# From Sachdev–Ye–Kitaev Model to Strongly Correlated Metals

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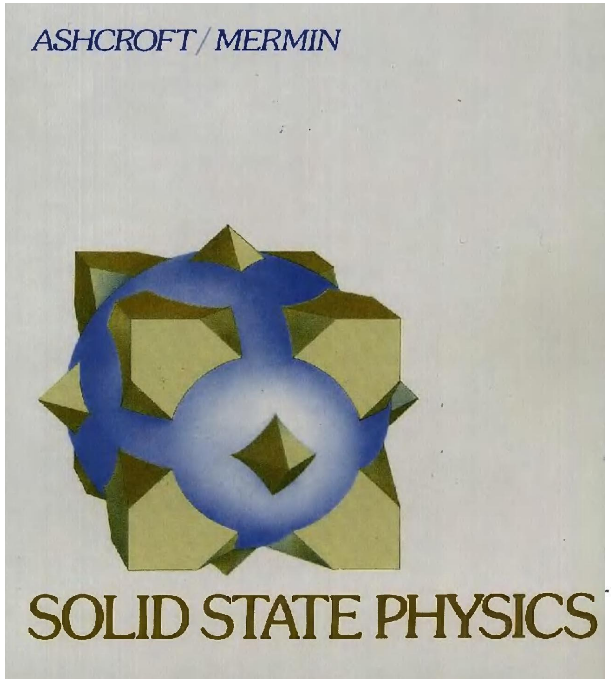
HKU-UCAS Young Physicist Forum

# Conventional Metal

- A conventional metal is described by the Fermi liquid theory.
- Adiabatically connected to a Fermi gas

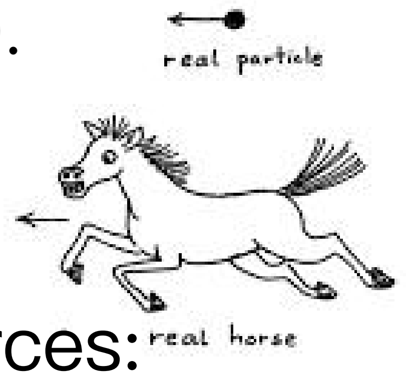


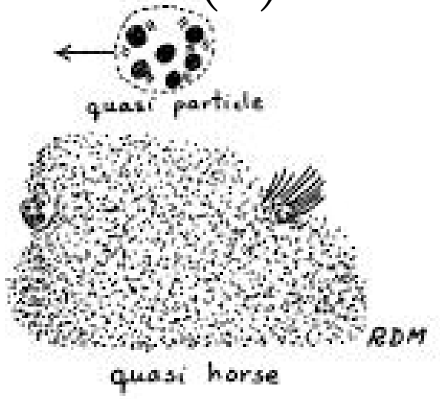

# 1
## *The Drude Theory of Metals*

# Fermi Liquid and their cousins at T=0 (single particle property)

A conventional metal is described by the Fermi liquid theory.

Fermi Liquid: fermionic quasiparticles with density of states $\mathcal{N}(\varepsilon) \sim$ constant and a lifetime $1/\tau(\varepsilon) \ll |\varepsilon|$ as $|\varepsilon| \to 0$.



