Department of Physics
The University of Hong Kong

Physics Laboratory
PHYS3850 Wave and Optics
Experiment No.3850-2: Diffraction, Interference and Diffraction Grating

Name: **University No.**

Laser Safety

The lasers used in this experiment and are of low power (i.e. around 5mW) but the narrow beam of light is still of high intensity. Consequently,

1. Wear Safety Goggle
2. Never look directly into the unexpanded laser beam or at its reflection from a mirror surface.
3. Do Not use the laser without getting instructions from the demonstrator.

WARNING! The beam of laser pointers is so concentrated that it can cause *real* damage to your retina if you look into the beam either directly or by reflection from a shiny object. Do NOT shine them at others or yourself.

A. AIMS

1. To find the experimental patterns of interference and diffraction for double-slit and multi-slit diaphragms and compare the theoretical values.
2. Differences and similarities between interference and diffraction patterns are examined.

B. INTRODUCTION

Electromagnetic radiation propagates as a wave, and as such can exhibit interference and diffraction. This is most strikingly seen with laser light, where light shining on a piece of paper looks speckled (with light and dark spots) rather than evenly illuminated, and where light shining through a small hole makes a pattern of bright and dark spots rather than the single spot you might expect from your everyday experiences with light.

When interference of light occurs as it passes through two slits, the angle from the central maximum (bright spot) to the side maxima in the interference pattern is given by

$$d\sin\theta = n\lambda,\ n = 0,1,2,3...$$

where d is the slit separation, θ is the angle from the center of the pattern to the n^{th} maximum, λ is the wavelength of the light, and the order (0 for the central maximum, 1 for the first side maximum, 2 for the second side maximum ...counting from the center out).

In this lab we will use laser light to investigate the phenomena of interference and diffraction.

C. PRE-LAB READING MATERIAL: Theory - Background Information

1. Superposition Principle of Wave:

   If two or more traveling waves are moving through a medium, the resultant wave function at any point is the algebraic sum of the wave functions of the individual waves.

2. Conditions for Observable Interference:
   2.1. The sources must be coherent – that is, they must maintain a constant phase with respect to each other.
   2.2. The sources should be monochromatic – that is, of a single wavelength.
3. Diffraction of Light
   - Diffraction is the bending of waves when they pass through apertures or around obstacles.
   - By Huygens's principle, there are centers of disturbance on the wavefront in plane of the obstacle.
   - This center sends out secondary circular wavelets which combine to form a diffracted wave and propagates forwards.
   - Behind the obstacles or apertures at which diffraction occurs, a diffraction pattern of dark and bright fringes is formed on the screen.

4. Single Slit Diffraction (Fraunhofer diffraction)

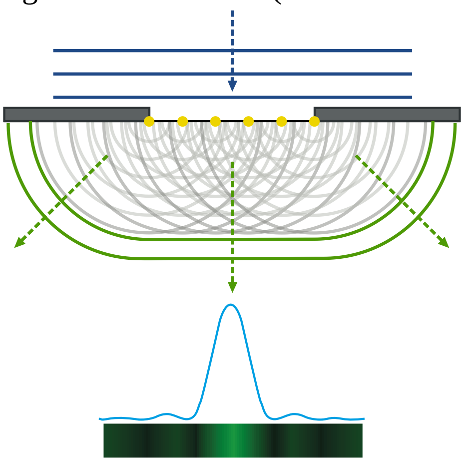


Fig. 1. Diffraction Pattern of Single Slit

   - The pattern is not equal spacing and the bright fringes are not equal width.
   - When white light source is used and color filters are placed in front of the slit in turns, the separation between the color bands varies.
   - The color bands correspond to the blue filter are closer together than that of the red one. It means that the diffraction pattern depends on the wavelength of the light.
   - The diffraction pattern becomes wider if the slit is made narrower or light of longer wavelength is used. The narrower the slit, the dimmer the pattern and the wider the central band.

   4.1. Mathematical Expression of Single Slit Diffraction:
   - When diffraction of light occurs as it passes through a slit, the angle to the minima (dark fringes) in the diffraction pattern is given by

$$a \sin\theta_m = m\lambda$$
$$m = 1, 2, 3\ldots$$

     where "a" is the slit width, θ_m is the angle from the central maximum of the pattern to the a minimum, λ is the wavelength of the light, and m is the order (1 for the first minimum, 2 for the second minimum, ...counting from the center out).
   - It should be highly mentioned

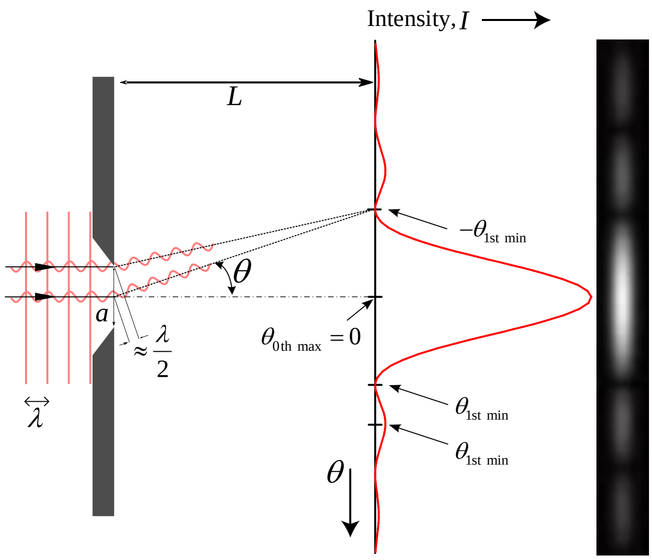


Fig. 2. Intensity Distribution of Single Slit Diffraction

that $m = 0$ is not included since $\theta_m = 0$ corresponds to the central maximum intensity.

4.2. Energy Spread of Single Slit Diffraction

- The angle between the center of diffraction pattern and the positron of the first minimum is given by $\sin\theta_1 = \frac{\lambda}{a}$.
- This angle θ_1 can indicate the angular spread = $2\theta_1$, which contains most of the energy of the wave.
- $\because \sin\theta_1 = \frac{\lambda}{a}$, if the width of the slit, a is large compared with the wavelength λ, the angular spread (or called angular width) of the center maximum ($2\theta_1$) becomes very small.
- The direction of the first minima is extremely near to the center maxima which imply minimum diffraction effect.

4.3. Intensity of Single-Slit Diffraction Pattern

- The intensity single-slit diffraction pattern is $I_{\text{1-slit diffraction}} = I_o \left[\frac{\sin\left(\frac{\pi}{\lambda} a \sin\theta\right)}{\frac{\pi}{\lambda} a \sin\theta} \right]^2$

where a is the width of the slit

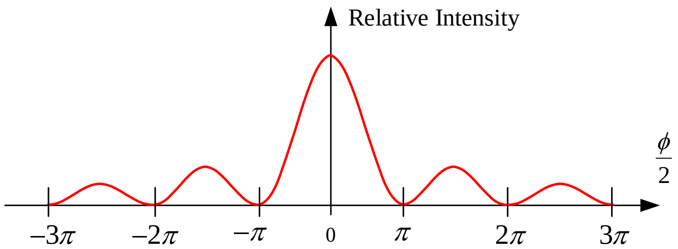


Fig. 3. Intensity Distribution of Single Slit Diffraction

5. Two-slit Interference

5.1. Assumptions:

- Use laser light or strong light source
- Use monochromatic light (i.e. single wavelength)
- The slits should be as narrow as possible.
- The slit separation must be very small (i.e. ~0.5mm)
- Screen-slit separation must be at least 1-2m.
- Assume $\lambda << d << D$ or $\frac{d}{D} \approx \frac{1}{2000}$

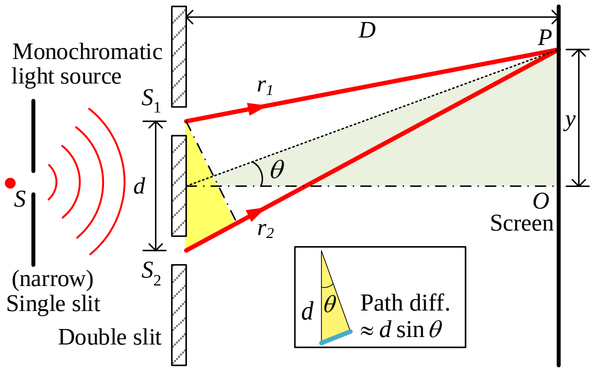


Fig. 4. Setup of Young Double-slit Experiment

5.2. Mathematical Expression of Double-slit Interference

Light from the laser hits two very narrow slits, which then act like in-phase point sources of light. In traveling from the slits to the screen, however, the light from the two slits travel different distances. In the picture at left light hitting point P from the bottom slit travels further than the light from the top slit. i.e. $r_2 > r_1$.

The extra path difference (i.e. $|r_2 - r_1|$) introduces a phase shift between the two waves and leads to a position dependent interference pattern on the screen. Here the extra path length is $\delta = d\sin\theta$, leading to a phase shift θ given by $\frac{\delta}{\lambda} = \frac{\theta}{2\pi}$. Realizing that phase shifts that are multiples of 2π give us constructive interference while odd multiples of π lead to destructive interference leads to the following conditions:

	Constructive interference	Destructive interference
Path difference for in phase source, $d\sin\theta$	$n\lambda$	$\left(n \pm \frac{1}{2}\right)\lambda$
	$n = 0,1,2,3...$	
Resultant amplitude	Max ($\lvert E_{\text{resultant}} \rvert = 2E_{each}$)	Min (i.e. $E_{\text{resultant}} = 0$)
Resultant intensity	$I_{\text{resultant}} = 4I_{each}$ $\left(\because I \propto E^2\right)$	$\lvert I_{\text{resultant}} \rvert = 0$
Distance between the successive interference	$\Delta y = \frac{\lambda D}{d}$	Where d is slits separation, D is slits-to-observer distance,
For light wave	Bright fringe	Dark fringe

5.3. The Fringe Separation of Double-slit Interference Pattern:

- If all assumptions are fulfilled, fringe separation, $\Delta y = \frac{\lambda D}{d}$ is obeyed

  where $\lambda =$ wavelength used
  $D =$ Screen-slit separation
  $d =$ slit-separation

- Derivation of fringe separation is shown in the appendix.

