

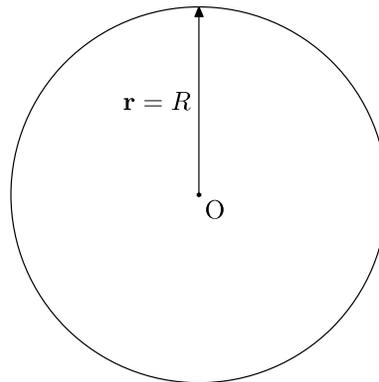
**Laboratory Manual**  
**Experiment NE01 - Centripetal Force**  
**Department of Physics**  
**The University of Hong Kong**

Background

Introduction

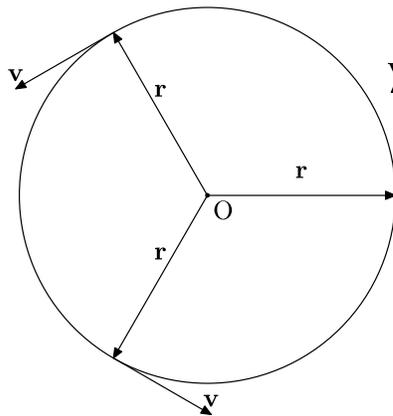
Circular Motion is one of the simplest forms of 2-dimensional motion in which the locus of the object is a circle.

The displacement ( $\vec{r}$ ) measured from the centre (O) of circle has a fixed magnitude (R), which is the radius. The displacement vector rotates as the object moves along the path.



*Figure 1: Circular path.*

The velocity ( $\vec{v}$ ) is tangential to the circle, which is always perpendicular to the displacement  $\vec{r}$ .



*Figure 2: Velocity and displacement.*

Since the velocity changes its direction with time, there must be acceleration ( $\vec{a}$ ) on the object. In uniform circular motion, where the speed is constant with time, the acceleration does not contain tangential component. Acceleration is then perpendicular to velocity and is anti-parallel to displacement.

**Centripetal Force** ( $\vec{F}_c$ ) is the force acting on a body such that the direction of the force always points towards a fixed point (centre). By Newton's Second Law,  $\vec{F}_c = m\vec{a}$ , and  $\vec{a}$  is the **centripetal acceleration**.

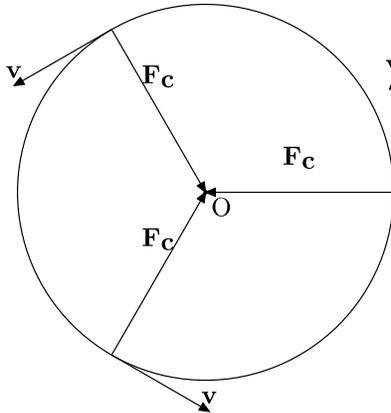


Figure 3: Velocity and centripetal force.

### Experiment

At this point, one may wonder how the magnitude of centripetal force varies with mass of object, radius of the circular path, and the speed of object. In this experiment, their relations will be investigated. The magnitudes of centripetal force will be measured by economy force sensor. The economy force sensor then sends the data to the data logger. Different masses and radius can be chosen in the Centripetal Force Apparatus. The linear speed of the object, or the angular speed of rotation, will be measured by the photogate and recorded in the data logger. The software *DataStudio*® plots the relations between centripetal force and mass, radius, linear speed or angular speed.

The expression of centripetal force in terms of mass, radius and speeds will be derived below.

### Derivation

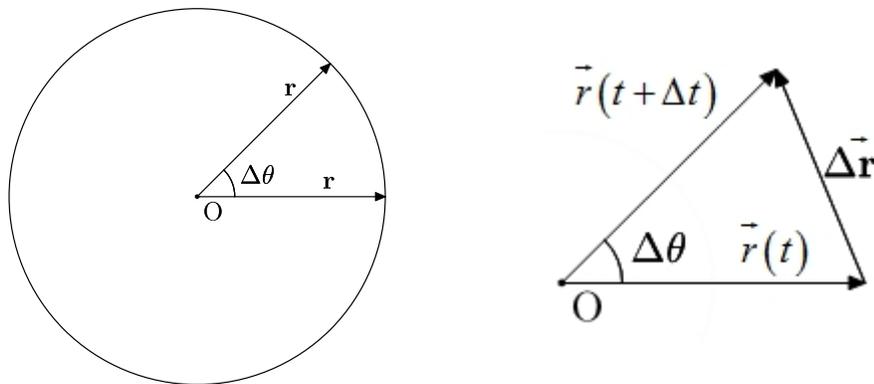


Figure 4: Change in displacement vector.