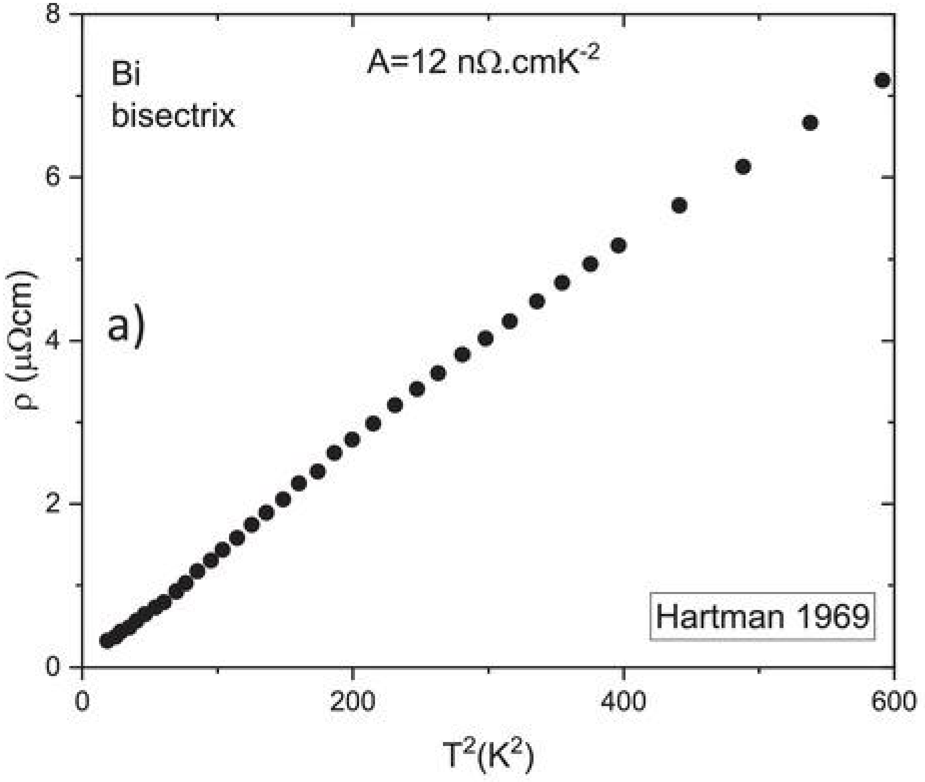

- Bound on resistivity for short-ranged forces:

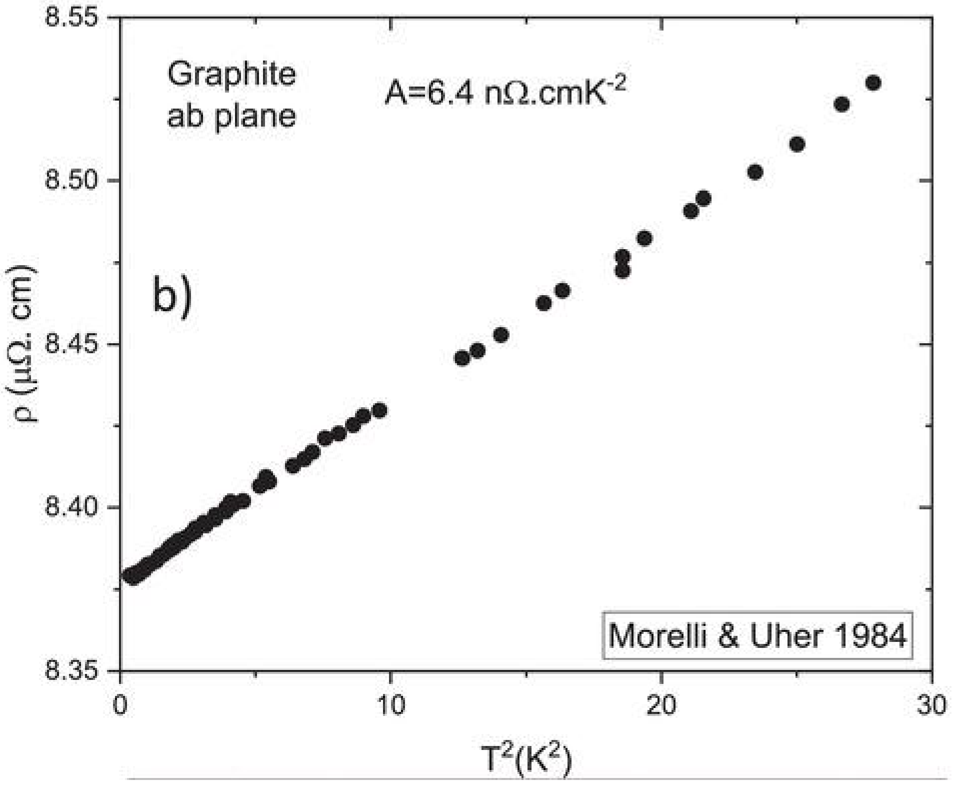
$$\frac{1}{\tau(\varepsilon_1)} \propto \int d\varepsilon_2 d\varepsilon_3 d\varepsilon_4 \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) n_F(\varepsilon_2) n_F(-\varepsilon_3) n_F(-\varepsilon_4)$$

