Aligned neutron-proton pairs in $N=Z$ nuclei

P. Van Isacker, GANIL, France

Motivation
Shell-model analysis
A model with high-spin bosons
Experimental tests
Spin-aligned $T=0$ np pairs

Motivation: A simple description of the $N=Z$ nuclei $^{98}_{\text{In}}, ^{96}_{\text{Cd}}, ^{94}_{\text{Ag}}, ^{92}_{\text{Pd}}, ^{90}_{\text{Rh}}$.

Starting point: Shell-model interpretation in terms of spin-aligned $T=0$ np pairs (Blomqvist).

Experiments have been proposed and carried out at GANIL (Cederwall, de France, Wadsworth...).

[Also, analysis of $N=Z$ nuclei in the $1f_{7/2}$ shell ($^{42}_{\text{Sc}}, ^{44}_{\text{Ti}}, ^{46}_{\text{V}}, ^{48}_{\text{Cr}}, ^{50}_{\text{Mn}}, ^{52}_{\text{Fe}}, ^{54}_{\text{Co}}$).]
stretch scheme, a shell-model description of deformed nuclei

michael danos and vincent gillet

service de physique théorique, centre d'études nucléaires de saclay, gif-sur-yvette, seine et oise, france

and

national bureau of standards, washington, d.c.

(Received 23 March 1967)

A good angular-momentum wave function containing the maximum possible intrinsic angular momenta leads to a microscopic description of the nuclear rotational spectra in terms of spherical shell-model states. The rotational excitation energies arise from the residual two-body force. In the actual model calculations, the only approximation was a partial violation of the exclusion principle. The computed departures from the $I(I+1)$ law are consistent with experiment. Reasons are given for the preference of positive over negative intrinsic deformations.
HIGH MULTIPole Proton-Neutron PAIRing in Nuclei

by

M.J. DALEY, SRRC Daresbury Laboratory

To be submitted to Nucl. Phys. A

NOVEMBER, 1987

Science and Engineering Research Council
DARESBURY LABORATORY
Daresbury, Warrington WA4 4AD
Nuclear belly dancer

B. Cederwall et al., Nature 469 (2011) 68
A new coupling scheme?

Our results reveal evidence for a spin-aligned, isoscalar neutron–proton coupling scheme. This coupling scheme replaces normal superfluidity (characterized by seniority coupling) in the ground and low-lying excited states of the heaviest $N=Z$ nuclei.

B. Cederwall et al., Nature 469 (2011) 68
Hypothesis: $N=Z$ nuclei can be described in the (spherical) shell model, in an appropriate model space and with an appropriate interaction.

Approximations:
(A) Truncate shell model to a single high-$j$ shell.
(B) Truncate single-$j$ shell space to one written in terms of aligned-spin $B$ ($J=9$) pairs.
(C) Replace aligned-spin $B$ pairs by $b$ bosons.
Shell-model interaction: $1g_{9/2}$

E.J.D. Serduke et al., Nucl. Phys. A 256 (1976) 45
Pair analysis in the shell model

Define different types of nucleon pairs:

\[ B_{JT}^+ = \left( a_{j_{1/2}}^+ \times a_{j_{1/2}}^+ \right)^{(JT)} \]

\[ S^+ : J = 0, T = 1; \quad D^+ : J = 2, T = 1; \quad B^+ : J = 9, T = 0. \]

Calculate overlap with shell-model wave functions with the nucleon-pair shell model in an isospin-invariant formulation.

$B$-pair analysis of $^{96}$Cd

\[ \langle J_1 | B^2; J \rangle^2 \]
Spectrum of $^{96}\text{Cd}$

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- $^{14+}$
- $^{16+}$
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- $^{2+}$
- $^{0+}$

$^{96}\text{Cd}$

$^{48}\text{Cd}$
$B$-pair analysis of $^{94}$Ag

\[ \left\langle J_1 | B^3; J \right\rangle^2 \]

$^{94}_{47}$Ag

\[ J \]

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|
|  1 | 0 | 3 | 0 | 1 | 2 | 3 | 3 | 2 | 1 |  2 |  4 |  8 |  9 |  4 |  2 |  2 |  0 |  1 | 0 |  1 | 0 |  1 |
Spectrum of $^{94}\text{Ag}$