From vector addition, we know that

$$\Delta \vec{r} = \vec{r}(t + \Delta t) - \vec{r}(t)$$

Using simple trigonometry and figure 4,

$$\begin{aligned} \frac{|\Delta \vec{r}|}{2} &= |\vec{r}| \sin\left(\frac{\Delta\theta}{2}\right) \\ |\Delta \vec{r}| &= 2|\vec{r}| \sin\left(\frac{\Delta\theta}{2}\right) \approx |\vec{r}| \Delta\theta \end{aligned}$$

For simplicity in notation, let  $R = |\vec{r}|$ . Then from the definition of velocity,

$$\begin{aligned} |\vec{v}| &= \left| \frac{d\vec{r}}{dt} \right| = \left| \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} \right| \\ &= \left| \lim_{\Delta t \rightarrow 0} \frac{R \Delta\theta}{\Delta t} \right| \\ &= R \left| \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} \right| \\ &= R |\omega| \end{aligned}$$

where  $\omega$  is the **angular velocity** [rad s<sup>-1</sup>] of the object, describing the rate of change of direction.

Therefore, we have  $|\vec{v}| = R|\omega|$  for uniform circular motion. Similarly, we can also derive the

acceleration  $|\vec{a}| = R|\omega|^2 = \frac{|\vec{v}|^2}{R}$ .

In conclusion, the magnitude of centripetal force is  $|\vec{F}_c| = mR|\omega|^2 = m \frac{|\vec{v}|^2}{R}$ .

### Apparatus

- 1 ME-8088 frame with mounted electric motor
- 2 Nuts for mass holders (on ME-8088 frame)
- 1 Thumbscrew for photogate (on ME-8088 frame)
- 1 *ScienceWorkshop*® 500 Interface data logger
- 1 Photogate head
- 1 Photogate cable (Part No. 514-06266)
- 1 Economy force sensor (CI-6746)
- 1 Connecting cable (with 2 holes at both ends and a clip)
- 1 Ball bearing swivel

- 2 5 g mass
- 2 10 g mass
- 2 20 g mass
- 1 45 cm steel rod
- 1 120 cm steel rod
- 1 Large rod stand
- 1 Multi clamp (ME-9507)
- 2 Banana plugs (1 red and 1 black)
- 1 Power supply
- 1 AC adapter
- 1 AC socket connecting cable
- 1 USB/Serial converter (CI-6759A)
- 1 Computer with *DataStudio*® Software installed

### Setup Procedures

1. Mount the 120 cm steel rod into the large rod stand vertically.
2. Setup the photogate as follows:
  - 2.1. Connect the photogate head and *ScienceWorkshop*® 500 Interface data logger (plug into digital channel 1) with the photogate cable.
  - 2.2. Remove the small rod clamp and thumbscrew at the side of photogate head and do not lose it.

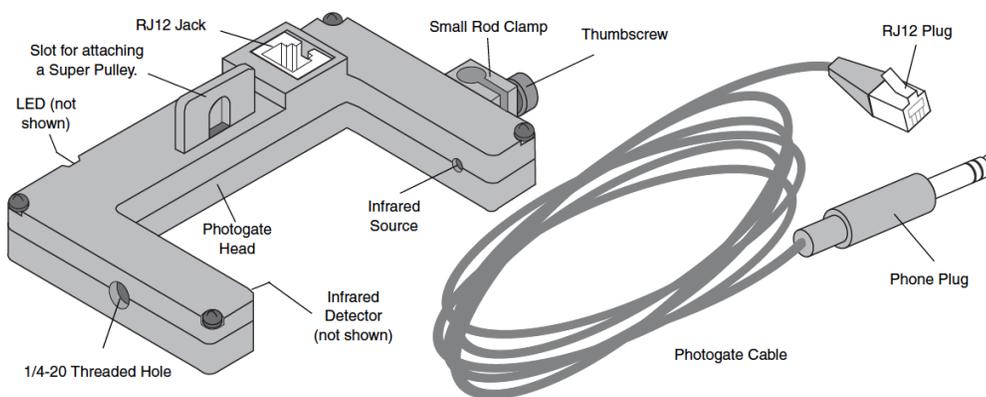


Figure 5: Photogate head.

