THE UNIVERSITY OF HONG KONG
Department of Physics

PHYS2265 Modern physics

Laboratory manual 2265-2: Emission spectrum of atomic hydrogen

Our goal in this experiment is to measure the Rydberg constant R_{H}. Nowadays, the Rydberg constant can already be measured with a percentage error less than 7 parts out of a trillion, making it one of the most precise measurements made for physical constants. Modern measurements of Rydberg constant requires consideration of quantum electrodynamics, hyperfine splitting, etc., which are beyond the scope of this course. Nevertheless, its earliest measurement was made empirically in the nineteenth century, and later it was explained by using Bohr's model, a semi-classical model for hydrogen proposed in the 1910s.

1 Bohr's model for hydrogen atom

A hydrogen atom consists of a hydrogen nucleus (proton) and an electron bound by Coulomb attraction. The huge mass difference between these two particles allows us to approximate the system by assuming a stationary nucleus, leaving us the 'orbit' of electron to be investigated. To derive its wave function, we should solve the Schrödinger with Coulomb potential. But if orbital energy of electron is the only concern, the Bohr's model is all we need.

In Bohr's model, it was postulated that an electron orbits in circular motion around the hydrogen nucleus due to the presence of Coulomb force. Suppose we assume the nucleus is at rest, then we can obtain the total energy

$$E = \text{K.E.} + \text{P.E.} = -\frac{1}{2} \cdot \frac{e^2}{4\pi\epsilon_0 r} \; .$$

Another postulate that Bohr made was that angular momentum of the electron is quantized, i.e.

$$p_e = m_e v r_n = \frac{nh}{2\pi} \; ,$$

where $n = 1, 2, 3, \ldots$ is called the *principal quantum number*; we denote the radius as r_n to emphasize its quantization. By considering the centripetal force acting upon the electron, we can eliminate v from this expression and arrive at

$$r_n = \frac{\epsilon_0 h^2}{\pi m_e e^2} \equiv a_0 n^2 \; ,$$

where $a_0 \approx 0.529\,\text{Å}$ is called the *Bohr radius*. Since the total energy is a function of radius, quantization also takes place, i.e.

$$E_n = -\frac{e^2}{8\pi\epsilon_0 a_0 n^2} = -\frac{m_e e^4}{8h^2\epsilon_0^2 n^2} \; .$$

All these energy levels form stationary states of electron, so if there was no external perturbation, an electron will stay at the same energy level forever. However, there are always some perturbations due to the zero-point energy of electromagnetic field[1]. When an electron makes a transition downward from E_i to E_f, where $E_i > E_f$, a photon with energy $hc/\lambda = E_i - E_f$ will emit. In other words, we have

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \; ,$$

where *Rydberg constant* is introduced to be

$$R_H \equiv \frac{m_e e^4}{8h^3 c \epsilon_0^2} \approx 1.097 \times 10^7\,\mathrm{m}^{-1} \; .$$

[1]To explain spontaneous transition of electron, we must invoke quantum electrodynamics, which is beyond the scope of this course.

Task 1: Visible spectrum of hydrogen

Hydrogen spectral series are named according to the final states, for example, Lyman series ($n_f = 1$), Balmer series ($n_f = 2$) and Paschen series ($n_f = 3$). In this experiment, we are going to observe the spectrum with our naked eyes. Make some simple calculations to determine the series that will be visible to us. Also, try to write down the colors of spectral lines that you expect to observe.

2 Diffraction grating

To project the emission spectrum on some screen, we need to use a diffraction grating to separate the lights being emitted. Hence, we should first spend some time to understand the behaviour of our grating setup.