R.D. Mattuck

$$\propto \varepsilon_1^2 + T^2 \quad \longrightarrow \quad \rho(T) \leq \rho(0) + BT^2$$

# Resistivity of FLs



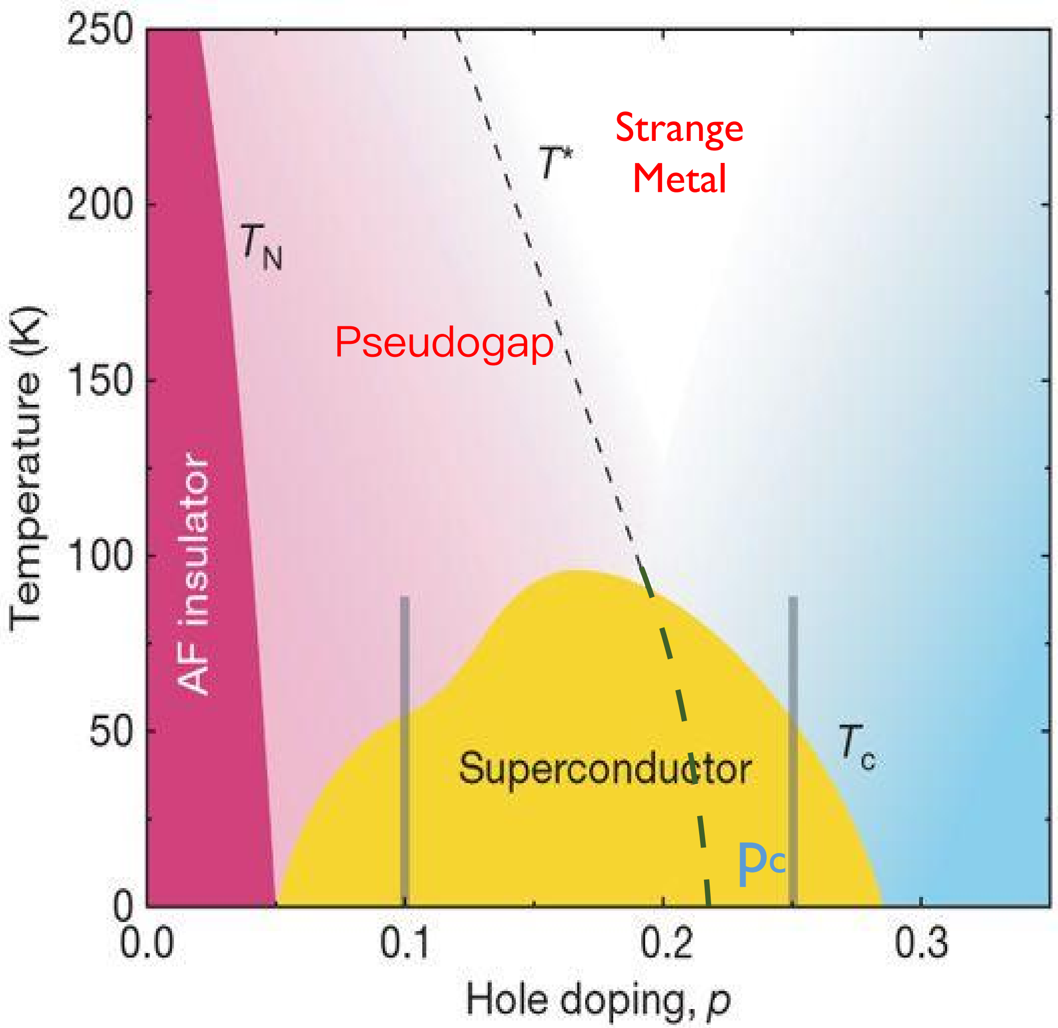



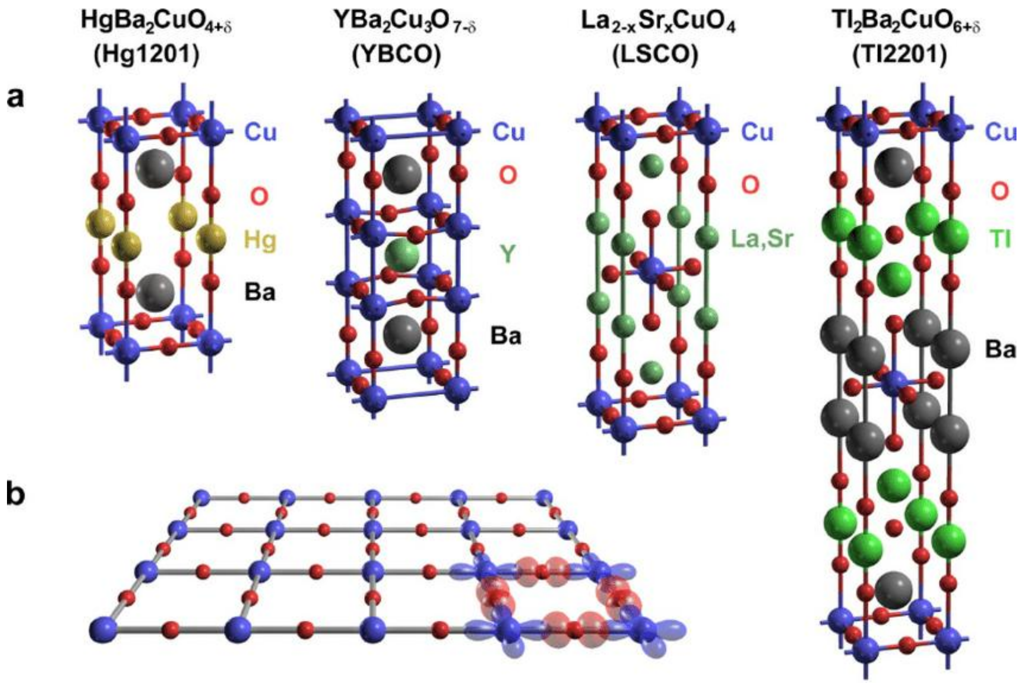

Kamran Behnia, *ANNALEN DER PHYSIK* 2022, 534, 21005

# Fermi Liquid and their cousins at T=0 (single particle property)

- Fermi Liquid: fermionic quasiparticles with density of states $\mathcal{N}(\varepsilon) \sim$ constant and a lifetime $1/\tau(\varepsilon) \ll |\varepsilon|$ as $|\varepsilon| \to 0$.
- Non-Fermi Liquid: No quasiparticle. A would-be fermionic quasiparticle has density of states $\mathcal{N}(\varepsilon) \sim$ constant and a lifetime $1/\tau(\varepsilon) \gg |\varepsilon|$ as $|\varepsilon| \to 0$.
- Marginal Fermi Liquid: fermionic quasiparticles with density of states $\mathcal{N}(\varepsilon) \sim$ constant and a lifetime $1/\tau(\varepsilon) \sim |\varepsilon|$ as $|\varepsilon| \to 0$.