3. Setup the Centripetal Force Apparatus as follows:
  - 3.1. Check the 2 nuts for mass holders are in the screws on the arms.

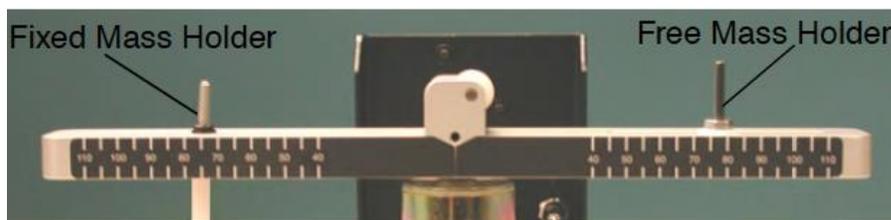


Figure 6: Mass holders.

- 3.2. Check the thumbscrew for photogate is in the screw at the base of ME-8088 frame.
- 3.3. Connect the ball bearing swivel to the clip of the connecting cable.
- 3.4. Thread the connecting cable to the free mass holder through the pulley.

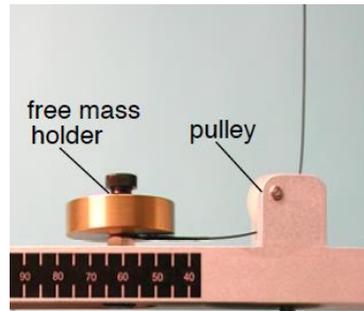


Figure 7: Connecting cable through the pulley.

- 3.5. Mount the connected ball bearing swivel into the economy force sensor.
- 3.6. Mount the photogate into the screw at the base of ME-8088 frame.

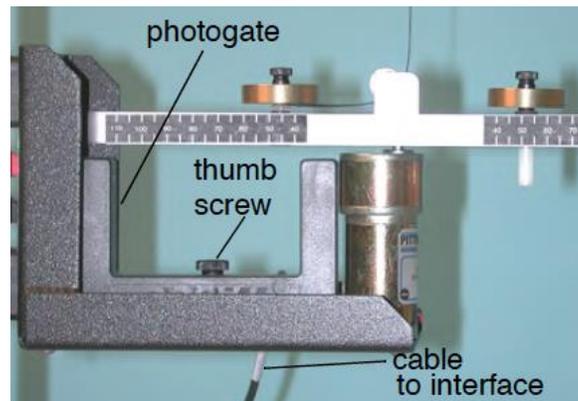


Figure 8: Centripetal Force Apparatus.

4. Mount the Centripetal Force Apparatus onto the vertical 120 cm steel rod.
5. Mount the multi clamp onto the vertical 120 cm steel rod.
6. Insert the 45 cm steel rod horizontally through the multi clamp.
7. Mount the economy force sensor onto the horizontal 45 cm steel rod.
8. Adjust the positions (both vertical and horizontal) of the economy force sensor such that the connecting cable is vertical.
9. Plug the cable of economy force sensor into *ScienceWorkshop*® 500 Interface data logger (plug into analog channel A).
10. Connect the AC adapter and AC socket connecting cable.
11. Plug the AC adapter into terminal '9 VDC 500mA' on *ScienceWorkshop*® 500 Interface data logger.
12. Connect the *ScienceWorkshop*® 500 Interface data logger with computer using USB/Serial converter. Plug the converter cable into terminal 'TO COMPUTER'.
13. Connect the Centripetal Force Apparatus and the power supply with red and black banana plugs.
14. Connect the power supply and AC socket connecting cable to the sockets.
15. Turn the Voltage of the power supply to zero before proceed.

