Nucleon Pair Approximation to the nuclear Shell Model

Yu-Min Zhao
(Speaker: Yi-Yuan Cheng)

2\textsuperscript{nd} International Workshop & 12\textsuperscript{th} RIBF Discussion on Neutron-Proton Correlations, Hong Kong
July 6-9, 2015
Outline

- A brief introduction of the NPA
- Validity and application of the NPA (nn & pp pairs)
- The NPA with isospin symmetry
  
  Spin-aligned isoscalar pair correlation in \(^{96}\text{Cd}\)
  
  \(T=0\) states of 8 nucleons in the \(j=9/2\) shell
  
  Pair condensation in g.s. of even-even \(N=Z\) nuclei

- Summary & future and perspective
Shell-model method

Projected shell model

Mean-field approximations

Nucleon-pair approximation

Coherent states method

Algebraic Models (IBM)
The NPA of the shell model

shell model basis:

\[ |\varphi\rangle = \left[ \left( C_{j_1}^+ C_{j_2}^+ \right)_{J_1} C_{j_3}^+ \ldots C_{j_n}^+ \right]_{J_n M_n} |0\rangle. \]

nucleon pair:

\[ A^r = \sum_{ab} y(ab) A^r(ab), \quad A^r(ab) = \left( C_a^+ \times C_b^+ \right)^r \]

pair basis:

\[ |\varphi\rangle = \left[ \left( A^{(r_1)} + A^{(r_2)} + \ldots A^{(r_{n/2})} \right)^{J_2} A^{(r_3)} \ldots A^{(r_{n/2})} \right]_{M_{n/2}}^{(J_{n/2})} |0\rangle. \]

If all kinds of nucleon pairs are considered, the NPA space = the SM space.
If a few important pairs are considered, the NPA space is very small.

The same Hamiltonian and transition operators as in the SM.
The NPA of the shell model

- The key technique of the NPA is the Wick theorem of coupled nucleon pairs developed by Chen.

  Recursive formulas for calculating the matrix elements

- This method was refined by Zhao et. al., and calculations for odd-system become practical.

- The generalized version of the NPA with isospin symmetry has been recently presented by Fu et. al.

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Validity: semi-magic nucleus $^{45}$Ca

the GXPF1 effective interaction

1. $pf$ shell model space

2. NPA space

$SDf_{7/2}, SDGf_{7/2}, SDI_{7/2}, SDGI_{7/2}$

$SD$ pairs: $J=0,2$ pairs

$G$ pair: $J=4$ pair

$I$ pair: $J=6$ pair

odd nucleon confined on $f_{7/2}$ orbit

<table>
<thead>
<tr>
<th>Dimension of different model spaces</th>
</tr>
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<tbody>
<tr>
<td>SM</td>
</tr>
<tr>
<td>----</td>
</tr>
<tr>
<td>$^{45}$Ca</td>
</tr>
<tr>
<td>$1/2^-$</td>
</tr>
<tr>
<td>$3/2^-$</td>
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<tr>
<td>$5/2^-$</td>
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<td>$7/2^-$</td>
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<td>$17/2^-$</td>
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<tr>
<td>$19/2^-$</td>
</tr>
<tr>
<td>$21/2^-$</td>
</tr>
</tbody>
</table>

Validity: semi-magic nucleus $^{45}\text{Ca}$

Overlap integrals between the NPA WF and the SM WF

<table>
<thead>
<tr>
<th></th>
<th>$SDf_{7/2}$</th>
<th>$SDGf_{7/2}$</th>
<th>$SDf_{7/2}$</th>
<th>$SDGf_{7/2}$</th>
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<tbody>
<tr>
<td>$^{45}\text{Ca}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$7/2^+_1$</td>
<td>0.990</td>
<td>0.997</td>
<td>0.991</td>
<td>0.999</td>
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<tr>
<td>$5/2^-_1$</td>
<td>0.975</td>
<td>0.986</td>
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<td>$3/2^-_1$</td>
<td>0.984</td>
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<td>0.995</td>
<td>0.997</td>
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<td>0.996</td>
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<tr>
<td>$1/2^-_1$</td>
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<td>0.935</td>
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<tr>
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<tr>
<td>$17/2^-_1$</td>
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<td>0.982</td>
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<tr>
<td>$3/2^-_2$</td>
<td>0.566</td>
<td>0.847</td>
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<td>$5/2^-_2$</td>
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<td>$7/2^-_2$</td>
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<td>$11/2^-_2$</td>
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<tr>
<td>$19/2^-_2$</td>
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<td>0.870</td>
<td>0.540</td>
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Validity: semi-magic nuclei $^{45}$Ca

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<th>$-{\frac{17}{2}}$</th>
<th>$-{\frac{13}{2}}$</th>
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<td>$-{\frac{17}{2}}$</td>
<td>$-{\frac{17}{2}}$</td>
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<tr>
<td>$S D I f_{r/2}$</td>
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</table>