5.3.1. Methods to Increase the Fringe Separation,

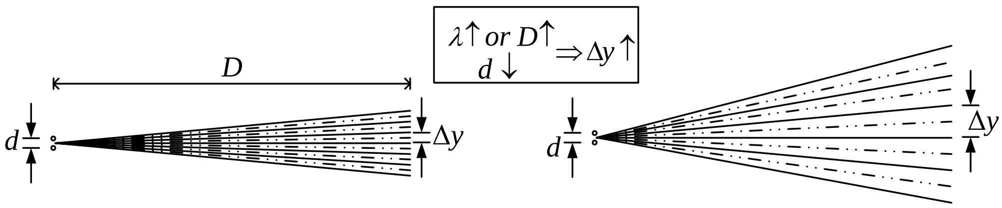


Fig. 5. Methods to Increase the Fringe Separation

- If d is decreasing, (or ≡ increasing λ or ≡ increasing D or ≡ decreasing f), the following changes occur:
  i) The fringe separation $\Delta y = \frac{\lambda D}{d}$ increases;
  ii) The fringes become more loosely packed;
  iii) Fewer fringes are observed; and
  iv) The brightness of the fringes decreases.

5.4. Intensity of Double-Slit (Pure) Interference Patterns:

5.4.1. Mathematical Expression

- According to the derivation from appendix, the intensity of double-slit (pure) interference patterns is

$$I = I_o \cos^2\left(\frac{\pi}{\lambda} d \sin\theta\right) \approx I_o \cos^2\left(\frac{\pi}{\lambda} d\theta\right) = I_o \cos^2\left(\frac{\pi d}{\lambda L} y\right)$$

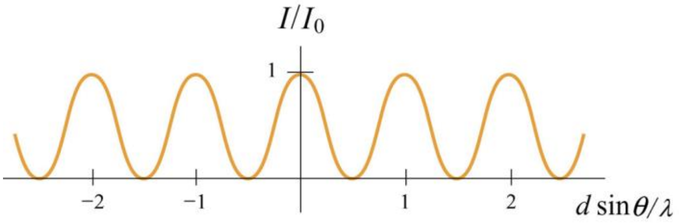


Fig. 6. Intensity as a function of dsinθ/λ

5.5. Graphical Expressions of Double Slit Interference with Diffraction Effect:

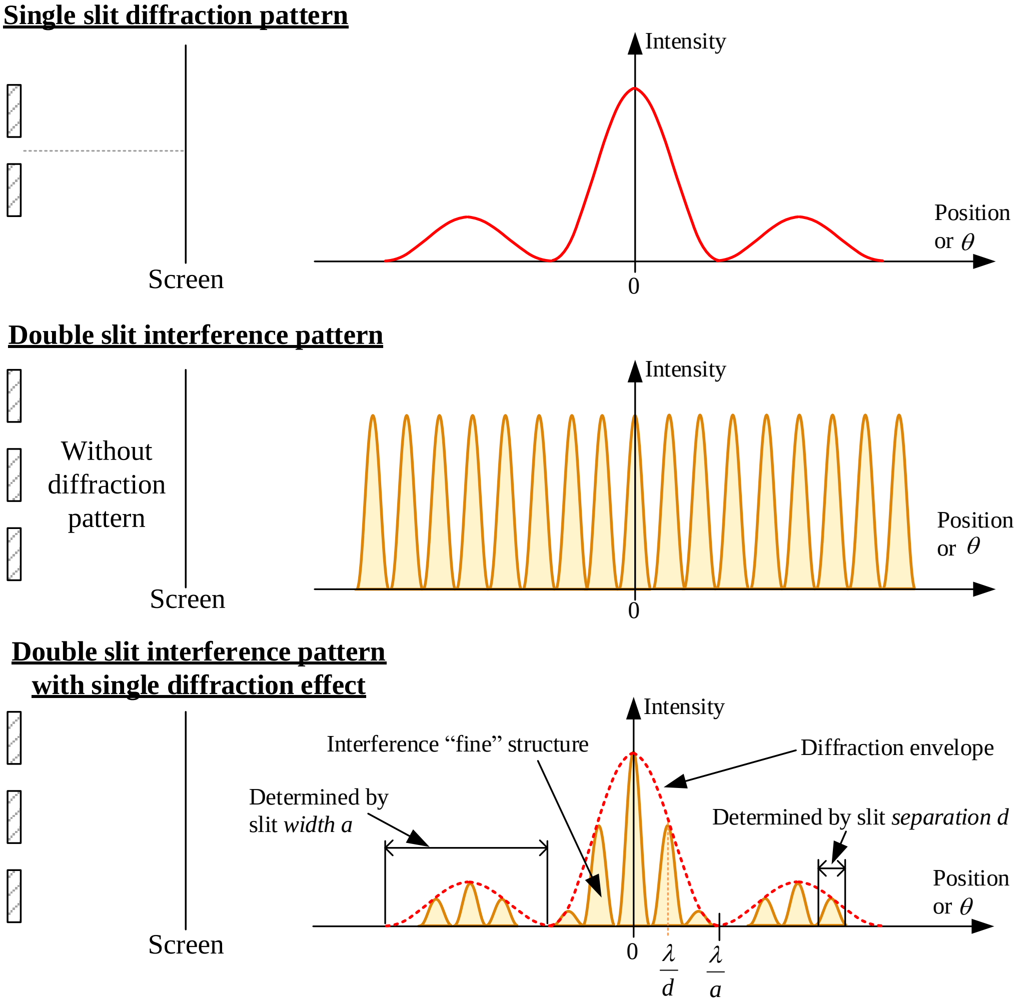


Fig. 7. Double-slit Interference Pattern with Single Diffraction Effect

- Bright fringes due to 2-slit interference: $\theta_{\text{max, 2-slit}} \approx n\frac{\lambda}{d}$
- Minimum (Zero) due to single-slit diffraction: $\theta_{\text{min,1-slit}} \approx \frac{\lambda}{a}, \frac{2\lambda}{a}, \ldots\ldots$
- All double-slit interference pattern **should be included** the single diffraction effect unless a specification of pure interference effect.

5.6. Intensity of Double-Slit Interference with Diffraction Effect:

$$I_{\text{1-slit diffraction}} = I_o \left[\frac{\sin\left(\frac{\pi}{\lambda} a \sin\theta\right)}{\frac{\pi}{\lambda} a \sin\theta} \right]^2 \qquad \text{Single-slit diffraction}$$

$$I_{\text{2-slit Pure Interference}} = I_o \cos^2\left(\frac{\phi}{2}\right) = I_o \cos^2\left(\frac{\pi}{\lambda} d \sin\theta\right) \qquad \text{Double-slit interference}$$

Suppose we now have two slits, each having a width *a*, and separated by a distance *d*. The resulting interference pattern for the double-slit will also include a diffraction pattern due to the individual slit. The intensity of the total pattern is simply the product of the two functions:

$$I_{\text{2-slit interference}} = I_o \underbrace{\cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)}_{\text{Interference factor}} \underbrace{\left[\frac{\sin\left(\frac{\pi}{\lambda} a \sin\theta\right)}{\frac{\pi}{\lambda} a \sin\theta} \right]^2}_{\text{Diffraction factor}}$$

5.6.1. Missing Orders of Double-Slit Interference with Diffraction Effect:

We have seen that the interference maxima occur when $d\sin\theta = n\lambda$. On the other hand, the condition for the first diffraction minimum $a\sin\theta = (1)\lambda = \lambda$. Thus, a particular interference maximum with order number *n* may coincide with the first diffraction minimum. The value of *n* may be obtained as:

$$\frac{d\sin\theta}{a\sin\theta} = \frac{n\lambda}{\lambda} \rightarrow \boxed{\frac{d}{a} = n}$$

For example, $\frac{d}{a} = \frac{0.3 \text{ mm}}{0.1 \text{ mm}} = 3$. Thus, the third interference maximum (if we count the central maximum as $n = 0$) is aligned the first diffraction minimum and cannot be seen. Since the n^{th} fringe is not seen, the number of fringes on each side of the central fringe is $n-1$. e.g. 2 fringes in the example. Thus, the total number of fringes in the central diffraction maximum is $N_{total} = 2(n-1)+1 = 2n-1$. e.g. 5 fringes in the example.

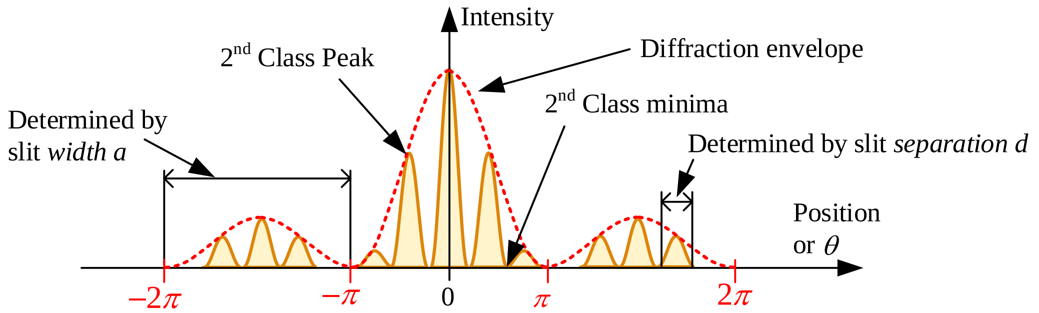


Fig. 8. Intensity Distribution of Double-slit Interference Pattern with Single Diffraction Effect

6. Multiple-slit Interference

If instead of two identical slits separated by a distance *d* there are multiple identical slits, each separated by a distance *d*, the same effect happens. For example, at all angles θ satisfying $d\sin\theta = n\lambda$. We find constructive interference, now from all of the holes.

