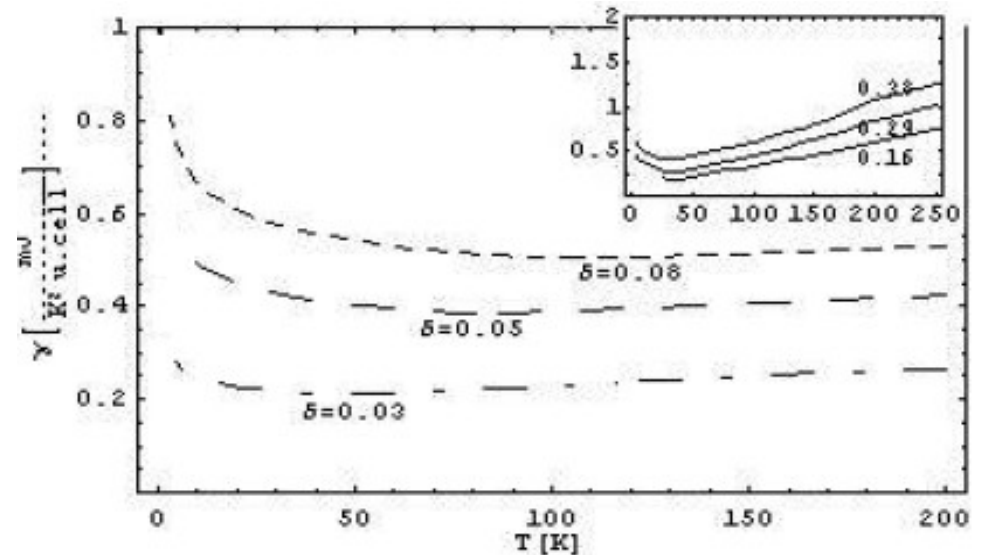
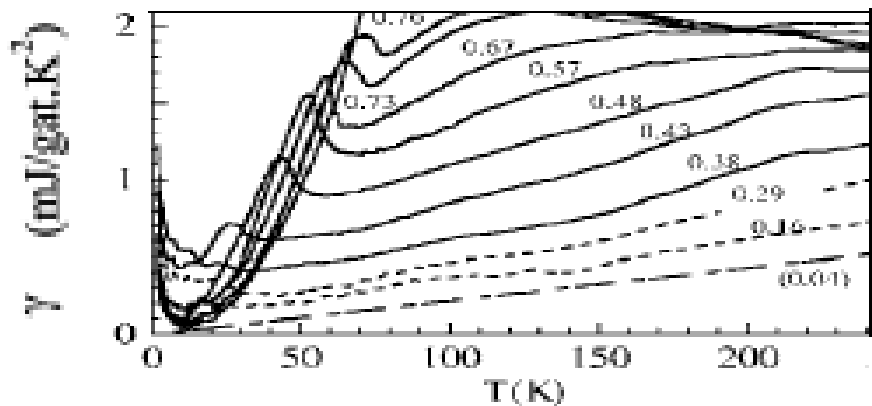
$$S \sim -T / v_F + c T^2 / v_F^2$$

- First term: constraint  $\rightarrow$  negative intercept of entropy at  $T=0$  in PG because dominating at moderate  $T$



# Gauge interpretation of rising $\gamma$

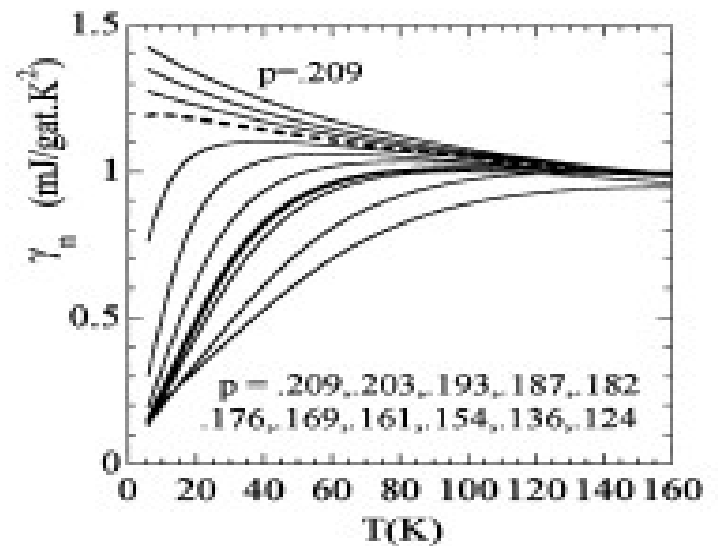
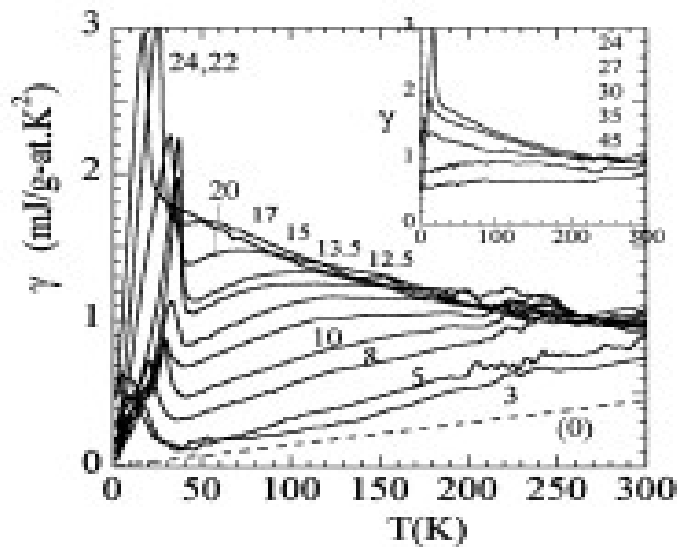
- $S \sim -T / v_F + c T^2 / v_F^2$  , second term:  
thermally relaxing constraint  $\rightarrow$  linear T rising in  $\gamma$  with doping -independent slope
- replaces the spin-wave contribution of undoped AF, removed by spinon gap



- YBCO JW Loram et al  
JPCS 2001

# Not Just DOS increase with T

- Presumably effect of linear rising in T of  $\gamma$  is enhanced by increasing DOS with T, due to destruction of holon-holon pairing (not taken into account here), but this cannot be the only source: curves in  $\gamma$  are approximately parallel not converging near  $T \sim 0$  as for DOS increase.



DOS increase-extrapolation  
JW Loram et al JPCS 2001

# Conclusions

- Many detailed features both of Metal–Insulator Crossover and of entropy in the normal state of hole–doped cuprates can be interpreted in terms of a composite structure of the electron given by a massive spinon bound to a gapless fermionic holon by a string of emergent “gauge glue”.
- Gauge fluctuations allow Short Range AF without spin gap
- Many crossovers in PG are attributed to a competition between SRAF and thermal diffusion induced by the gauge string
- The crossover PG–SM could be explained as a change of holon FS inducing one on the electron FS (perhaps partially masked by pairing effects)