$B(E2, \leftarrow)$

![Graph showing $B(E2, \leftarrow)$ for $^{45}$Ca]
Applications of the NPA

- Backbending in yrast states of even–even nuclei (Yoshinaga, Lei et al.)
- Negative parity levels of even–even nuclei (Lei et al.)
- Chiral bands in odd–odd nuclei (Yoshinaga et al.)
- B(E2) and g factors of even–even Tin isotopes (Jiang et al.)
- Shape phase transitions in the NPA (Luo, Lei et al.)
- Beta decay within the NPA (Lu et al.)
- Nucleon-pair approximation with random interactions (Zhao et al.)
Outline

- A brief introduction of the NPA
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- Summary & future and perspective
Proton-neutron pair correlation

A proton and a neutron are the same kind of nucleons, with different $z$-components of isospin $T = 1/2$, $T_z = +1/2$ for

$T = 1/2$, $T_z = -1/2$ for

Analogizing singlet and triplet states in spin space, one have

$T = 1, T_z = 1$
$T = 1, T_z = -1$ → isospin triplet states

$T = 1, T_z = 0$

$T = 0, T_z = 0$ → isospin singlet state

Nuclear force is charge independent

The Coulomb interaction doesn’t play an important role in low-lying states
Proton-neutron pair correlation

--attractive and short range

\[ V_{12}(\delta) = -V_0 \delta (r_1 - r_2) = \frac{-V_0}{r_1 - r_2} \delta (\cos \theta_1 - \cos \theta_2) \delta (\Phi_1 - \Phi_2) \]

\[ T = 1 \]

\[ j = 9/2 \]

R. F. Casten, “Nuclear structure from a simple perspective”
Evidence for a spin-aligned neutron–proton paired phase from the level structure of $^{92}$Pd


$^{92}$Pd: a system of four-proton holes and four-neutron holes below $^{100}$Sn
Single-\textit{j} shell-model calculation

\begin{center}
\textit{Spin-aligned neutron-proton pair mode in atomic nuclei}
\end{center}

C. Qi, J. Blomqvist, T. Bäck, B. Cederwall, A. Johnson, R. J. Liotta, and R. Wyss
Department of Physics, Royal Institute of Technology, SE-10691 Stockholm, Sweden
(Received 20 January 2011; revised manuscript received 28 June 2011; published 10 August 2011)

IBM calculation

\begin{center}
\textit{Spin-aligned neutron-proton pairs in} \( N = Z \) \textit{nuclei}
\end{center}

S. Zerguine\textsuperscript{1} and P. Van Isacker\textsuperscript{2}
Many-\(j\) shells calculation within the NPA

PHYSICAL REVIEW C 87, 044312 (2013)
Spin-aligned isoscalar pair correlation in \(^{96}\)Cd, \(^{94}\)Ag, and \(^{92}\)Pd
G. J. Fu,\(^1\) J. J. Shen,\(^1\) Y. M. Zhao,\(^1,\,*\) and A. Arima\(^1,\,2\)

- perform calculations in the \textit{pfg} shell, with the JUN45 interaction
- study pair approximations:
  - \(S\) pair: \(J=0, T=1\)
  - \(D\) pair: \(J=2, T=1\)
  - \(K\) pair: \(J=8, T=1\)
  - \(A^{(9)}\) pair: \(J=9, T=0\)
$^{96}\text{Cd}: \text{the lowest } T=0 \text{ states}$

- $S \text{ pair: } J=0, T=1$
- $D \text{ pair: } J=2, T=1$
- $K \text{ pair: } J=8, T=1$
- $A^{(9)} \text{ pair: } J=9, T=0$

Dual description: Non-orthogonality feature of nucleon-pair basis!
$^{96}$Cd: the lowest $T=0$ states

The lowest $8^+$ state cannot be well described by the $A^{(9)}$ pair. It can be described by the seniority two configuration.

Exclusive pair basis!
\[ T=0 \text{ states of 8 nucleons in the } j=9/2 \text{ shell} \]

Truncated space:

The lowest seniority scheme (LS)
\[ |\Psi_{\text{LS}}(I)\rangle = \left( ((S^\dagger \times S^\dagger)^{(0,0)} \times S^\dagger)^{(0,1)} \times A^{(I,1)^\dagger}\right)^{(I,0)} |0\rangle \]

The spin-aligned isoscalar pair approximation (SA)
\[ |\Psi_{\text{SA}}\rangle = \left( ((A^{(9,0)^\dagger} \times A^{(9,0)^\dagger})^{(J_2,0)} \times A^{(9,0)^\dagger})^{(J_3,0)} \times A^{(9,0)^\dagger}\right)^{(I,0)} |0\rangle \]

The isoscalar \( P \) \((J=1)\) pairs (SO1)
\[ |\Psi_{\text{SO1}}\rangle = \left( ((P^\dagger \times P^\dagger)^{(J_2,0)} \times P^\dagger)^{(J_3,0)} \times P^\dagger\right)^{(I,0)} |0\rangle \]

Three \( P \) pairs plus another isoscalar pair (SO2)
\[ |\Psi_{\text{SO2}}\rangle = \left( ((P^\dagger \times P^\dagger)^{(J_2,0)} \times P^\dagger)^{(J_3,0)} \times A^{(J,0)^\dagger}\right)^{(I,0)} |0\rangle \]