The difference in the resulting interference pattern lies in those regions that are neither maxima or minima but rather in between. Here, because more incoming waves are available to interfere, the interference becomes more destructive, making the minima appear broader and the maxima sharper.

6.1. Analysis of Phasor Diagram for 3 Slits Interference:

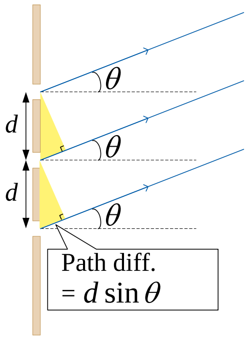


Fig. 9. 3-slit Interference

Consider all rays coming from same angle, θ for all maximum and minimum:

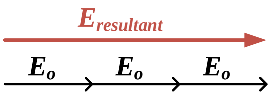
Path difference between rays from adjacent slits:

$$\Delta L = d\sin\theta \approx \theta d \rightarrow \theta \approx \frac{\Delta L}{d}$$

Phase difference between rays from adjacent slits:

$$\phi = 2\pi\frac{\Delta L}{\lambda} = 2\pi\frac{\theta d}{\lambda}$$

- Case 1: Central Maximum



Phase difference between rays from adjacent slits, $\phi = 0$

Path difference between rays from adjacent slits, $d\sin\theta = 0$

Amplitude of resultant ray at central maximum,

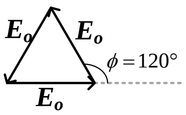
$E_{\text{resultant}} = 3E_o$

Intensity of resultant ray at central maximum, $I_{\text{resultant}} = 9I_o$

- Case 2: First Minimum

Consider all rays coming from same angle, θ for 1^{st} minimum

➔ Equal angle for 3 phasors ➔ Equilateral Triangle



Phase difference between rays from adjacent slits, $\phi = \frac{2\pi}{3}$

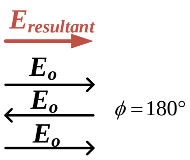
Path difference between rays from adjacent slits, $d\sin\theta = \frac{\lambda}{3}$

$$\rightarrow \theta \approx \frac{\lambda}{3}\frac{1}{d}$$

Amplitude of resultant ray at central maximum, $E_{\text{resultant}} = 0$

Intensity of resultant ray at central maximum, $I_{\text{resultant}} = 0$

- Case 3: Next Maximum



Phase difference between rays from adjacent slits, $\phi = \pi$

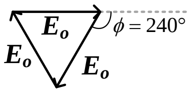
Path difference between rays from adjacent slits, $d\sin\theta = \frac{\lambda}{2}$

$$\rightarrow \theta \approx \frac{\lambda}{2}\frac{1}{d}$$

Amplitude of resultant ray at central maximum, $E_{\text{resultant}} = E_o$

Intensity of resultant ray at central maximum, $I_{\text{resultant}} = I_o$

- Case 4: Next Minimum



Phase difference between rays from adjacent slits, $\phi = \frac{2}{3}2\pi$

Path difference between rays from adjacent slits,

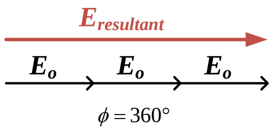
$$d\sin\theta = \frac{2}{3}\lambda$$

$\Rightarrow \theta \approx \frac{2\lambda}{3}\frac{1}{d}$

Amplitude of resultant ray at central maximum, $E_{\text{resultant}} = 0$

Intensity of resultant ray at central maximum, $I_{\text{resultant}} = 0$

- Case 5: Next Maximum



Phase difference between rays from adjacent slits, $\phi = 2\pi$

Path difference between rays from adjacent slits, $d\sin\theta = \lambda$

$\Rightarrow \theta \approx \frac{\lambda}{2}\frac{1}{d}$

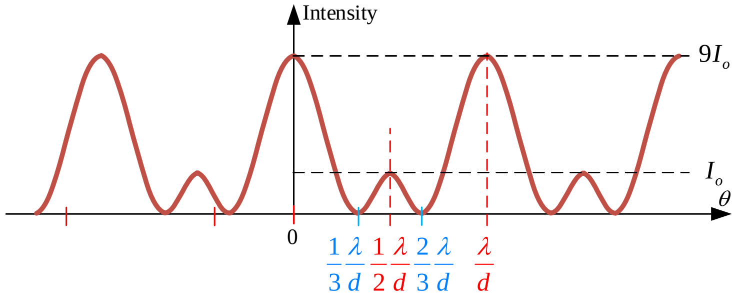
Amplitude of resultant ray at central maximum,

$E_{\text{resultant}} = 3E_o$

Intensity of resultant ray at central maximum, $I_{\text{resultant}} = 9I_o$

- Summary:

$$\boxed{\theta_{\max} \approx n\frac{\lambda}{d}} \qquad \text{where } n = 0, \pm1, \pm2, \pm3\ldots$$



	Central Max	1st min	1st max	2nd min	2nd max
ΔL	0	$\frac{\lambda}{3}$	$\frac{\lambda}{2}$	$\frac{2\lambda}{3}$	λ
$\theta \approx \frac{\Delta L}{d}$	0	$\frac{1}{3}\frac{\lambda}{d}$	$\frac{1}{2}\frac{\lambda}{d}$	$\frac{2}{3}\frac{\lambda}{d}$	$\frac{\lambda}{d}$

6.2. Diffraction Grating:

- An optical transmission diffraction grating consists of a large number of fine, close equidistant parallel lines ruled onto a transparent plate (such as glass).
  - Fine grating: 300 – 600 slits per mm
  - Coarse grating: 100 – 200 slits per mm
- Constructive interference produces sharp lines of maximum intensity at set angles either side of a sharp, central maximum.
- In between, destructive interference gives zero or near-zero intensity. To identify the lines, they are each given an order number (0, 1, 2, …… etc).

6.3. Mathematical expressions of path difference of diffraction Grating:

- For the bright fringe, $d\sin\theta = m\lambda$

6.4. Diffraction grating is a multi-slit plate, the maxima occur exactly at the same position as double slits interference.

The bright fringes are much narrower ($\because$ sharpness $\uparrow$) and much brighter ($\because$ sharpness $\uparrow$).

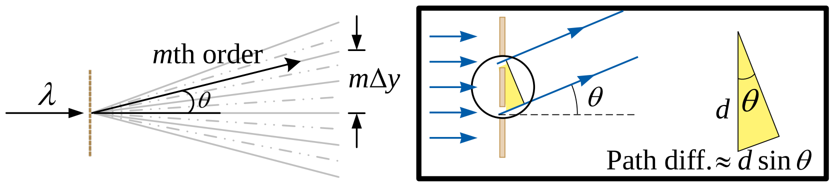


Fig. 11. Path Difference of Diffraction Grating

- Generally, the maxima occur at the position, where

  $d\sin\theta_m = m\lambda$ m = the number of order of maximum and $m = 0, \pm 1, \pm 2, \pm 3...$

  where $d = \dfrac{1}{N}$ = grating separation and N per mm = grating density

- The maximum order is given by $m_{\max} = \dfrac{d\sin\theta_m}{\lambda} \le \dfrac{d}{\lambda}$ ➔ $m_{\max}$ = Integral part of $\left(\dfrac{d}{\lambda}\right)$

6.5. Mathematical Expression of Intensity of Multiple-slit Diffraction Grating

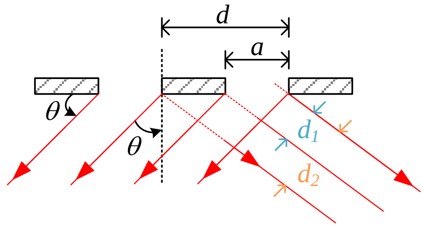


Fig. 12. Path Difference of Diffraction Grating

- The diffraction pattern after a grid can be obtained through superposition of the diffraction patterns of the single slits of the grid. A grid consists of a regular array of N equidistant slits, so that N parallel interfering light beams are obtained.
- The path difference of the border beams of a slit of width a is $d_1 = a\sin\theta$
- This gives a phase difference of $\phi_1 = \dfrac{2\pi d_1}{\lambda} = \dfrac{2\pi}{\lambda} a\sin\theta$
- The path difference of beams coming from two slits is

  $d_2 = d\sin\theta$ where d is the distance between slits (grid constant).
- This yields a phase difference of $\phi_2 = \dfrac{2\pi d_2}{\lambda} = \dfrac{2\pi}{\lambda} d\sin\theta$
- When N beams are deviated to observation point D under a diffraction angle θ, the following intensity is found with the amplitude E_θ of a diffracted beam:

$$I_\theta = I_o \frac{\sin^2\phi}{\phi^2} \;\;➔\;\; I_\theta \approx E_\theta^2 \cdot \frac{\sin^2\left(N\dfrac{\phi_2}{2}\right)}{\sin^2\left(\dfrac{\phi_2}{2}\right)}$$

- Now E_θ^2 is the intensity of the beam where diffracted by a single slit in the direction θ

$$E_\theta^2 \approx \frac{\sin^2\left(\dfrac{\phi_1}{2}\right)}{\left(\dfrac{\phi_1}{2}\right)^2}$$

- The diffraction intensity of the total grid is obtained through combination of equations above:

$$I_\theta \approx E_\theta^2 \cdot \frac{\sin^2\left(N\frac{\phi_2}{2}\right)}{\sin^2\left(\frac{\phi_2}{2}\right)} \Rightarrow \qquad I_\theta \approx \underbrace{\frac{\sin^2\left(\frac{\pi}{\lambda}a\sin\theta\right)}{\left(\frac{\pi}{\lambda}a\sin\theta\right)^2}}_{\text{Single-slit Diffraction Factor}} \cdot \underbrace{\frac{\sin^2\left(N\frac{\pi}{\lambda}d\sin\theta\right)}{\sin^2\left(\frac{\pi}{\lambda}d\sin\theta\right)}}_{\text{Interaction Factor due to } N \text{ slits}}$$

The first part of the product in the above equation is thus the intensity distribution of a single slit, the second part is the result of the interaction of diffraction due to *N* slits. This shows that the minima of the single slits also are maintained in the case of a grid, because if the first factor becomes zero, the product also becomes zero.