# Cuprate and its Phase diagram

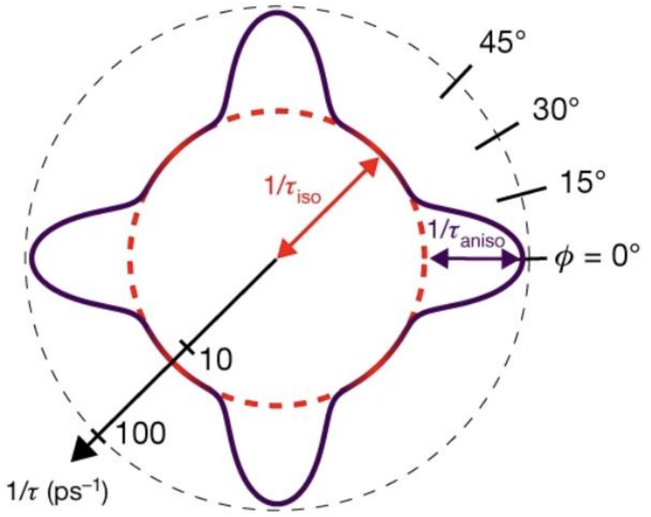


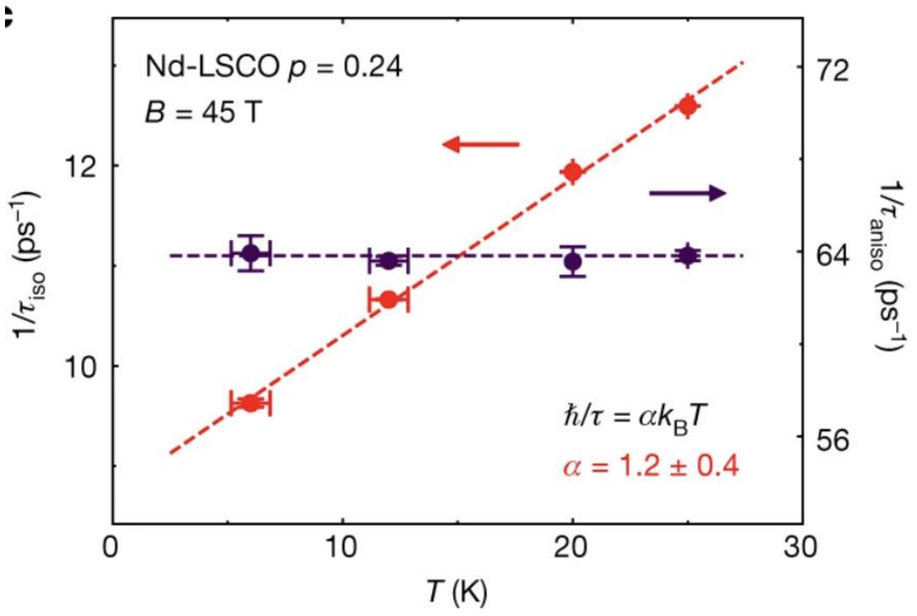



# Linear-in temperature resistivity from an isotropic Planckian scattering rate

Gaël Grissonnanche, Yawen Fang, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Jianshi Zhou, David Graf, Paul A. Goddard, Louis Taillefer & B. J. Ramshaw

*Nature* **595**, 667–672 (2021) | Cite this article



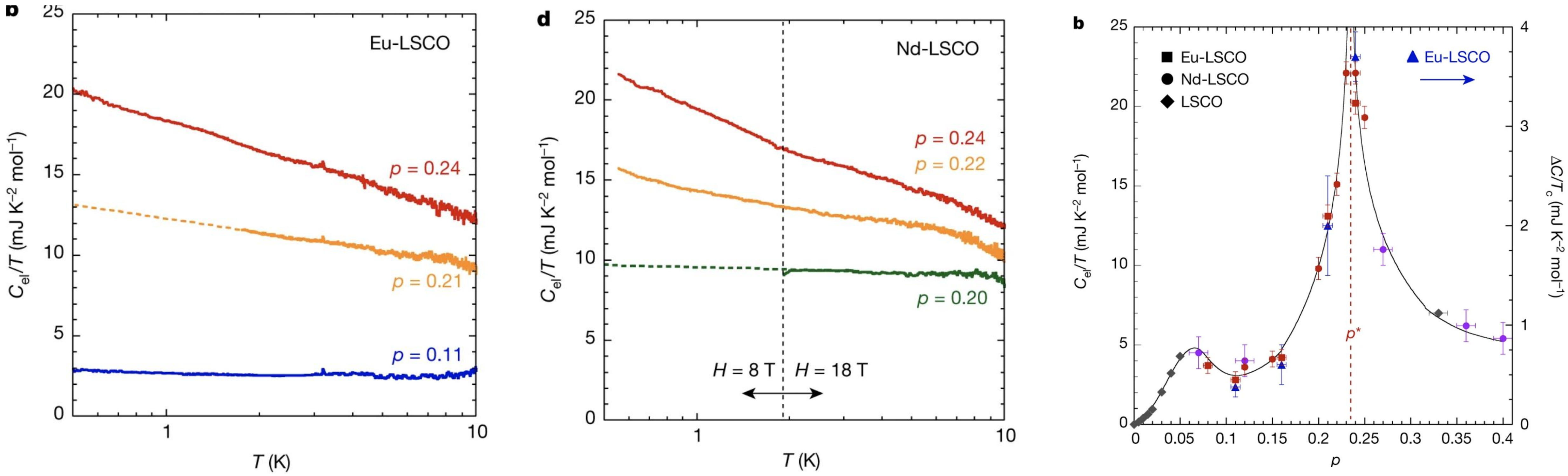




$$\rho(T) = \frac{m_*}{ne^2\tau}$$

$$\tau^{-1} \sim \frac{k_B T}{\hbar}$$

# Singular specific heat

$C \sim T \ln(1/T)$ (In a Fermi liquid $C \sim T$)