Schematic interaction:

\[ H(a,b) = (1 - a - b)V_{J=0} + aV_{J=2j} + bV_{J=1}, \]

where \( a \) and \( b \) are adjustable parameters ranging 0–1, and

\[ V_J = - \sum_{M=-J}^{M} \sum_{\tau=-T}^{T} A_{M\tau}^{(JT)} A_{M\tau}^{(JT)}, \]

\[ A_{M\tau}^{(JT)} = \frac{1}{\sqrt{2}} (a_j^{(JT)} \times a_j^{(JT)})_{M\tau}, \]
$T=0$ states of 8 nucleons in the $j=9/2$ shell: schematic interaction

Squared overlap in the limit

$H(a,b) = (1-a-b)V_{J=0} + aV_{J=2} + bV_{J=1}$

$L_S$ Lowest seniority

Isoscalar spin-aligned pairs

Isoscalar spin-one pairs

$J=0, 2 \approx 0.6$

$J=4 \approx 1$

Three spin-one pairs

& another isoscalar pair
**T=0 states of 8 nucleons in the j=9/2 shell : schematic interaction**

\[ H(a,b) = (1 - a - b)V_{J=0} + aV_{J=2j} + bV_{J=1} \]

**SM space**

(b) \( B(E2; I \rightarrow I-2) \) (W.u.)

(c) \( Q \) (e fm\(^2\))

Different patterns depending on Hamiltonian (or we say pair-truncated scheme)
Ten sets of effective interactions

- **LS**: good for $0^+$, $2^+$
- **SO2**: not so good
- **SA**: remarkably good for $0^+$, $2^+$, $4^+$
$T=0$ states of 8 nucleons in the $j=9/2$ shell: realistic interaction
Pair condensations in g.s. of even-even $N = Z$ nuclei

$^{20}$Ne, $^{24}$Mg, $^{32}$S, $^{36}$Ar : USDB
$^{44}$Ti, $^{48}$Cr : GXPF1
$^{60}$Zn, $^{64}$Ge, $^{92}$Pd, $^{96}$Cd : JUN45

Semi-magic:
$S$-pair condensation WF remarkably good

$N=Z$ nuclei:
$S$-pair: reasonably good
$P$-pair: not good as $S$-pair
$SA$-pair: reasonably good for 2p2n nuclei ($^{44}$Ti, $^{96}$Cd).

not good for 4p4n nuclei ($^{48}$Cr, $^{92}$Pd), due to the contribution from orbits other than $f_{7/2}$ ($^{48}$Cr) and $g_{9/2}$ ($^{92}$Pd) in SM WF.

Pair condensations in g.s. of even-even $N = Z$ nuclei

\[
\epsilon_{nlj}' = \epsilon_{nl} - \chi_{SO} V_{SO} \frac{j(j+1) - l(l+1) - 3/4}{2},
\]
\[
\epsilon_{nl} = \frac{2\epsilon_{nl} - \epsilon_{n\ell\ell}}{2l+1},
\]
\[
V_{SO} = \frac{l\epsilon_{n\ell\ell} + (l+1)\epsilon_{n\ell\ell}}{2l+1}.
\]

0.5 < $\chi_{SO}$ < 1.0,
P-pair condensation is suppressed

$\chi_{SO}$ = 0,
P-pair condensation is better

$\chi_{SO}$ > 0.5,
S-pair condensation is better
Summary

- Many-\(j\) shell calculation confirms the importance of the spin-aligned isoscalar pair on the \(1g_{9/2}\) orbit in \(^{96}\text{Cd}\).
- The traditional isovector \(SD\) pairs also provide a good description for \(0^+\) and \(2^+\). The dual description is due to the non-orthogonality feature of the pair basis.

- The \(S/P/\)spin-aligned pair is very important if the \(J=0/J=1/J=2j\) pairing interaction is relatively very strong, with an exception: \(P\) pair condensation does not exist for eight nucleons under the \(J=1\) pairing interaction.
- The single-\(j\) shell calculation with the realistic interactions shows that the spin-aligned pair approximation is very good.

- The \(S\) pair condensation is extremely dominant in g.s. of semi-magic even-even nuclei, and reasonably relevant for \(N=Z\) nuclei. The \(P\) pair condensation is not as good as the \(S\) pair condensation. Spin-aligned pairs are important in g.s. of \(^{96}\text{Cd}\) and \(^{44}\text{Ti}\), but irrelevant for other nuclei.
- The effect of the spin-orbit potential is discussed.
Future and perspective

- Description of heavy nuclei
- The interplay between the $T = 1$ and $T = 0$ pairing interactions
- Quartet (or alternatively like-$\alpha$) correlations in self-conjugate nuclei
- Multiple particle–hole excitations across closed shells
- The effect of unbound states in the low-energy structure of exotic nuclei

Nucleon-pair approximation to the nuclear shell model

Y.M. Zhao$^{a,*}$, A. Arima$^{a,b}$

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Thanks for your attentions!