According to Fraunhofer, the peaks and minima of <u>one</u> slit are called 1st class interference (i.e. pure single slit diffraction), whereas peaks and minima due to the simultaneous effect of <u>several</u> slits are called 2nd class interference.

6.5.1. <u>Special case:</u> *N* = 1 (single slit)

$$I_\theta = I_o \frac{\sin^2\phi}{\phi^2} \text{ provided that } \phi = \frac{2\pi}{\lambda}a\sin\theta$$

- Considering $\frac{dI_\theta}{d\beta} = 0$ to find the extreme points of I_θ occurring at values of ϕ
- It leads to two equations: $\sin\phi = 0$ or $\phi\cos\phi - \sin\phi = 0$
  ➔ $\phi = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$ or $\tan\phi = \phi$
- $\therefore \phi = \pm 1.4303\pi, \pm 2.4590\pi, \pm 3.4707\pi, \ldots$ (maximum)
  or
  $\phi = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$ (minimum)

6.5.2. <u>Special case:</u> *N* = 2 (double slit)

The following intensity distribution is obtained for diffraction at a slit:

$$I_\theta \approx \underbrace{\frac{\sin^2\left(\frac{\pi}{\lambda}a\sin\theta\right)}{\left(\frac{\pi}{\lambda}a\sin\theta\right)^2}}_{\text{Intensity distribution of a single slit}} \cdot \underbrace{\frac{\sin^2\left(2\frac{\pi}{\lambda}d\sin\theta\right)}{\sin^2\left(\frac{\pi}{\lambda}d\sin\theta\right)}}_{\text{Interaction of diffraction due to 2 slits}}$$

a. 1st class minima (zeros of the numerator of the first factor **only**)

$$\sin^2\left(\frac{\pi}{\lambda}a\sin\theta\right) = 0 \Rightarrow \left(\frac{\pi}{\lambda}a\sin\theta_k\right) = k\pi$$

$$\boxed{\sin\theta_k = \frac{k\lambda}{a}} \qquad k = 1, 2, 3, \ldots\ldots$$

The particular case of the central 1st class peak for $\theta_k = 0$/ $k = 0$ is not seized by above equation.

For $k = 0$, one obtains the central 1st class peak.

For the secondary 1st class peaks one obtains approximately:

$$\boxed{\sin\theta_{k^*} = \frac{2k^*+1}{2}\frac{\lambda}{a}} \qquad k^* = 1, 2, 3, \ldots\ldots$$

b. 2nd class minima (zeros of the numerator of the second factor **AND <u>not</u>** simultaneously zero of the denominator):

$$\sin^2\left(2\frac{\pi}{\lambda}d\sin\theta\right)=0 \text{ and } \sin^2\left(\frac{\pi}{\lambda}d\sin\theta\right)\neq 0$$

$$\Rightarrow \sin\theta_q=\frac{q\lambda}{2d} \text{ and } \sin\theta_q\neq\frac{q\lambda}{d} \text{ where } q=0,1,2,3,\ldots\ldots$$

$q=$	0	1	2	3	4
$\sin\theta_q=\frac{q\lambda}{2d}$	$\sin\theta_q=0$	$\sin\theta_q=\frac{\lambda}{2d}$	$\sin\theta_q=\frac{2\lambda}{2d}=\frac{\lambda}{d}$	$\sin\theta_q=\frac{3\lambda}{2d}$	$\sin\theta_q=\frac{4\lambda}{2d}=\frac{2\lambda}{d}$
$\sin\theta_q\neq\frac{q\lambda}{d}$	$\sin\theta_q\neq 0$	$\sin\theta_q\neq\frac{\lambda}{d}$	$\sin\theta_q\neq\frac{2\lambda}{d}$	$\sin\theta_q\neq\frac{3\lambda}{d}$	$\sin\theta_q\neq\frac{4\lambda}{d}$

The solutions for 2nd class minima are $\sin\theta_q=\frac{\lambda}{2d},\frac{3\lambda}{2d},\frac{5\lambda}{2d},\frac{7\lambda}{2d},\ldots\ldots$

To sum up, it is $\boxed{\sin\theta_k=\frac{(2k+1)\lambda}{2d}}$ $\quad k=0,1,2,3,\ldots\ldots$

c. The common zeros of numerator **AND** denominator of the second factor are the 2nd class peaks (main peaks):

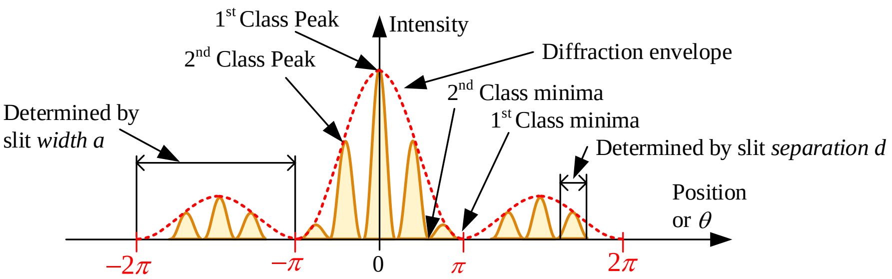
$$\sin^2\left(2\frac{\pi}{\lambda}d\sin\theta\right)=0 \text{ and } \sin^2\left(\frac{\pi}{\lambda}d\sin\theta\right)=0$$

$$\Rightarrow \boxed{\sin\theta_q=\frac{q\lambda}{2d} \text{ and } \sin\theta_q=\frac{q\lambda}{d}} \text{ where } q=0,1,2,3,\ldots\ldots$$

$q=$	0	1	2	3	4
$\sin\theta_q=\frac{q\lambda}{2d}$	$\sin\theta_q=0$	$\sin\theta_q=\frac{\lambda}{2d}$	$\sin\theta_q=\frac{2\lambda}{2d}=\frac{\lambda}{d}$	$\sin\theta_q=\frac{3\lambda}{2d}$	$\sin\theta_q=\frac{4\lambda}{2d}=\frac{2\lambda}{d}$
$\sin\theta_q=\frac{q\lambda}{d}$	$\sin\theta_q=0$	$\sin\theta_q=\frac{\lambda}{d}$	$\sin\theta_q=\frac{2\lambda}{d}$	$\sin\theta_q=\frac{3\lambda}{d}$	$\sin\theta_q=\frac{4\lambda}{d}$

The solutions for 2nd class peak are $\sin\theta_q=0,\frac{\lambda}{d},\frac{2\lambda}{d},\frac{3\lambda}{d},\frac{4\lambda}{d},\ldots\ldots$

$\boxed{\sin\theta_k=\frac{k\lambda}{d}}$ $\quad k=0,1,2,3,\ldots\ldots$



6.5.3. Multiple slit, grid:

The main peaks (2nd class peaks) become more pronounced with increasing number of slits N, because the intensities of these peaks are proportional to N^2.

The envelope of the diffraction pattern is identical to the diffraction pattern of a single slit of width *a*. Between the occurring main peaks, there always are (*N*-2) secondary peaks

6.6. Intensity vs position in multiple-slit interference

6.6.1. Assuming that the slit spacing, *a*, remains unchanged. As the number of slits increases; the positions of the major peaks remain unchanged but the number of subpeaks increases (With lower intensities).

No. of slits | **Intensity vs position**

$N = 2$

$N = 3$

$N = 4$

$N = 5$

$N = \infty$
Diffraction grating

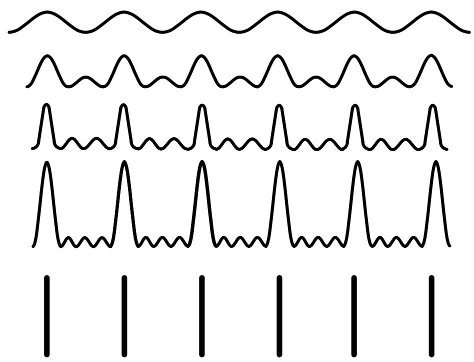
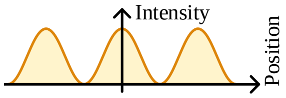
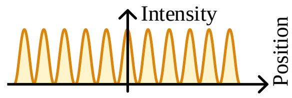


Fig. 14. Intensity vs. Position of Multiple-slit Interference

6.6.2. As a result, diffraction grating = an interference of coherent sources from closely packed multiple slits.

6.6.3. For your information, since the slit width, *d*, is not much longer than the wavelength λ, the formula $\Delta y = \frac{\lambda D}{d}$ is **NOT** applicable for diffraction grating.