## Experiment 1: Centripetal force and linear velocity

### Introduction

In this experiment, you will vary the velocity by changing the voltage to the electric motor as the centripetal force is continuously measured by the Force Sensor. The radius and mass are held constant as the velocity is increased. A “Force vs linear velocity” curve is obtained for 15g, 25g, 20 g and 30 g of free mass and fixed mass respectively.

### Experimental Procedures

1. Open the *DataStudio*® working file ‘NE01\_EXP1\_UID.ds’ in the NE01 folder. Click ‘Save Activity As’ function to save your own file where UID should be your/your partner University no.(e.g. NE01\_EXP1\_3333123456.ds)

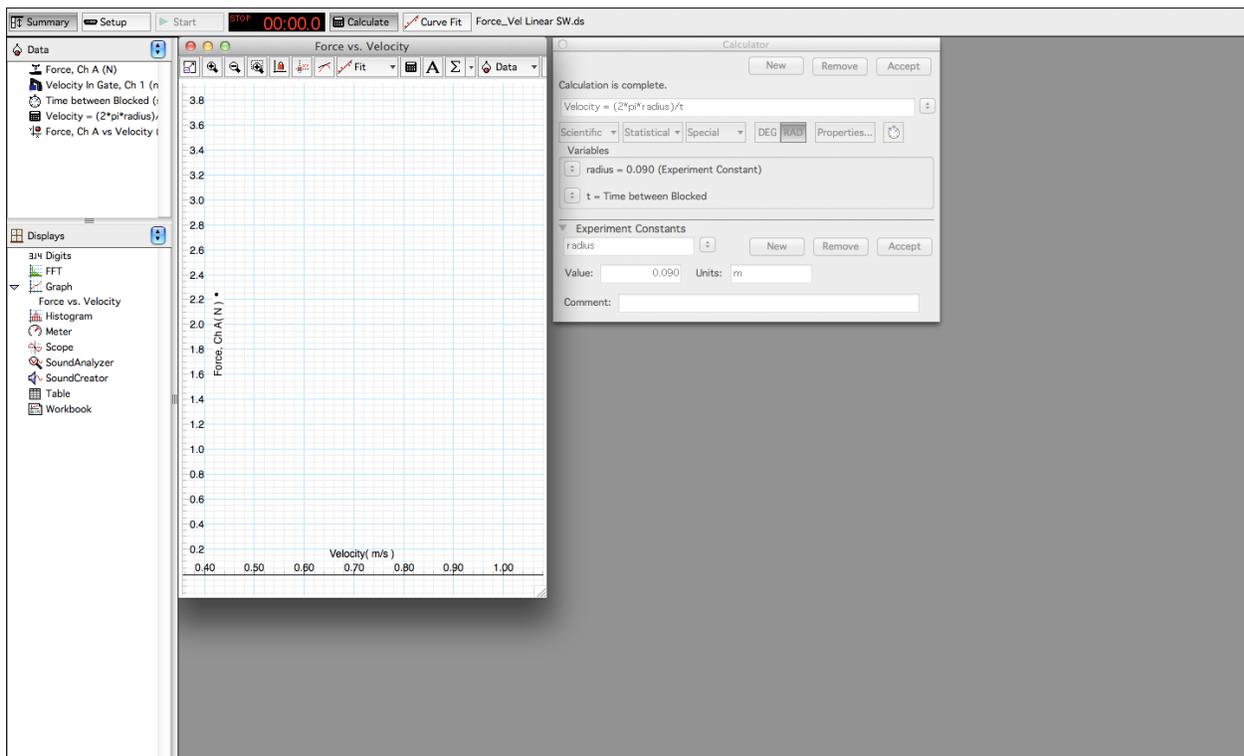


Figure I.1: *DataStudio*® file ‘EXP1\_UID.ds’.

2. Adding a 15g mass to the free mass holder. Then add equal amount of mass to the fixed mass holder.
3. Using 90 mm for the position of free mass (i.e. radius). Then, in the ‘Calculator’ window, input the ‘Value’ for radius and click ‘Accept’.