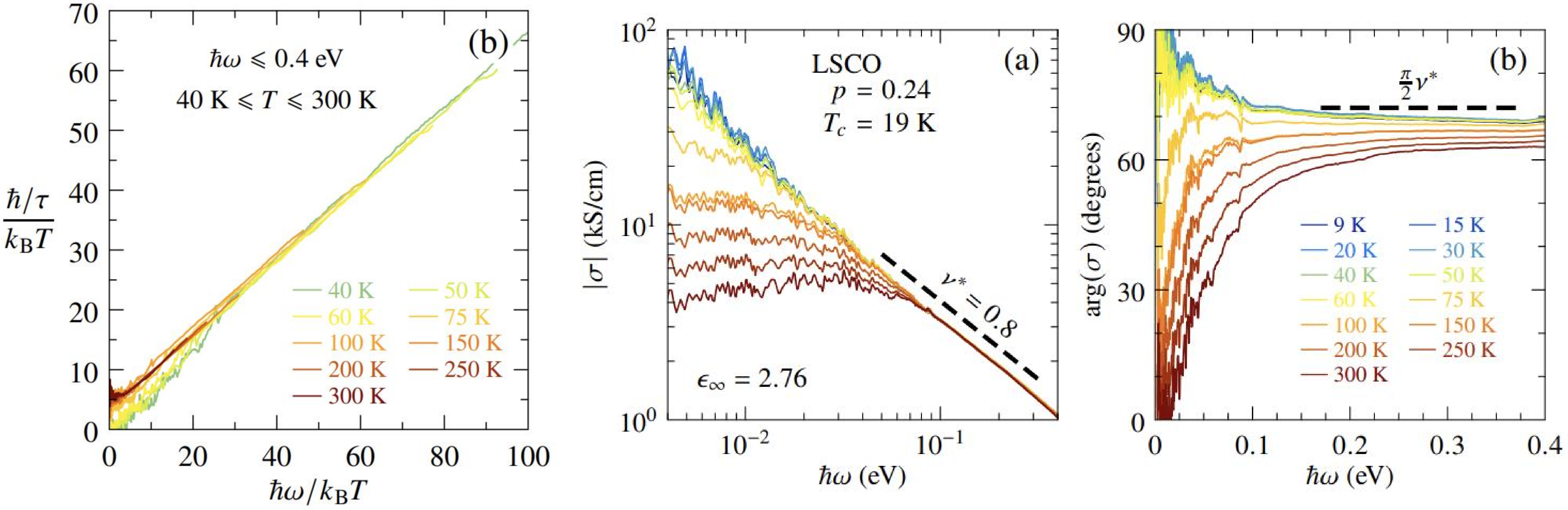



B. Michon et.al. *Nature* (2019)

# Scaling collapse of optical conductivity

$$\sigma(\omega) = \frac{K}{\tau(\omega)^{-1} - i\omega\frac{m^*(\omega)}{m}}, \ \frac{1}{\tau(\omega)} = \frac{k_B T}{\hbar} S\left(\frac{\hbar\omega}{k_B T}\right)$$

B. Michon……A. Georges, arXiv:2205.0403



# Properties of a strange metal

- Resistivity: $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$, and $\rho(T) < h/e^2$ (in 2D). Metals with $\rho(T) > h/e^2$ is a bad metal.
- A marginal Fermi liquid self energy. $\mathrm{Im}\Sigma_R(\omega) \sim |\omega| \quad (T = 0)$.
- Optical conductivity $\sigma(\omega) = \frac{K}{\tau(\omega)^{-1} - i\omega\frac{m^*(\omega)}{m}}$, $\frac{1}{\tau(\omega)} = \frac{k_B T}{\hbar} S\left(\frac{\hbar\omega}{k_B T}\right)$

Want a theory to explain some or all the properties

# Outline

- A toy model of Non-Fermi Liquid: SYK model
- Lattice SYK model and random t-J Model
- SYK ideas in 2+1D: Yukawa-SYK model