Expt | SM | $B^3_{18^+}$ | $b^3 [2]$ | $b^3 [3]_{18^+}$
---|---|---|---|---
$21^+$ | $18^+$ | $19^+$ | $21^+$ | $21^+$
$21^+$ | $19^+$ | $16^+$ | $19^+$ | $16^+$
$16^+$ | $17^+$ | $17^+$ | $17^+$ | $17^+$
$17^+$ | $14^+$ | $15^+$ | $14^+$ | $14^+$
$15^+$ | $15^+$ | $1^+$ | $1^+$ | $1^+$
$14^+$ | $3^+$ | $3^+$ | $3^+$ | $3^+$
$12^+$ | $12^+$ | $13^+$ | $13^+$ | $13^+$
$13^+$ | $4^+$ | $4^+$ | $4^+$ | $4^+$
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$10^+$ | $10^+$ | $11^+$ | $11^+$ | $11^+$
$11^+$ | $5^+$ | $5^+$ | $5^+$ | $5^+$
$6^+$ | $8^+$ | $8^+$ | $8^+$ | $8^+$
$5^+$ | $7^+$ | $7^+$ | $7^+$ | $7^+$

$^{94}_{47}\text{Ag}_{47}$
Mapping to bosons

Transform to a much simpler problem in terms of interacting bosons: $B^+ \rightarrow b^+$

Boson energies and boson-boson interactions are derived from the shell model (i.e., not fitted).

T. Otsuka et al., Nucl. Phys. A 309 (1978) 1
L.D. Skouras et al., Nucl. Phys. A 516 (1990) 255
Magnetic dipole moments

For any state in a single-$j$ shell

$$g(\alpha J) = \frac{1}{2}(g_\nu + g_\pi) \approx \begin{cases} 
0.52 \text{ to } 0.55 \, \mu_N & (1f_{7/2}) \\
0.51 \text{ to } 0.54 \, \mu_N & (1g_{9/2})
\end{cases}$$

The same result is obtained with $b$-IBM mapped from a single-$j$ shell model.

.: Magnetic dipole moments test approximation (A) but are insensitive to (B) and (C).

In $^{46}\text{V}$:

$$\mu\left(3^+_1\right) = 1.64(3) \, \mu_N$$

In $^{50}\text{Mn}$:

$$\mu\left(5^+_1\right) = 2.76(1) \, \mu_N$$

**Q** moment of $21^+$ isomer in $^{94}$Ag

Shell model in $pf_{5/2}g_{9/2}$ space ($M=21 \rightarrow \text{dim}=2$):
\[ Q(21^+) = 0.44 \text{ b} \]

Shell model in $1g_{9/2}$ ($J=21 \rightarrow \text{dim}=1$):
\[ Q(21^+) = \sqrt{\frac{196}{9}} (e_v + e_\pi)(\ell_{ho})^2 \approx 0.42 \text{ b} \]

Expression in terms of $b$ bosons:
\[ Q(b^3_{\infty}21) = \sqrt{\frac{81949367824}{3489855625}} (e_v + e_\pi)(\ell_{ho})^2 \approx 0.44 \text{ b} \]

∴ Measurement of $Q(21^+)$ tests (A). Calculation confirms (B+C).

A. Poves, private communication LSSM
**Q** moment of $7^+$ isomer in $^{94}$Ag

Shell model in $pf_{5/2}g_{9/2}$ space ($M=7 \rightarrow \text{dim}=37327$): 
\[ Q(7^+_1) = 0.62 \text{ b} \]

Shell model in $1g_{9/2}$ ($J=7 \rightarrow \text{dim}=84$): 
\[ Q(7^+_1) = 6.60 (e_\nu + e_\pi)(\ell_{ho})^2 \approx 0.60 \text{ b} \]

Expression in terms of $b$ bosons:
\[ Q(b^3[16]7) = \sqrt{\frac{30930277300923364}{627253477610841}} (e_\nu + e_\pi)(\ell_{ho})^2 \approx 0.64 \text{ b} \]

\[ \therefore \] Measurement of $Q(7^+)$ tests (A). Calculation confirms (B+C).
A discussion between physicists

What is the correct coupling scheme for these nuclei? Seniority or aligned pairs?