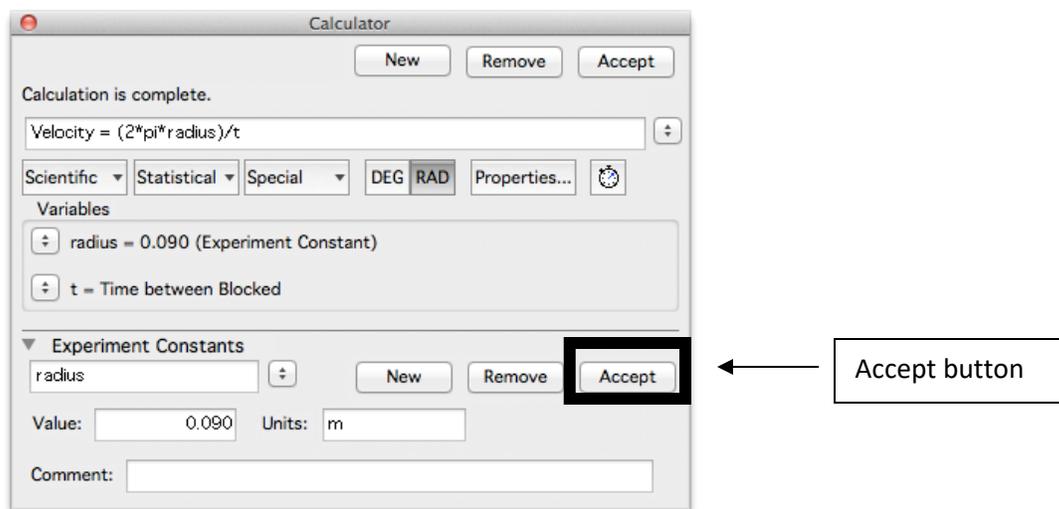


Figure I.2: 'Calculator' window.

4. Adjust the position of fixed mass holder to the 90 mm.
5. Make sure the positions of fixed mass and free mass are symmetric.
6. Move the economy force sensor upward or downward to ensure the string connecting the free mass holder in tension at position = 90 mm.
7. Fix the position of both mass holders by the nuts. **Record the positions used in free mass holder and fixed mass holder in the table 1.1 of worksheet.**
8. Click 'Start' in the *DataStudio*® window to start data logging.
9. Push the 'Tare' button on the economy force sensor to set zero.
10. Slowly increase the Voltage from 0V to 8V such that the Centripetal Force Apparatus starts rotating gradually.
 

**WARNING: Do not stand next to the motor or rotating arm or look at the rotating arm at eye level. To avoid possible injury from the rotating arm hitting the body, keep at least 1 foot distance from the motor and rotating arm when running the motor.**

**[NOTE: The maximum voltage is 8V!!! Otherwise, the motor burns out and the fast rotating arm will injure you or your classmates]**
11. Plots will be shown on the 'Force vs. Velocity' graph.
12. When the voltage is adjusted to 8V, click 'Stop' in the *DataStudio*® window to stop data logging.
13. Turn off the power supply by adjusting the Voltage of the power supply to 0V.
14. Try a suitable fitting function in  so as to verify the derived expression of force vs. linear velocity (speed).
 

**[NOTE: Please read the Derivation part if you are not sure what kind of fitting function you should use]**
15. Fill the fitting parameter, A, B, C, mean squared error and root MSE in the table 1.2 of worksheet.
16. Repeat the measurement through steps (3) to (1) with different mass 15g, 20g, 25g and 30g respectively.
17. Save your experiment result and exit the program.

## Experiment 2: Centripetal force and angular velocity

### Introduction

In this experiment, you will vary the angular velocity by changing the voltage to the electric motor as the centripetal force is continuously measured by the Force Sensor. The radius and mass are held constant as the angular velocity is increased. A “Force vs linear velocity” curve is obtained for position of the free mass holder and fixed mass holder at 80 mm, 90 mm and 100 mm respectively.

