Investigation of alpha cluster states using alpha knockout reactions

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Outline of this talk

• Introduction: quasifree $\alpha$ knockout reaction
• Formalism: Distorted Wave Impulse Approximation (DWIA)
• Input, result and discussion
• Summary
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α clustering on the surface of Sn isotopes

- Relativistic mean-field model suggests the existence of α on the surface.
- The relationship between neutron skin thickness $r_{\text{skin}}$ and slope coefficient $L$ will be changed.

“α-particles abundances should be studied experimentally, e.g., by quasifree ($p,p\alpha$) reactions.” → Experiment at RCNP is scheduled.

quasifree $\alpha$ knockout reaction

initial state  quasifree $p$-$\alpha$ collision  final state

Features

- $\sim 400$ MeV beam energy $\rightarrow$ simpler reaction mechanism
- Applicable to unstable nuclei using inverse kinematics
- Good probe for the single particle state of knocked out particle
Purpose of this study

• Understanding how \((p,p\alpha)\) reaction probes \(\alpha\) clustering on the surface
• How the distortion affects the observables
• The first attempt for heavy nuclei such as Sn isotopes
  (Distortion effect of \((p,p\alpha)\) reaction is already discussed for \(A\sim 20\) targets: c.f. N. S. Chant and P. G. Roos, Phys. Rev. C15 (1977).)
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Distorted Wave Impulse Approximation (DWIA)

\[ T = \langle \chi_1 \chi_\alpha | t_{p\alpha} | \chi_0 \varphi \rangle \]

- \( \chi_i \): scattering wave of particle \( i \)
- \( \varphi \): \( \alpha \)-cluster wave function
- \( t_{p\alpha} \): free \( p-\alpha \) t-matrix
Distorted Wave Impulse Approximation (DWIA)

\[ T = \langle \chi_1(R_1) \chi_\alpha(R_\alpha) | t_{p\alpha} | \chi_0(R_1) \varphi(R_\alpha) \rangle \]

Asymptotic momentum Approximation:
Propagation of \( \chi \) for short distance \( \Delta R \) is replaced by plane wave

\[ \chi_K(R + \Delta R) \approx \chi_K(R) e^{iK \cdot \Delta R} \]

\[ T \approx \int dR \chi_0(R) \chi_1^*(R) \chi_\alpha^*(R) \varphi(R) \tilde{t}_{p\alpha}(\kappa', \kappa) \]

\[ \tilde{t}_{p\alpha}(\kappa', \kappa) \equiv \int ds e^{-i\kappa' \cdot s} t_{p\alpha} e^{i\kappa \cdot s} \leftrightarrow \frac{d\sigma_{p\alpha}}{d\Omega_{p\alpha}} \]

\( \kappa (\kappa') \) is the initial (final) relative momentum of \( p-\alpha \).
Distorted Wave Impulse Approximation (DWIA) simple understanding by considering plane waves (PWIA):

\[ T^{PW} \approx \int dR e^{i\mathbf{q} \cdot \mathbf{R}} \varphi(\mathbf{R}) \tilde{t}_{p\alpha}(\kappa', \kappa), \]

where \( q \) is defined by,

\[ q \equiv K_0 - K_1 - K_\alpha. \]

Due to the momentum conservation,

\[ q = K_B \approx -K_{\text{cluster}}, \]

\[ T^{PW} \approx \tilde{\varphi}(K_{\text{cluster}}) \tilde{t}_{p\alpha}(\kappa', \kappa) \]

\( T \) has the shape like the Fourier transform of \( \varphi(\mathbf{R}) \).
Distorted Wave Impulse Approximation (DWIA)

\[ T \approx \int d\mathbf{R} \chi_0(\mathbf{R}) \chi_1^*(\mathbf{R}) \chi_\alpha^*(\mathbf{R}) \varphi(\mathbf{R}) \tilde{t}_{p\alpha}(\kappa', \kappa) \]

For \( t_{p\alpha} \), we employ a microscopic folding model with the Melbourne g matrix that reproduce \( p-\alpha \) cross sections.

\[
\begin{align*}
E_p &= 297 \text{ MeV} \\
E_p &= 500 \text{ MeV}
\end{align*}
\]
Distorted Wave Impulse Approximation (DWIA)

\[ T \approx \int dR \chi_0(R) \chi_1^*(R) \chi_\alpha^*(R) \varphi(R) \tilde{t}_{p\alpha}(\kappa', \kappa) \]

In this calculation, we assume isotropic approximation:

\[ C|\tilde{t}_{p\alpha}(\kappa', \kappa)|^2 \approx \frac{\sigma_{p\alpha}}{4\pi}, \]

\[ |\tilde{t}_{p\alpha}(\kappa', \kappa)|^2 \approx \frac{\sigma_{p\alpha}}{4\pi C} \equiv \bar{\sigma}_{p\alpha}. \]