Physicist 1: The $7^+$ state in $^{94}$Ag can be written as

$$| 7^+ \rangle = | B^2 [16] B; J = 7 \rangle$$

Physicist 2: Not at all, this $7^+$ state should be written as

$$| 7^+ \rangle = | B^2 [4] B; J = 7 \rangle$$
(Anti)-symmetric amnesia
(Anti)-symmetric amnesia

$|b^2[16]b; J = 7\rangle$
(Anti)-symmetric amnesia

$| b^2 [16] b; J = 7 \rangle$
(Anti)-symmetric amnesia

$| b^2[16] b; J = 7 \rangle$
(Anti)-symmetric amnesia
(Anti)-symmetric amnesia

\[ |b^2[16]\ b; J = 7\rangle \]

\[ |b^2[4]\ b; J = 7\rangle \]
(Anti)-symmetric amnesia

\[ | b^2 [16] b; J = 7 \rangle \]

\[ | b^2 [4] b; J = 7 \rangle \]
(Anti)-symmetric amnesia

\[ | b^2 [16] b; J = 7 \rangle \quad \text{and} \quad | b^2 [4] b; J = 7 \rangle \]
(Anti)-symmetric amnesia
Fermion pairs versus bosons

In the $1g_{9/2}$ nucleon-pair shell model:

$$\left\langle B^2 [4] B; J = 7 \right| B^2 [16] B; J = 7 \right\rangle = \sqrt{\frac{112919600563049280}{139849953265085321}} \approx 0.899$$

In the $b$-IBM:

$$\left\langle b^2 [4] b; J = 7 \right| b^2 [16] b; J = 7 \right\rangle = \sqrt{\frac{7012200}{8733503}} \approx 0.896$$

∴ The $B$ pair behaves as a boson.
Conclusions

(A) Truncation of the shell model to a single-$j$ shell. (?)

(B) Truncation of the single-$j$ shell space to one written in terms of aligned-spin $B$ ($J=7$ or 9) pairs. (✓)

(C) Replacement of aligned-spin $B$ pairs by $b$ bosons. (✓)

Shell-model interaction: $1f_{7/2}$
Shell-model interaction: $1f_{7/2}$
$B$-pair analysis of $^{44}\text{Ti}$ and $^{52}\text{Fe}$
$B$-pair analysis of $^{46}$V and $^{50}$Mn

$$\langle J_1 | B^3; J \rangle^2$$
$B$-pair analysis of $^{48}\text{Cr}$

\[
\langle J_1 \mid B^4; J \rangle^2
\]
Structure of $^{96}\text{Cd}$
Spectrum of $^{44}$Ti

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Spectrum of $^{52}\text{Fe}$

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Spectrum of $^{46}\text{V}$
Spectrum of $^{50}$Mn

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$^{50}_{25}$Mn$_{25}$
### Spectrum of $^{48}\text{Cr}$

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$^{48}\text{Cr}$
### Spectrum of $^{92}\text{Pd}$

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$^{92}\text{Pd}_{46}$
$Q$ moment of $5^+$ state in $^{50}$Mn

Experimental value: $Q(5^+_1) = 0.80(12)$ b

Large-scale shell model: $Q(5^+_1) = 0.58$ b

Numerical result in the $1f_{7/2}$ shell:

$$Q(5^+_1) = 4.2(e_\nu + e_\pi)(\ell_{ho})^2$$

Expression in terms of $b$ bosons:

$$Q(b^3[12]5) = \frac{649485}{150241}(e_\nu + e_\pi)(\ell_{ho})^2 \approx 4.3(e_\nu + e_\pi)(\ell_{ho})^2$$

Parameter-free test:

$$\frac{Q(5^+_1; ^{46}V)}{Q(7^+_1; ^{42}Sc)} \approx \frac{Q(b^3[12]5)}{Q(b)} = \frac{216495}{150241} \approx 1.44$$
E2 properties in $b$-IBM

The E2 operator in the shell model:

$$\hat{T}_\mu^F(E2) = e_\nu \sum_{i \in \nu} r_i^2 Y_{2\mu}(\theta_i, \phi_i) + e_\pi \sum_{i \in \pi} r_i^2 Y_{2\mu}(\theta_i, \phi_i)$$

The E2 operator in terms of $b$ bosons:

$$\hat{T}_\mu^B(E2) = e_b \left( b^+ \times \tilde{b} \right)_\mu^{(2)}$$

The mapping implies the relation:

$$\langle (g_{9/2})^2; 9^+ | \hat{T}_\mu^F(E2) | (g_{9/2})^2; 9^+ \rangle = \langle b | \hat{T}_\mu^B(E2) | b \rangle$$

$$\Rightarrow e_b = \sqrt{\frac{55}{3\pi}} \left( \ell_{ho} \right)^2 \times \sqrt{\frac{266}{187}} (e_\nu + e_\pi)$$
B(E2) values in $^{96}$Cd
B(E2) values in $^{96}$Cd