### Experimental Procedures

1. Open the DataStudio® working file ‘NE01\_EXP2\_UID.ds’ in the NE01 folder. Click ‘Save Activity As’ function to save your own file where UID should be your/your partner University no.(e.g. NE01\_EXP2\_3333123456.ds)

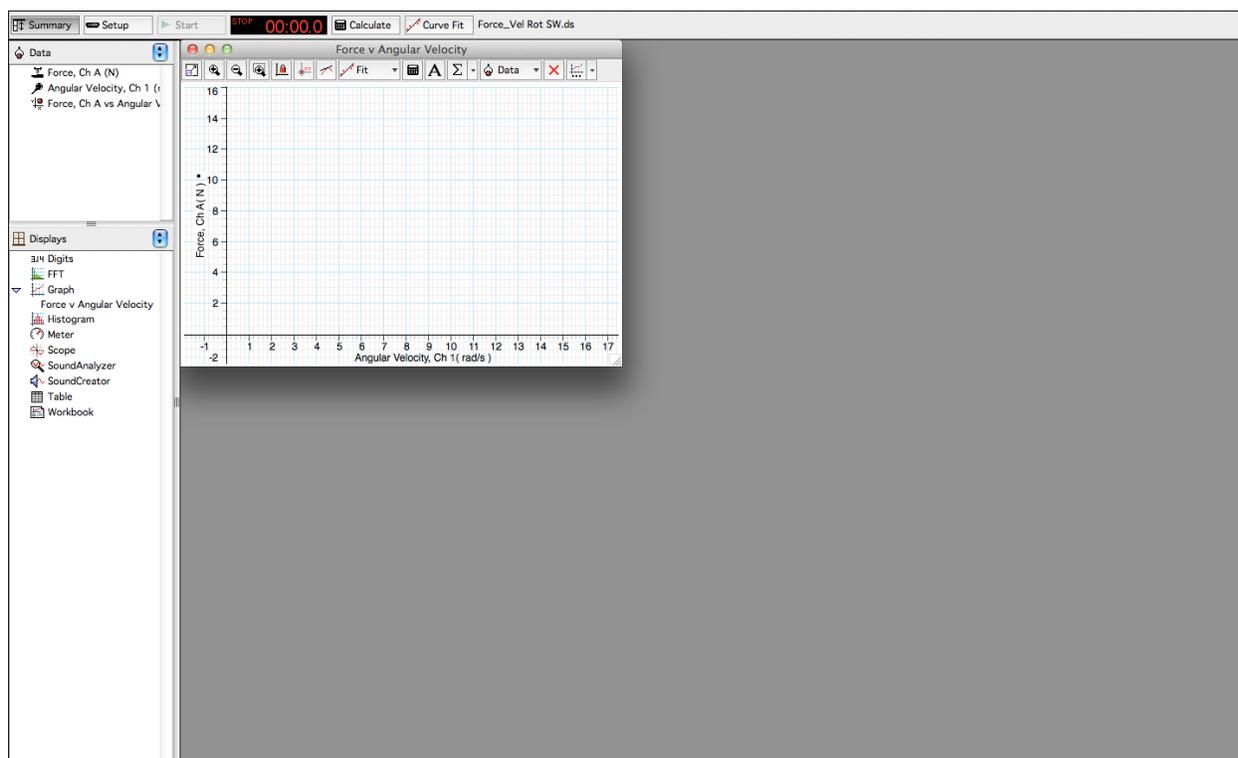
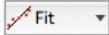


Figure II.1: DataStudio® file ‘EXP2\_UID.ds’.

2. Adding a 20g mass to the free mass holder. Then add equal amount of mass to the fixed mass holder.
3. Using 90 mm for the position of free mass (i.e. radius).
4. Adjust the position of fixed mass holder to the selected radius in step (3). **Record the masses used in free mass holder and fixed mass holder in the table 2.1 of worksheet.**
5. Move the economy force sensor upward or downward to ensure the string connecting the free mass holder in tension at position used in step (3).
6. Fix the position of both mass holders by the nuts.
7. Click ‘Start’ in the DataStudio® window to start logging.