6.7. Compare between Double-slit Interference and diffraction grating by using monochromatic visible light source

	Double-slit interference	**Diffraction grating**
Nature	Interference	Diffraction
Graph of intensity-position (Pure interference)		
Mathematical expression	$\Delta s = \frac{\lambda D}{d}$	$a\sin\theta = m\lambda$
Approximation?	Yes. As $\theta \to 0$, $\sin\theta \approx \tan\theta \approx \theta$	No
Fringe separation	Evenly distributed and evenly spaced (Near the central maximum)	Wider as m increases $\because \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{a\cos\theta}$
Slit separation, d	Large	Small
Slit width, a	Large	Small
Number of fringe	Many. About 1000	A few, less than 10.
Width of bright fringe	Wide	Narrow
Intensity	$I_{2\text{-slit}} = I_o \cos^2\left(\frac{\pi}{\lambda} d \sin\theta\right)$	$I_{1\text{-slit}} = I_o \left[\frac{\sin\left(\frac{\pi}{\lambda} a \sin\theta\right)}{\frac{\pi}{\lambda} a \sin\theta}\right]^2$

Applications of diffraction:

a. X-ray diffraction and crystallography

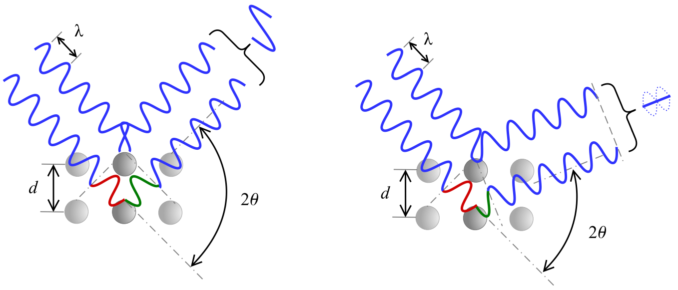


Fig. 16. Bragg's law: $2d\sin\theta = m\lambda$

X rays are light waves that have very short wavelengths. When they irradiate a solid, crystal material they are diffracted by the atoms in the crystal. But since it is a characteristic of crystals to be made up of equally spaced atoms, it is possible to use the diffraction patterns that are produced to determine the locations and distances between atoms. Simple crystals made up of equally spaced planes of atoms diffract x rays according to Bragg's Law.

This is the most commonly used of the structural biology disciplines. Surprisingly, when proteins, DNA, RNA, or complexes of these macromolecules are forced to precipitate out of solution, sometimes the individual macromolecules orientate into an ordered repeating lattice, or crystal. The technique of X-ray crystallography relies on this ability to grow crystals of the protein (or macromolecule) of interest. This is because the ordered lattice of a crystal allows scattering of X-rays (diffraction), which the crystallographer measures and then manipulates by computer to discern the three-dimensional atomic-level structure of the protein or macromolecule.

b. Holography

When two laser beams mix at an angle on the surface of a photographic plate or other recording material, they produce an interference pattern of alternating dark and bright lines. Because the lines are perfectly parallel, equally spaced, and of equal width, this process is used to manufacture holographic diffraction gratings of high quality.

In fact, any hologram (holos—whole: gram—message) can be thought of as a complicated diffraction grating. The recording of a hologram involves the mixing of a laser beam and the unfocused diffraction pattern of some object. In order to reconstruct an image of the object (holography is also known as wavefront reconstruction) an illuminating beam is diffracted by plane surfaces within the hologram, following Bragg's Law, such that an observer can view the image with all of its three-dimensional detail.

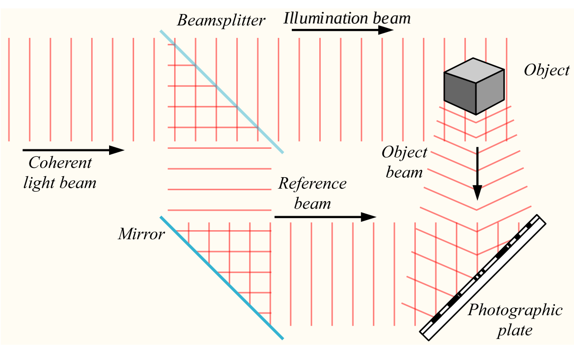


Fig. 17. Recording a hologram

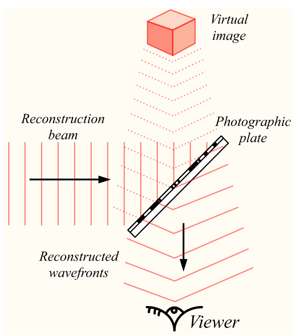


Fig. 18. Reconstructing a hologram

D. SETUP

1. Experimental Apparatus

- HeNe Laser ($\lambda = 633nm$)
- 4 sets of Double Slits
  Width of slits, *a*/ Distance of slits, *d*:
  0.2mm/0.25mm
  0.1mm/0.25mm
  0.1mm/0.5mm
  0.1mm/1.0mm

- Diffraction Grating, 4lines/mm
  N = 2, *d* = 0.25 mm, *a* = 0.1mm
  N = 3, *d* = 0.25 mm, *a* = 0.1mm
  N = 4, *d* = 0.25 mm, *a* = 0.1mm

- Adjusting support
- Mirror
- Photocell
- Sliding device
- Measurement amplifier
- Voltmeter
- Connecting cables

2. Set-up and Procedure

1. The experimental set up is shown in below.

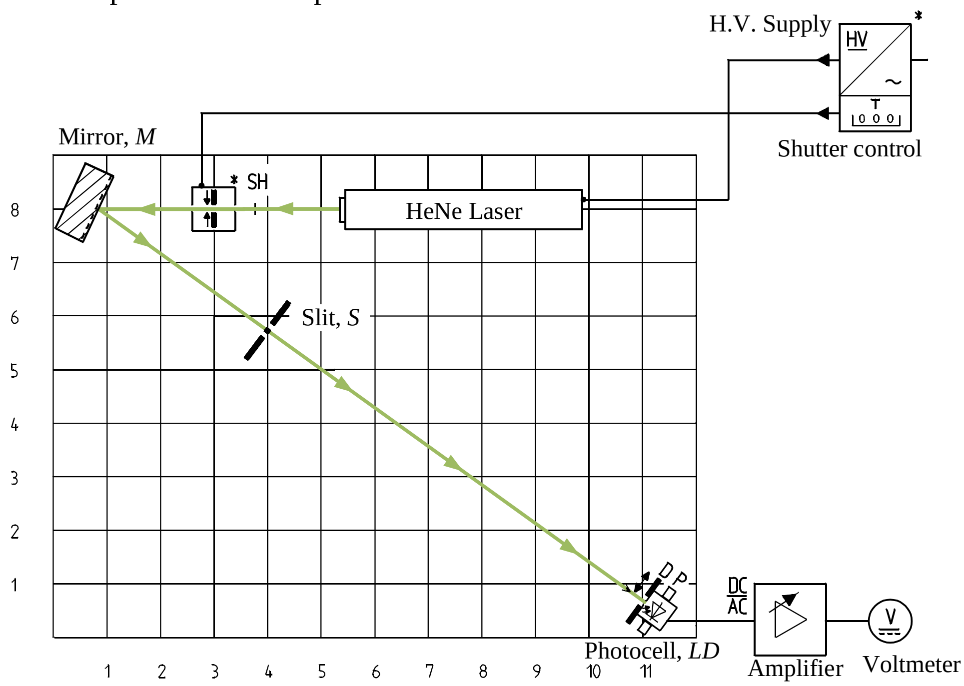


Fig. 19. Experimental set up for diffraction through a double slit or by a grid

2. The laser beam is directed by mirror *M* to the aperture screen with multiple slits *S*.

3. The laser and the measurement amplifier should be warmed up for about 15 minutes before work starts, so as to avoid bothersome intensity fluctuations during measurements.
4. A diffraction pattern is formed after the multiple slits, the intensity distribution (Only extreme points) of which is measured for different numbers of slits, for different slit widths and for different grid constants (exact indications on the multiple aperture screen).

5. Diffraction angle θ is obtained from the relation:

$$\sin\theta = \frac{x}{\sqrt{x^2 + L^2}} = \frac{1}{\sqrt{1 + \left(\frac{L}{x}\right)^2}}$$

where x = distance to the central peak
L = distance between slit and observation plane.

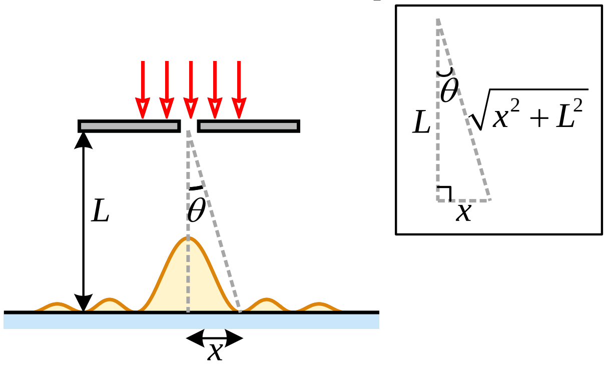
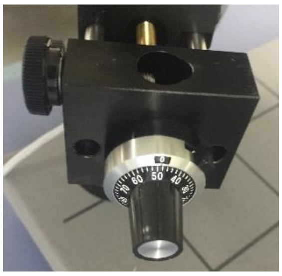


Fig. 20. Geometry of the Setup and the Diffraction Angle

6. A photocell *LD* is used to measure the intensity distribution. For this, the zero of the measurement amplifier is adjusted with the laser switched off to start with. Furthermore, amplification should be adjusted in such a way that the maximum voltage does not increase to more than the peak voltage of about 10 V in the

area of largest intensity (central peak). Amplification of 10^3 and Time constant = 0.1 s is the recommended setting at the beginning.