Finally,

\[ |T|^2 \approx \bar{\sigma}_{p\alpha} \left| \int dR \tilde{\zeta}(R) \varphi(R) \right|^2, \]

where the distortion factor is defined by,

\[ \tilde{\zeta}(R) \equiv \chi_0(R) \chi_1^*(R) \chi_\alpha^*(R). \]
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Validity of the DWIA calc.

\[ S = 1.72 \ (e,e'p) \]

\[ ^{12}\text{C}(p,2p)^{11}\text{B} \ @ \ 392 \text{ MeV} \]

T. Noro et al., J. Phys. Conf. Ser. 20, 101 (2005);
T. Noro, private communication (2014).
Kinematics of $^{120}\text{Sn}(p,p\alpha)^{116}\text{Cd}$ reaction

- The initial and final momenta of proton is fixed.
- $p_\alpha$ and $p_B$ can change, satisfying energy and momentum conservation.
- Dirac phenomenology for incoming and outgoing proton.


- Global optical potential for $\alpha$ that reproduces the elastic scattering data.

\( \alpha \) wave function and Triple Differential Cross Section

- Single-particle potential of Bohr and Mottelson \((r_0 = 1.27 \text{ fm}, a = 0.67 \text{ fm})\)
- Potential depth is adjusted to reproduce \(^{120}\text{Sn} \rightarrow ^{116}\text{Cd} + \alpha\) separation energy \(S_\alpha = 4.8 \text{ MeV}\)
- Assuming 4S state.
\( \alpha \) wave function and Triple Differential Cross Section

\[ |\mathcal{F}(R) \varphi(R) R^2| \text{ at } P_B = 0 \]

\[ T = \int dR \mathcal{F}(R) \varphi(R) Y_{00}(\hat{R}) \]

\[ = \int dR R^2 \varphi(R) \int d\hat{R} \mathcal{F}(R) Y_{00}(\hat{R}) \]

Triple differential CS
Peripherality of $^{120}\text{Sn}(p,p\alpha)^{116}\text{Cd}$ reaction

$|\tilde{\mathcal{G}}(R) \varphi(R) R^2|$ at $P_B = 0$

$T = \int dR \tilde{\mathcal{G}}(R) \varphi(R) Y_{00}(\hat{R})$

$= \int dR R^2 \varphi(R) \int d\hat{R} \tilde{\mathcal{G}}(R) Y_{00}(\hat{R})$

Triple differential CS

$(p,p\alpha)$ reaction probes $\alpha$ clustering on the surface of heavy nuclei.
Comparison between DWIA with PWIA

\[ |\mathcal{F}(R) \varphi(R) R^2| \text{ at } P_B = 0 \]

\[ T = \int d\mathbf{R} \mathcal{F}(\mathbf{R}) \varphi(\mathbf{R}) Y_{00}(\hat{\mathbf{R}}) \]
\[ = \int d\mathbf{R} R^2 \varphi(\mathbf{R}) \int d\mathbf{R} \mathcal{F}(\mathbf{R}) Y_{00}(\hat{\mathbf{R}}) \]

\( (p,p\alpha) \) reaction probes \( \alpha \) clustering on the surface of heavy nuclei.
Comparison between DWIA with PWIA

The distortion (absorption) effects of the scattering waves...

- decrease the cross section by a factor of 200.
- widen the momentum distribution due to the difference of probed range.

\[ \Gamma_{\text{DWIA}} = 125 \text{ MeV/c, while } \Gamma_{\text{PWIA}} = 79 \text{ MeV/c.} \]
Sensitivity to the range of cluster w.f.

- Extending the $\alpha$ particle wave function $\varphi(R)$ by changing the Woods-Saxon range parameter $r_0 = 1.27$ fm $\rightarrow 1.1 \times 1.27$ fm.
Sensitivity to the range of cluster w.f. 

\( |\mathcal{F}(R) \varphi(R) R^2| \) at \( P_B = 0 \)

**Woods-Saxon potential**

- \( r_0 = 1.27 \) fm
- \( r_0 = 1.1 \times 1.27 \) fm

**\( \alpha \)-cluster wave function**

- \( r_0 = 1.27 \) fm
- \( r_0 = 1.1 \times 1.27 \) fm

**Triple differential CS**

- \( r_0 = 1.27 \) fm
- \( r_0 = 1.1 \times 1.27 \) fm
Sensitivity to the range of cluster w.f.

- Magnitude of the cross section changes drastically.
- Width of the momentum distribution changes slightly.

\[ \Gamma = 125 \text{ MeV/c} \rightarrow 113 \text{ MeV/c} \]
Summary

• DWIA calculation has done for \((p, p\alpha)\) reaction.
• Very strong distortion (absorption) effects enable us to probe the \(\alpha\) clustering on the surface of heavy nuclei.
• Not only the magnitude of the momentum distribution, but also the width of that is sensitive to the range of \(\alpha\)-cluster w.f.

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