8. Push the ‘Tare’ button on the economy force sensor to set zero.
9. Slowly increase the Voltage from 0V to 8 V such that the Centripetal Force Apparatus starts rotating gradually.  
**[NOTE: The maximum voltage is 8V!!! Otherwise, the motor burns out and the fast rotating arm will injure you or your classmates]**
10. Plots will be shown on the ‘Force vs. Angular Velocity’ graph.
11. When the voltage is adjusted to 8V, click ‘Stop’ in the *DataStudio*® window to stop data logging.
12. Turn off the power supply by adjusting the Voltage of the power supply to 0V.
13. Try a suitable fitting function in  so as to verify the derived expression of force versus angular velocity.  
**[NOTE: Please read the Derivation part if you are not sure what kind of fitting function you should use]**
14. Fill the fitting parameter, A, B, C, mean squared error and root MSE in the table 2.2 of worksheet.
15. Repeat the measurement through steps (8) to (11) with changing another position of free mass, at 80 mm and 100 mm respectively in step (3). Remember to change the position of fixed mass to ensure the positions of fixed mass and free mass are symmetric in step (4).
16. Save your experiment result and exit the program.

**WARNING:** Do not stand next to the motor or rotating arm or look at the rotating arm at eye level. To avoid possible injury from the rotating arm hitting the body, keep at least 1 foot distance from the motor and rotating arm when running the motor.

### Experiment 3: Centripetal force and mass

#### Introduction

In this experiment, the radius and velocity are held constant as the mass is varied. By adding additional drilled masses to the holder, the mass of the system is increased. Equal amounts of mass must also be added to the “fixed mass” to balance the arm as it rotates. Centripetal force is directly measured by the Force Sensor.

#### Experimental Procedures

1. Open the *DataStudio*® working file ‘NE01\_EXP3\_UID.ds’ in the NE01 folder. Click ‘Save Activity As’ function to save your own file where UID should be your/your partner University no.(e.g. NE01\_EXP3\_3333123456.ds)

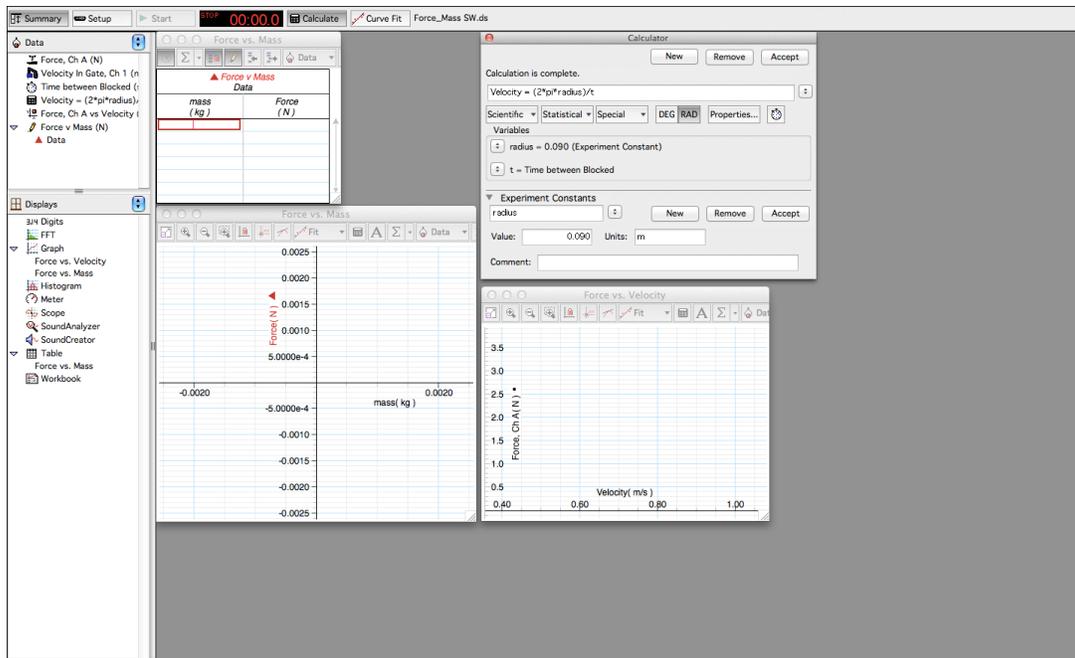


Figure III.1: DataStudio® file 'Force\_Mass SW.ds'.