7. Intensity distribution in the observation plane can be measured by means of the scale on the sliding device (1 unit $\cong 500\mu$m). i.e. experimental values of x



8. **In the case of double slit systems, <u>only the positions of extremes and the intensities of peaks must be determined and linked by straight lines</u>, as the experimental determination of the intermediary intensities is too complicated.**
9. The intensity and location of extreme points have to be measured.
10. General steps to measure the intensity and location of 1st class and 2nd class extreme points:
    1. Measure the distance between slit to screen distance
    2. Set Amplification to 10^3 and Time constant = 0.1 s of the amplifier
    3. Voltmeter sent to 10V first, 10V full scale division = 100 or 30
    4. Otherwise use 30V as scale
    5. Find 0^{th} maximum first:
    6. Locate its relative position.
    7. Find $+1^{st}$ minimum
    8. Find $+2^{nd}$ maximum
    9. Find $+3^{rd}$ minimum
    10. Find -1^{st} minimum
    11. Find -2^{nd} maximum
    12. Find -3^{rd} minimum
    13. Find 75% Intensity of 0^{th} maximum
    14. Find 50% Intensity of 0^{th} maximum
    15. Find 25% Intensity of 0^{th} maximum
11. The obtained distributions are compared to theoretical values. For finding the theoretical values, the following equations should be utilized:

$$I_{\theta,\text{within 1st Class}} \propto a^2 \cdot \frac{\sin^2\left(\frac{\pi}{\lambda} a \sin\theta\right)}{\left(\frac{\pi}{\lambda} a \sin\theta\right)^2}$$

$$\sin\theta = \frac{x}{\sqrt{x^2+L^2}} = \frac{1}{\sqrt{1+\left(\frac{L}{x}\right)^2}}$$

$$N_{total} = 2\frac{d}{a} - 1$$

1st class peaks: $\boxed{\sin\theta_k = \frac{k\lambda}{a}}$ $\quad k = 1, 2, 3, \ldots\ldots$

Secondary 1st class peaks: $\boxed{\sin\theta_{k^*} = \frac{2k^*+1}{2}\frac{\lambda}{a}}$ $\quad k^* = 1, 2, 3, \ldots\ldots$

12. For the theoretical intensity distribution, you should make use of the following program from wolfram
http://demonstrations.wolfram.com/IntensityDistributionForMultipleSlitDiffraction/

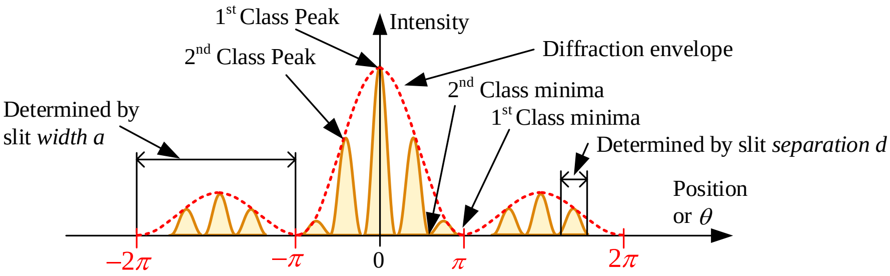
E. Data Collection:

1. 4 sets of Double Slits

$\lambda = 633nm$

$$N_{total} = 2\frac{d}{a} - 1$$

Double Slits:	Set 1	Set 2	Set 3	Set 4
Width of slits, *a*	0.2mm	0.1mm	0.1mm	0.1mm
Distance of slits, *d*	0.25mm	0.25mm	0.5mm	1.0mm
$\frac{a}{\lambda}$				
$\frac{d}{\lambda}$				
Fringe separation: $\Delta y = \frac{\lambda D}{d}$				



Set 1 (Example):

Width of slits, *a*/ Distance of slits, *d*: 0.2mm/0.25mm:

According to $I_\theta \propto a^2 \cdot \frac{\sin^2\left(\frac{\pi}{\lambda}a\sin\theta\right)}{\left(\frac{\pi}{\lambda}a\sin\theta\right)^2} \cdot \frac{\sin^2\left(2\frac{\pi}{\lambda}d\sin\theta\right)}{\sin^2\left(\frac{\pi}{\lambda}d\sin\theta\right)}$

1st class peaks: $\boxed{\sin\theta_k = \frac{k\lambda}{a}}$ $k = 1,2,3,......$

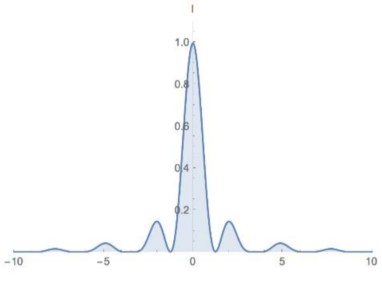
Secondary 1st class peaks: $\boxed{\sin\theta_{k^*} = \frac{2k^*+1}{2}\frac{\lambda}{a}}$ $k^* = 1,2,3,......$

For central 1st class peaks: $\sin\theta_k = 0 \rightarrow I_0 \propto a^2$ (Principle maximum)

For $\sin\theta_k = \frac{3}{2}\frac{\lambda}{a}$:			
Relative intensity:	$I_1 \propto \frac{\sin^2\left(\frac{\pi}{\lambda}a\frac{3}{2}\frac{\lambda}{a}\right)}{\left(\frac{\pi}{\lambda}a\frac{3}{2}\frac{\lambda}{a}\right)^2} = 0.045I_0$ (1st order peak)	Position:	$x = \frac{\pm L}{\sqrt{\left(\frac{2}{3}\right)^2(316)^2 - 1}} = \pm 0.00454689L$
For $\sin\theta_k = \frac{5}{2}\frac{\lambda}{a}$:			
Relative intensity:	$I_2 \propto \frac{\sin^2\left(\frac{\pi}{\lambda}a\frac{5}{2}\frac{\lambda}{a}\right)}{\left(\frac{\pi}{\lambda}a\frac{5}{2}\frac{\lambda}{a}\right)^2} = 0.016I_0$ (2nd order peak)	Position:	$x = \frac{\pm L}{\sqrt{\left(\frac{2}{5}\right)^2(316)^2 - 1}} = \pm 0.00791164L$

$n = \frac{d}{a} = \frac{0.25}{0.2} = 1.25 \rightarrow$ the minima of single and double slits do not coincide.

No. of 2nd class peaks within central band: $N_{total} = 2\times1.25 - 1 = 1.5$



Set 2:

Width of slits, *a*/ Distance of slits, *d*: 0.1mm/0.25mm

According to $I_\theta \propto a^2 \cdot \frac{\sin^2\left(\frac{\pi}{\lambda} a \sin\theta\right)}{\left(\frac{\pi}{\lambda} a \sin\theta\right)^2} \cdot \frac{\sin^2\left(2\frac{\pi}{\lambda} d \sin\theta\right)}{\sin^2\left(\frac{\pi}{\lambda} d \sin\theta\right)}$

1st class peaks: $\boxed{\sin\theta_k = \frac{k\lambda}{a}}$ $\quad k = 1, 2, 3, \ldots\ldots$

Secondary 1st class peaks: $\boxed{\sin\theta_{k^*} = \frac{2k^*+1}{2}\frac{\lambda}{a}}$ $\quad k^* = 1, 2, 3, \ldots\ldots$

For central 1st class peaks: $\sin\theta_k = 0 \rightarrow I_0 \propto a^2$ (Principle maximum)

For $\sin\theta_k = \frac{3}{2}\frac{\lambda}{a}$:			
Relative intensity:		Position:	
For $\sin\theta_k = \frac{5}{2}\frac{\lambda}{a}$:			
Relative intensity:		Position:	
For $\sin\theta_k = \frac{7}{2}\frac{\lambda}{a}$			
Relative intensity:		Position:	

$n = \frac{d}{a} =$

No. of 2nd class peaks within central band: $N_{total} =$

Set 3:
Width of slits, *a*/ Distance of slits, *d*: 0.1mm/0.5mm

According to $I_\theta \propto a^2 \cdot \dfrac{\sin^2\left(\dfrac{\pi}{\lambda} a \sin\theta\right)}{\left(\dfrac{\pi}{\lambda} a\sin\theta\right)^2} \cdot \dfrac{\sin^2\left(2\dfrac{\pi}{\lambda} d\sin\theta\right)}{\sin^2\left(\dfrac{\pi}{\lambda} d\sin\theta\right)}$

1st class peaks: $\boxed{\sin\theta_k = \dfrac{k\lambda}{a}}$ $k = 1, 2, 3, \ldots\ldots$

Secondary 1st class peaks: $\boxed{\sin\theta_{k^*} = \dfrac{2k^*+1}{2}\dfrac{\lambda}{a}}$ $k^* = 1, 2, 3, \ldots\ldots$

For central 1st class peaks: $\sin\theta_k = 0$ ➔ $I_0 \propto a^2$ (Principle maximum)

For $\sin\theta_k = \dfrac{3}{2}\dfrac{\lambda}{a}$:			
Relative intensity:		Position:	
For $\sin\theta_k = \dfrac{5}{2}\dfrac{\lambda}{a}$:			
Relative intensity:		Position:	
For $\sin\theta_k = \dfrac{7}{2}\dfrac{\lambda}{a}$			
Relative intensity:		Position:	