A simple consequence of the aligned-pair assumption:

$$B(E2; J + 2 \rightarrow J) = e_b^2 20 (2J + 1) \left[ \begin{array} {ccc} 9 & 9 & 2 \\ J + 2 & J & 9 \end{array} \right]^2$$
B(E2) values in $^{94}$Ag
B(E2) values in $^{94}\text{Ag}$
B(E2) values in $^{92}\text{Pd}$
Energy of $21^+$ isomer in $^{94}$Ag

How many independent states of three $b$-bosons can couple to angular momentum $21$?

$$d(\nu, \ell, J) = \frac{i}{2\pi} \oint_{|z|=1} \frac{(z^{2J+1} - 1)(z^{2\nu+2\ell-1} - 1)}{z^{\ell\nu+2J+2}} \prod_{k=1}^{2\ell-2} \left( z^{\nu+k} - 1 \right)$$

Answer: $d(3,9,21)=2$ one of which is spurious. After elimination of the spurious state, the energy of the physical $21^+$ state is

$$E(21^+) = 3\epsilon_b + \frac{6851}{20155} \nu_{12}^b + \frac{15488}{21545} \nu_{14}^b + \frac{1212882}{624805} \nu_{16}^b$$

Energy of $21^+$ isomer in $^{94}$Ag

In turn, we know the boson matrix elements in terms of the shell-model matrix elements:

\[
v_{12}^b = \frac{1218}{69355} v_3 + \frac{63423}{138710} v_4 + \frac{29957}{63050} v_5 + \frac{109881}{53350} v_6
\]
\[
+ \frac{1148337}{2358070} v_7 + \frac{15231}{31525} v_8 + \frac{10893}{535925} v_9
\]

\[
v_{14}^b = \frac{868}{8515} v_5 + \frac{1953}{1310} v_6 + \frac{46251}{57902} v_7 + \frac{1977}{1310} v_8 + \frac{2211}{22270} v_9
\]

\[
v_{16}^b = \frac{8}{17} v_7 + 3v_8 + \frac{9}{17} v_9
\]
Energy of $21^+$ isomer in $^{94}$Ag

Therefore, we know the energy of the $21^+$ isomer in $^{94}$Ag in terms of the shell-model interaction matrix elements:

$$E_b(21^+) = \frac{22134}{3707825} \nu_3 + \frac{1152549}{7415650} \nu_4 + \frac{1347751953}{5740387250} \nu_5$$
$$+ \frac{8606149749}{4857250750} \nu_6 + \frac{354940047213}{214690483150} \nu_7$$
$$+ \frac{1561553973}{220784125} \nu_8 + \frac{15411107094}{3753330125} \nu_9$$
Energy of $21^+$ isomer in $^{94}$Ag

The $21^+$ state is unique in the $1g_{9/2}$ shell model. Its energy is therefore known analytically:

$$E_f(21^+) = \frac{21}{65} \nu_5 + \frac{21}{10} \nu_6 + \frac{645}{442} \nu_7 + \frac{69}{10} \nu_8 + \frac{717}{170} \nu_9$$

Comparison tests the reliability of the mapping:

$$E_f(21^+) \approx 0.323 \nu_5 + 2.1 \nu_6 + 1.459 \nu_7 + 6.9 \nu_8 + 4.218 \nu_9$$

$$E_b(21^+) \approx 0.006 \nu_3 + 0.155 \nu_4 + 0.235 \nu_5 + 1.772 \nu_6 + 1.653 \nu_7 + 7.073 \nu_8 + 4.106 \nu_9$$
 Conservation of $n, J$ and $T$

A unique $n$-particle shell-model state with angular momentum $J$ and isospin $T$ has energy

$$E_f(j^nJT) = \sum_{\lambda} a_{\lambda} v_{\lambda}$$

The coefficients $a_{\lambda}$ satisfy

$$\sum_{\lambda=0}^{2j} a_{\lambda} = \frac{n(n-1)}{2},$$

$$\sum_{\lambda=0}^{2j} \lambda(\lambda+1)a_{\lambda} = J(J+1) + j(j+1) \times n(n-2)$$

$$\sum_{\lambda=0}^{2j} 2a_{\lambda} = T(T+1) + \frac{3}{4} n(n-2)$$

$$\text{even}$$