- Using 90 mm for the position of free mass (i.e. radius). Then, in the 'Calculator' window, input the 'Value' for radius and click 'Accept'.



Figure III.2: 'Calculator' window.

- Using a 20g mass for free mass holder. Then add equal amount of mass to the fixed mass holder.
- Adjust the position of fixed mass holder to the 90 mm. **Record the positions used in free mass holder and fixed mass holder in the table 3.1 of worksheet.**
- Make sure the positions of fixed mass and free mass are symmetric.
- Make sure the string connecting the free mass holder in tension at position = 90 mm by moving the economy force sensor upward or downward.
- Fix the position of both mass holders by the nuts.
- Click 'Start' in the DataStudio® window to start data logging.

9. Push the 'Tare' button on the economy force sensor to set zero.
10. Click 'Stop' in the *DataStudio*® window.
11. Turn on the power supply.
12. Slowly increase Voltage from 0V to 6V such that the Centripetal Force Apparatus starts rotating gradually.  
**[NOTE: The maximum voltage is 8V!!! Otherwise, the motor burns out and the fast rotating arm will injure you or your classmates]**
13. Keep the voltage fixed at 6V.
14. Click 'Start' in the *DataStudio*® window.
15. Plots will be shown on the 'Force vs. Velocity' graph.
16. When the mean value of the force is stable, click 'Stop' in the *DataStudio*® window.
17. Turn off the power supply by adjusting the Voltage of the power supply to 0V.
18. Calculate mean value for 'Force' by using the functions in .

**WARNING: Do not stand next to the motor or rotating arm or look at the rotating arm at eye level. To avoid possible injury from the rotating arm hitting the body, keep at least 1 foot distance from the motor and rotating arm when running the motor.**

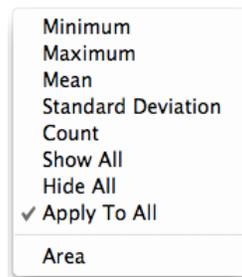


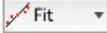
Figure III.3: Functions in .

19. Enter the value of 'Mass' you used in step (5) and mean value of 'Force' from step (16) in the 'Force vs. Mass' table, with suitable units. **Record the values in the table 3.2 of worksheet.**

Force vs. Mass	
Data	
mass (kg)	Force (N)

Figure III.4: 'Force vs. Mass' table.

20. A plot will be generated in the 'Force vs. Mass' graph.
21. Replace the free mass with 5g, 10g, 15g, 20g, 25g, 30g and 35g respectively in step (5), to repeat the measurement through steps (4) to (20).

22. After measuring all different masses, try a suitable fitting function in  so as to verify the derived expression of force vs. mass.

[NOTE: Please read the Derivation part if you are not sure what kind of fitting function you should use]

23. Save the working file and exit the program.

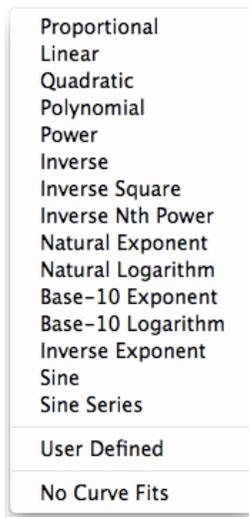
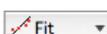


Figure III.5: Functions in .

### Appendix:

A Taylor series is a series expansion of a function about a point. A one-dimensional Taylor series is an expansion of a real function  $f(x)$  about a point  $x = a$  is given by

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \dots$$

Taylor series of some common functions include

$$\begin{aligned} \frac{1}{1-x} &\approx 1 + x + x^2 + x^3 + \dots && \text{for } |x| < 1 \\ \sin x &\approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \\ \cos x &\approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \\ e^x &\approx 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \end{aligned}$$

### References

PASCO scientific. (n.d.). 012-08478B Centripetal Force Manual. Roseville, CA, USA: PASCO scientific.