$n = \dfrac{d}{a} =$

No. of 2nd class peaks: $N_{total} =$

Set 4:
Width of slits, *a*/ Distance of slits, *d*: 0.1mm/1.0mm

According to $I_\theta \propto a^2 \cdot \dfrac{\sin^2\left(\dfrac{\pi}{\lambda} a \sin\theta\right)}{\left(\dfrac{\pi}{\lambda} a \sin\theta\right)^2} \cdot \dfrac{\sin^2\left(2\dfrac{\pi}{\lambda} d \sin\theta\right)}{\sin^2\left(\dfrac{\pi}{\lambda} d \sin\theta\right)}$

1st class peaks:

$$\boxed{\sin\theta_k = \frac{k\lambda}{a}} \qquad k = 1, 2, 3, \ldots\ldots$$

Secondary 1st class peaks:

$$\boxed{\sin\theta_{k^*} = \frac{2k^*+1}{2}\frac{\lambda}{a}} \qquad k^* = 1, 2, 3, \ldots\ldots$$

For central 1st class peaks: $\sin\theta_k = 0 \Rightarrow I_0 \propto a^2$ (Principle maximum)

For $\sin\theta_k = \frac{3}{2}\frac{\lambda}{a}$:			
Relative intensity:		Position:	
For $\sin\theta_k = \frac{5}{2}\frac{\lambda}{a}$:			
Relative intensity:		Position:	
For $\sin\theta_k = \frac{7}{2}\frac{\lambda}{a}$			
Relative intensity:		Position:	

$n = \dfrac{d}{a} =$

No. of 2nd class peaks: $N_{total} =$

2. Diffraction Grating with different number of slits, N

$N = 2$, $d = 0.25$ mm, $a = 0.1$mm
$N = 3$, $d = 0.25$ mm, $a = 0.1$mm
$N = 4$, $d = 0.25$ mm, $a = 0.1$mm
$N = 5$, $d = 0.25$ mm, $a = 0.1$mm

$$I_\theta \approx \underbrace{\frac{\sin^2\left(\frac{\pi}{\lambda}a\sin\theta\right)}{\left(\frac{\pi}{\lambda}a\sin\theta\right)^2}}_{\text{Intensity distribution of a single slit}} \cdot \underbrace{\frac{\sin^2\left(N\frac{\pi}{\lambda}d\sin\theta\right)}{\sin^2\left(\frac{\pi}{\lambda}d\sin\theta\right)}}_{\text{Interaction of diffraction due to } N \text{ slits}} \Rightarrow I_\theta \approx N^2$$

Diffraction Grating	$N = 2$	$N = 3$	$N = 4$	$N = 5$
Width of slits, a	0.1mm	0.1mm	0.1mm	0.1mm
Distance of slits, d	0.25mm	0.25mm	0.25mm	0.25mm
Theoretical relative central peak intensity (N=5 as benchmark)			$\left(\frac{5}{4}\right)^2 = 1.56$	/
Experimental absolute central peak intensity				
Experimental relative central peak intensity (N=5 as benchmark)				/

F. Discussion Questions

1. State the assumptions of this experiment
2. For both double slits and multiple slits, there are some locations where minimum (or zero amplitude) should be founded but the reading from photocell is non-zero. Explain this situation.
3. How does the distance from the central maximum to the first minimum in the single-slit pattern compare to the distance from the central maximum to the first diffraction minimum in the double-slit pattern?
4. How many interference maxima are actually in the central envelope for different diagraph of double slits and multiple slits?
5. For a single slit, as the slit width is increased, what happens to the pattern? Does the angle to the first diffraction minimum increase or decrease or stay the same?
6. For a double slit, as the slit width is increased, what happens to the pattern? Does the angle to the first diffraction minimum increase or decrease or stay the same? Does the angle to the first interference maximum increase or decrease or stay the same?

7. For a double slit, as the slit separation is increased, what happens to the pattern? Does the angle to the first diffraction minimum increase or decrease or stay the same? Does the angle to the first interference maximum increase or decrease or stay the same?

G. References:

1. Specification sheet of He-Ne-laser, basic set 08656.93 of PHYWE
2. Specification sheet of Diffraction of Light Through a Double Slit or by a Grid (LP 1.3) of PHYWE
3. Specification sheet of Double slits (https://www.phywe.com/en/geraetehierarchie/physics/optics/optical-components/08523-00#tabs1)
4. Hecht, E. (2001). *Optics 4th edition*. *Optics 4th edition by Eugene Hecht Reading MA AddisonWesley Publishing Company 2001* (Vol. 1, p. 122). Addison Wesley. Retrieved from http://adsabs.harvard.edu/abs/2001opt4.book.....H
5. Jenkins, F. A., White, H. E., & Jenkins, F. A. (1976). *Fundamentals of optics. New York: McGraw-Hill.*

H. APPENDIX:

1. Intensity of Single-Slit Diffraction Pattern

- Let's divide the single slit into N small zones each of width $\Delta w = \frac{a}{N}$. We assume that $\Delta w \ll \lambda$ so that all the light from a given zone is in phase. Two adjacent zones have a relative path difference $\Delta L = \Delta w \sin\theta$. The relative phase difference $\Delta\phi$ is given by the ratio:

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta L}{\lambda} = \frac{\Delta w \sin\theta}{\lambda} \Rightarrow \Delta\phi = \frac{2\pi}{\lambda}\Delta w \sin\theta$$

- Since each successive component has the same phase difference relative the previous one, the electric field from point N is $E_N = E_o \sin\left(\omega t + (N-1)\Delta\phi\right)$
- The total electric field is the sum of each individual contribution:

$$E_{total} = \sum_{k=1}^{k=N} E_k = E_o\left[\sin\omega t + \sin(\omega t + \Delta\phi) + \ldots + \sin\left(\omega t + (N-1)\Delta\phi\right)\right]$$

- Note that total phase difference between the point N and the point 1 is

$$\phi = N\Delta\phi = \frac{2\pi}{\lambda}(N\Delta w)\sin\theta = \frac{2\pi}{\lambda}a\sin\theta$$

- After making use of geometry identities and some simplification,

$$E_{total} = E_o\left[\frac{\sin\left(\frac{\phi}{2}\right)}{\sin\left(\frac{\Delta\phi}{2}\right)}\right]\sin\left(\omega t + (N-1)\frac{\Delta\phi}{2}\right)$$

- By considering the proportionality of intensity I and the time average of E^2, i.e.

$$I \propto \langle E^2 \rangle, \text{ as a result, } I = \frac{I_o}{N^2}\left[\frac{\sin\left(\frac{\phi}{2}\right)}{\sin\left(\frac{\Delta\phi}{2}\right)}\right]^2$$

- In the limit where $\Delta\phi \to 0$, $N\sin\left(\frac{\Delta\phi}{2}\right) \approx N\left(\frac{\Delta\phi}{2}\right) = \frac{N\Delta\phi}{2} = \frac{\phi}{2}$
- So the intensity becomes,

$$I_{\text{1-slit diffraction}} = I_o\left[\frac{\sin\left(\frac{\phi}{2}\right)}{\frac{\phi}{2}}\right]^2 = I_o\left[\frac{\sin\left(\frac{1}{2}\frac{2\pi}{\lambda}a\sin\theta\right)}{\frac{1}{2}\frac{2\pi}{\lambda}a\sin\theta}\right]^2 = I_o\left[\frac{\sin\left(\frac{\pi}{\lambda}a\sin\theta\right)}{\frac{\pi}{\lambda}a\sin\theta}\right]^2$$

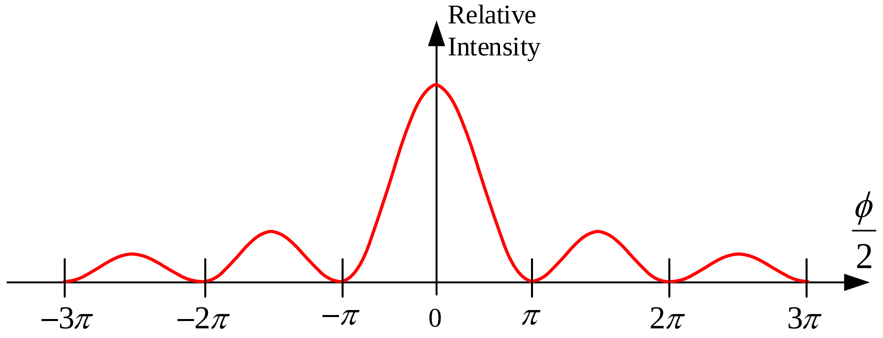


Fig. 3. Intensity Distribution of Single Slit Diffraction

2. Intensity of Double-Slit (Pure) Interference Patterns:
   - Let the two waves emerging from the slits be $E_1 = E_o \sin\omega t$ and $E_2 = E_o \sin(\omega t + \phi)$ and where the waves have the same amplitude E_o and angular frequency $\omega = 2\pi f$, but a constant phase difference $\phi = \frac{2\pi}{\lambda} d \sin\theta$.
   - The total electric field at the point P on the screen is
     $$E_{total} = \sum_{n=1}^{2} E_n = E_1 + E_2 = E_o\left[\sin\omega t + \sin(\omega t + \phi)\right]$$
     $$E_{total} = E_o\left[\sin\omega t + \sin(\omega t + \phi)\right]$$
     $$\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \therefore E_{total} = 2E_o\cos\left(\frac{\phi}{2}\right)\sin\left(\omega t + \frac{\phi}{2}\right)$$
   - The intensity is proportional to $\langle E^2 \rangle$:
     $$I \propto 4E_o^2\cos^2\left(\frac{\phi}{2}\right)\left\langle \sin^2\left(\omega t + \frac{\phi}{2}\right)\right\rangle = 2E_o^2\cos^2\left(\frac{\phi}{2}\right) \qquad \because \frac{\phi}{2} = \text{constant}$$
     or
     $$I = I_o\cos^2\left(\frac{\phi}{2}\right)$$ where $I_o = kE_o^2$ is the maximum intensity on the screen.
   - As a result, $I = I_o\cos^2\left(\frac{1}{2}\frac{2\pi}{\lambda} d\sin\theta\right) = I_o\cos^2\left(\frac{\pi}{\lambda} d\sin\theta\right)$