# A toy model of Non–Fermi Liquid

# Sachdev–Ye–Kitaev (SYK) model: a model of matter without quasiparticle

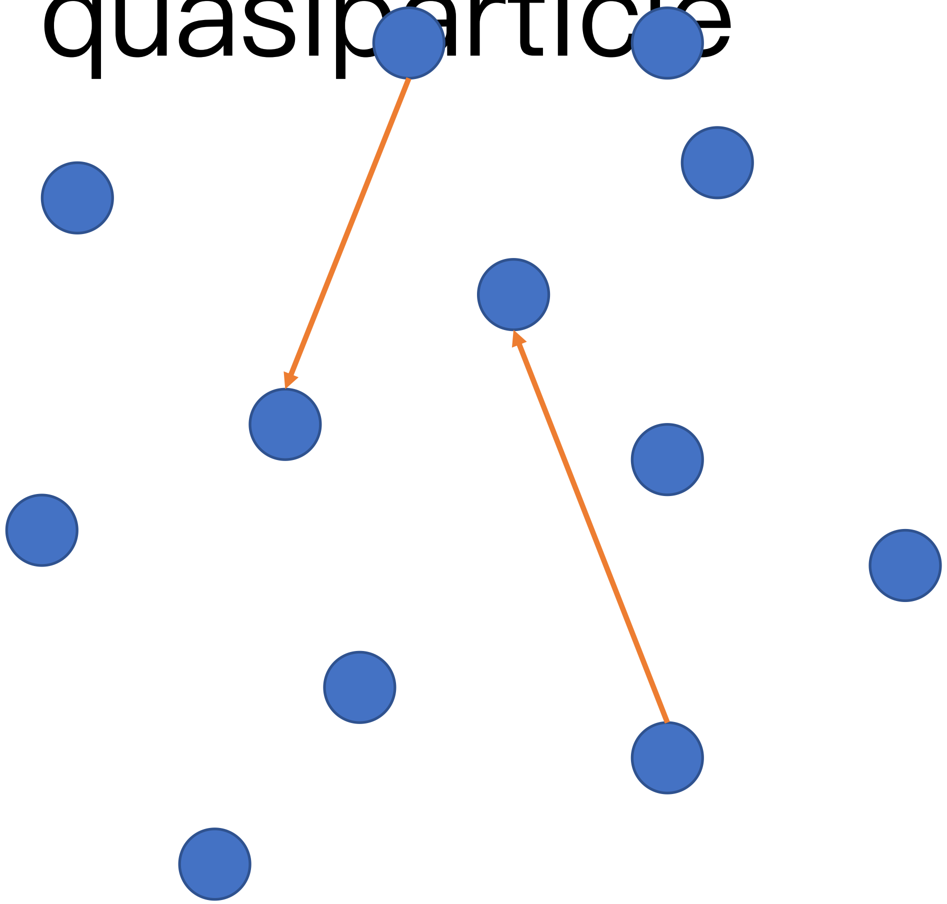
$$H_{\text{SYK}} = -\mu \sum_a c_a^\dagger c_a + \frac{1}{(2N)^{3/2}} \sum_{abcd} U_{abcd} c_a^\dagger c_b^\dagger c_c c_d$$

Take $N \to \infty$ limit

$U_{abcd}$: Gaussian random couplings,

$U_{abcd} = -U_{bacd} = -U_{abdc} = U^*_{cdab}$,

$\overline{U_{abcd}} = 0, \overline{|U_{abcd}|^2} = U^2$



S. Sachdev and J. Ye, PRL 70, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX 5, 041025 (2

# SYK Model: Large N limit

$$H_{\mathrm{SYK}} = -\mu \sum_a c_a^\dagger c_a + \frac{1}{(2N)^{3/2}} \sum_{abcd} U_{abcd} c_a^\dagger c_b^\dagger c_c c_d$$