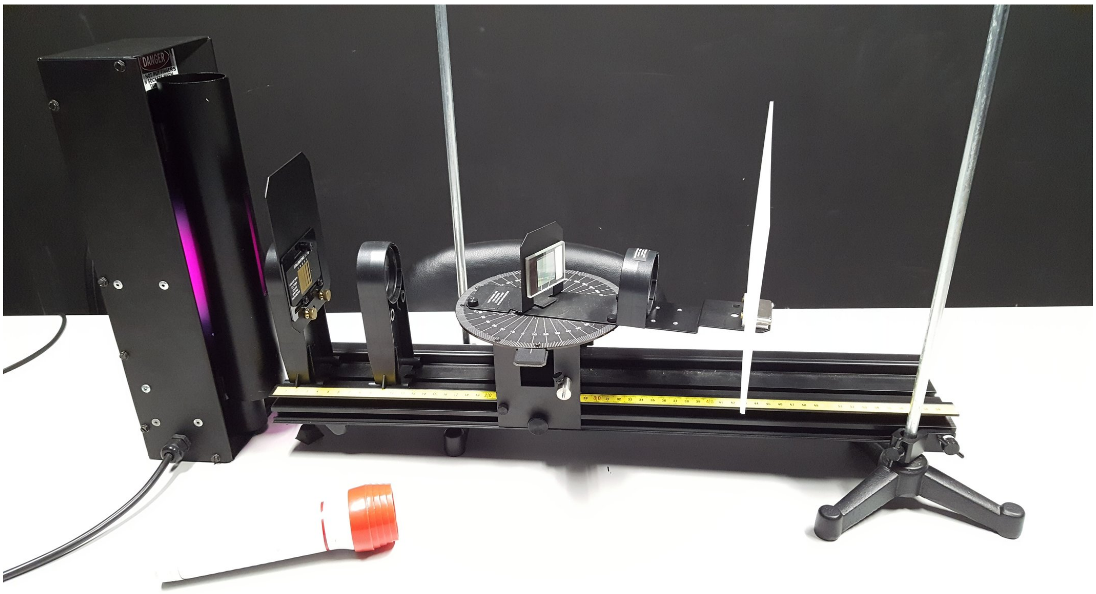


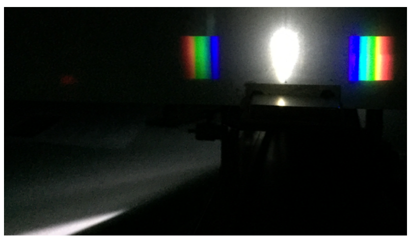
Figure 1: This experiment will be conducted inside a darkroom. It may take your eyes some time (around 20 - 30 minutes) to fully reach dark adaptation. After that, you might be able to observe the dim spectral lines. A torch is given to assist measurements in dark environment.

Task 2: Diffraction grating

Before you begin, please take some time to understand various components of the setup in Figure 1. Please use the Internet to make useful searches. You may also want to know the grating constant being provided.

In order to understand how a diffraction grating bends lights of different wavelengths, we put some light source that emits continuous spectrum over the range of visible light. Qualitatively describe the spectrum you see. To make things quantitative, you will need to derive a formula that relates diffraction angle θ and wavelength λ, then verify it experimentally. Of course, this is not a precise measurement but you can at least make a rough check by knowing the wavelengths of different colors, e.g. red light is about $620\,\mathrm{nm}$ to $740\,\mathrm{nm}$.

Hint: The formula should be in the form of $a \sin\theta = k\lambda$. It is your duty to figure out what does each variable mean, how can you measure/determine it from this experiment, and how can one derive it. For continuous spectrum, you would expect a spectrum similar to the right figure.



3 Measuring Rydberg constant R_{H}

Task 3: Measuring Rydberg constant R_{H}

Having understood the relation between diffraction angle θ and wavelength λ, you should now be able to come up with a method measure Rydberg constant; describe your method in the report. The number of data points you can take highly depends on how many spectral lines you can see. You should use at least three spectral lines for each order and measure up to the second order. Error analysis must not be omitted.

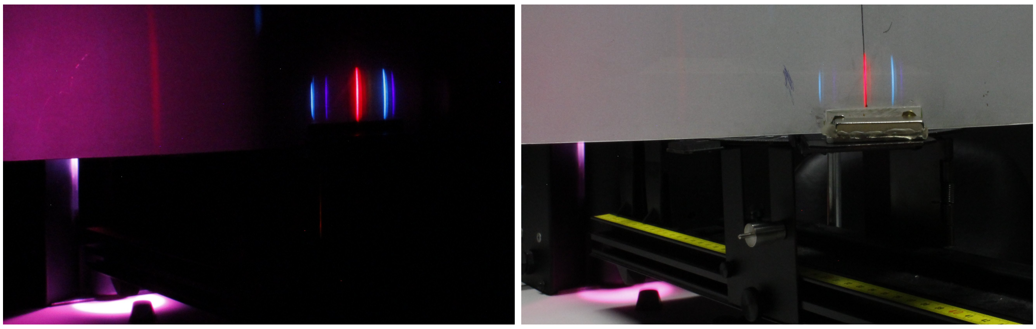


Figure 2: Left figure shows what you will expect to observe, except for the interference which only occured due to the aperture of camera; right figure was captured with double exposure, so the intensities of spectral lines are expected to be dimmer when being seen with naked eyes.

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Discussion

A few questions have been listed below for you to ponder:

- Alignment and focus of light are extremely crucial to make measurement accurate. Discuss all the details one should pay attention to when doing this experiment.
- Will using a prism instead of a grating yield better measurement?

Discuss them in your report under the "Discussion" section. Feel free to include more constructive comments.