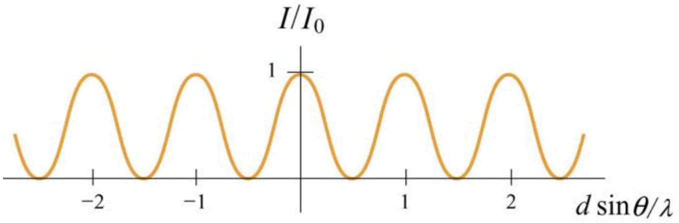


Fig. 8. Intensity as a function of dsinθ/λ

$$I = I_o\cos^2\left(\frac{\pi}{\lambda} d\sin\theta\right) \approx I_o\cos^2\left(\frac{\pi}{\lambda} d\theta\right) = I_o\cos^2\left(\frac{\pi d}{\lambda L} y\right)$$

3. Derivation of the Positions of Fringes and Fringe Separation:

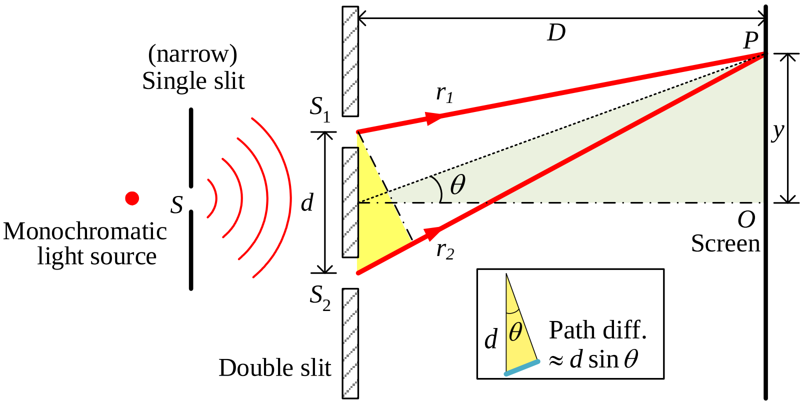


Fig. 6. Setup of Young Double-slit Experiment

- Assume the amplitudes of the two sources S_1 and S_2 are the same and equal to E_o
- If the distance SS_1 and SS_2 are the same, the wave came from two slits are coherent and in-phase.
- From the graph below, at O, the path difference of the two waves from S_1 and S_2 respectively is zero
- O is the 1st central maximum or bright fringe.
- From the graph, $\tan\theta = \dfrac{y}{D}$
- At point P, path difference = $S_2P - S_1P = a\sin\theta$
- For constructive interference of in-phase source,
  - i) path difference = $n\lambda$ i.e. $S_2P - S_1P = d\sin\theta = n\lambda$
  - ii) Resultant amplitude at n^{th} maximum bright fringe = $E_{\text{resultant}} = 2E_o$
  - iii) Resultant intensity at n^{th} maximum bright fringe = $I = 4I_o$
  - iv) There are $(n+1)^{th}$ bright fringe (including the central bright fringe)
- Small angle approximation: $\sin\theta \approx \tan\theta = \dfrac{y}{D}$
- For constructive interference/ bright fringe:

  $\because$ path difference = $S_2P - S_1P = d\sin\theta = n\lambda \rightarrow \therefore d\sin\theta \approx d\tan\theta = \dfrac{dy}{D}$
- At the n^{th} bright fringe, $d\sin\theta = n\lambda$, $d\sin\theta \approx \dfrac{dy_n}{D} = n\lambda$, $y_n = \dfrac{n\lambda D}{d}$
- At the $(n+1)^{th}$ bright fringe, $d\sin\theta = (n+1)\lambda$, $d\sin\theta \approx \dfrac{dy_{n+1}}{D} = (n+1)\lambda$,

  $$y_{n+1} = \frac{(n+1)\lambda D}{d}$$
- The separation between the $(n+1)^{th}$ and n^{th} fringes is

  $$\Delta y = y_{n+1} - y_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d}$$

$\therefore$ The separation of consecutive bright fringes on the screen is $\Delta y = \dfrac{\lambda D}{d}$

4. Special Analysis of diffraction Grating:
   - Suppose there are 10000 slits for the diffraction grating.

   a. **Case one: For constructive interference**
   - For the first order of maximum for the diffraction grating,

     the path difference between the 1st ray and 2nd ray:
     $= d\sin\theta_1 = \lambda$,

     the path difference between the 1st ray and 3rd ray:
     $= 2d\sin\theta_1 = 2\lambda$,

     the path difference between the 1st ray and 4th ray:
     $= 3d\sin\theta_1 = 3\lambda$,

     ……

     the path difference between the 1st ray and 5001st ray:
     $= 5000d\sin\theta_1 = 5000\lambda$,

     Similarly…
     the path difference between the 2nd ray and 5002nd ray:
     $= 5000d\sin\theta_1 = 5000\lambda$,

     the path difference between the 3rd ray and 5003rd ray:
     $= 5000d\sin\theta_1 = 5000\lambda$,

     the path difference between the 4th ray and 5004th ray:
     $= 5000d\sin\theta_1 = 5000\lambda$,

     ……

     the path difference between the 5000th ray and 10000th ray:
     $= 5000d\sin\theta_1 = 5000\lambda$,

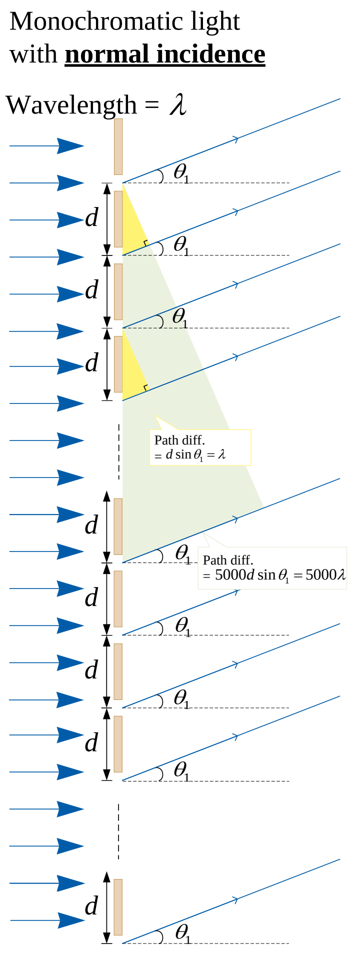


Fig. 9. Constructive Interference of Diffraction Grating

b. **Case two: For destructive interference**

- Consider the system of parallel rays inclined at an angle $\theta_1 + \Delta\theta$, which is slightly greater than the angle, θ_1

the path difference between the 1st ray and 5001st ray:

$$= 5000d\sin(\theta_1 + \Delta\theta) = \left(5000 + \frac{1}{2}\right)\lambda,$$

the path difference between the 2nd ray and 5002nd ray:

$$= 5000d\sin(\theta_1 + \Delta\theta) = \left(5000 + \frac{1}{2}\right)\lambda,$$

the path difference between the 3rd ray and 5003rd ray:

$$= 5000d\sin(\theta_1 + \Delta\theta) = \left(5000 + \frac{1}{2}\right)\lambda,$$

the path difference between the 4th ray and 5004th ray:

$$= 5000d\sin(\theta_1 + \Delta\theta) = \left(5000 + \frac{1}{2}\right)\lambda,$$

……

the path difference between the 5000th ray and 10000th ray:

$$= 5000d\sin(\theta_1 + \Delta\theta) = \left(5000 + \frac{1}{2}\right)\lambda,$$

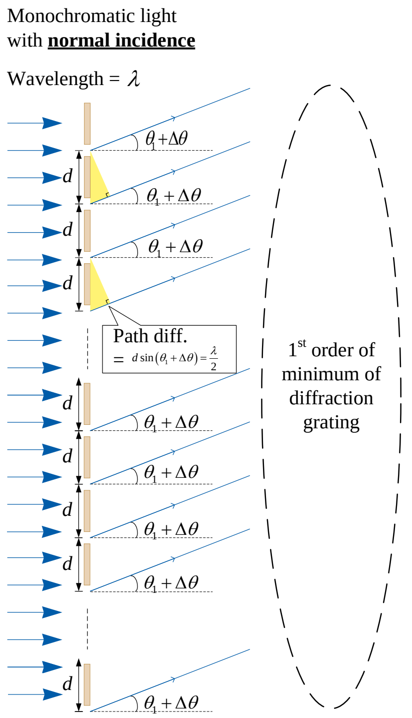


Fig. 10. Destructive Interference of Diffraction Grating

- the path difference between the 1st ray and 2nd ray:

$$= d\sin(\theta_1 + \Delta\theta) = \left(1 + \frac{1}{10000}\right)\lambda$$

- The increasing of angle, $\Delta\theta$, causes a destructive interference. This angle is called **angular half-width**.
- Similarly, by increasing the number of slits to 100000 (i.e. 10 times than 10000).

The path difference between 1st and 2nd ray $= d\sin(\theta_1 + \Delta\theta) = \left(1 + \frac{1}{100000}\right)\lambda$

c. As the number of slits, N, increases,
   i. the angular half-width, $\Delta\theta$, decreases;
   ii. the intensity of maxima increases; and
   iii. the sharpness fringes also increases

b. For N slits, the path difference between 1st and 2nd ray

$$= d\sin(\theta_1 + \Delta\theta) = \left(1 + \frac{1}{N}\right)\lambda$$