Take $N \to \infty$ limit

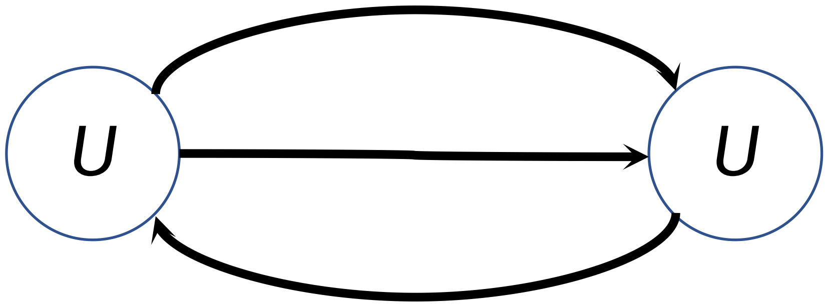
$U_{abcd}$: Gaussian random couplings,

$U_{abcd} = -U_{bacd} = -U_{abdc} = U^*_{cdab}$,

$\overline{U_{abcd}} = 0, \overline{|U_{abcd}|^2} = U^2$

$$G(\tau, \tau') \equiv \tfrac{-1}{N} \textstyle\sum_a \langle c_a(\tau) c_a^\dagger(\tau') \rangle$$

## Melon Diagrams:

$\Sigma =$ 


## Large N EoM

$$G^{-1}(i\omega_n) = i\omega_n + \mu - \Sigma(i\omega_n)$$

$$\Sigma(\tau, \tau') = -U^2 G(\tau, \tau')^2 G(\tau', \tau)$$

# Another way: $G - \Sigma$ action

- We can rewrite the problem exactly using bi-local fields $G$ and $\Sigma$

$$1 = \int DGD\Sigma \exp\left(\int \mathrm{d}\tau\mathrm{d}\tau'\Sigma(\tau',\tau)(NG(\tau,\tau') + \sum_a c_a(\tau)c_a^\dagger(\tau'))\right)$$

$$Z = \int DGD\Sigma \exp(-NS)$$

$$S = -\ln\det((\partial_\tau - \mu)\delta(\tau-\tau') + \Sigma(\tau,\tau')) - \int \mathrm{d}\tau\mathrm{d}\tau'\left[\Sigma(\tau',\tau)G(\tau,\tau') + \frac{U^2}{4}G(\tau,\tau')^2G(\tau',\tau)^2\right]$$

Melon Diagram = Saddle point

# Solving the saddle point

- We want to find gapless solutions, so $\Sigma(i\omega \to 0) = \mu$
- In the low-energy limit, we can ignore the UV terms $i\omega + \mu$

$$G^{-1}(i\omega_n) = \cancel{i\omega_n + \mu} - \Sigma(i\omega_n)$$

$$\Sigma(\tau) = -U^2 G(\tau)^2 G(-\tau)$$

- Emergent IR symmetries

U(1): $G(\tau_1, \tau_2) \to G(\tau_1, \tau_2) e^{i(\lambda(\tau_1) - \lambda(\tau_2))}$
$\Sigma(\tau_1, \tau_2) \to \Sigma(\tau_1, \tau_2) e^{i(\lambda(\tau_1) - \lambda(\tau_2))}$

Time reparameterization: $\tau \to f(\tau)$
$G(\tau_1, \tau_2) \to G(f(\tau_1), f(\tau_2)) f'(\tau_1)^{\Delta} f'(\tau_2)^{\Delta}$
$\Sigma(\tau_1, \tau_2) \to \Sigma(f(\tau_1), f(\tau_2)) f'(\tau_1)^{1-\Delta} f'(\tau_2)^{1-\Delta}$

$\Delta = 1/4$

- We get a CFT in the IR

# Conformal solution

$$G(z) = C\frac{e^{-i(\pi\Delta+\theta)\mathrm{sgn\ Im}z}}{z^{1-2\Delta_a}}$$

$$G(\tau) = -\frac{C\Gamma(2\Delta)}{\pi|\tau|^{2\Delta}}\sin(\theta+\pi\Delta\mathrm{sgn}\ \tau),$$

$$T = 0$$

Here $\theta$ is related to the U(1) charge

$$\langle f^\dagger f\rangle - \frac{1}{2} = -\frac{\theta}{\pi} - \left(\frac{1}{2}-\Delta\right)\frac{\sin(2\theta)}{\sin(2\pi\Delta)}$$

Yingfei Gu, A. Kitaev,
S. Sachdev, and G. Tarnopolsk
JHEP 02, 157 (2020)

# Absence of quasiparticle

There is $T = 0$ extensive entropy

$$\lim_{T\to 0}\lim_{N\to\infty}\frac{S}{N} = \mathcal{S}_0 > 0$$

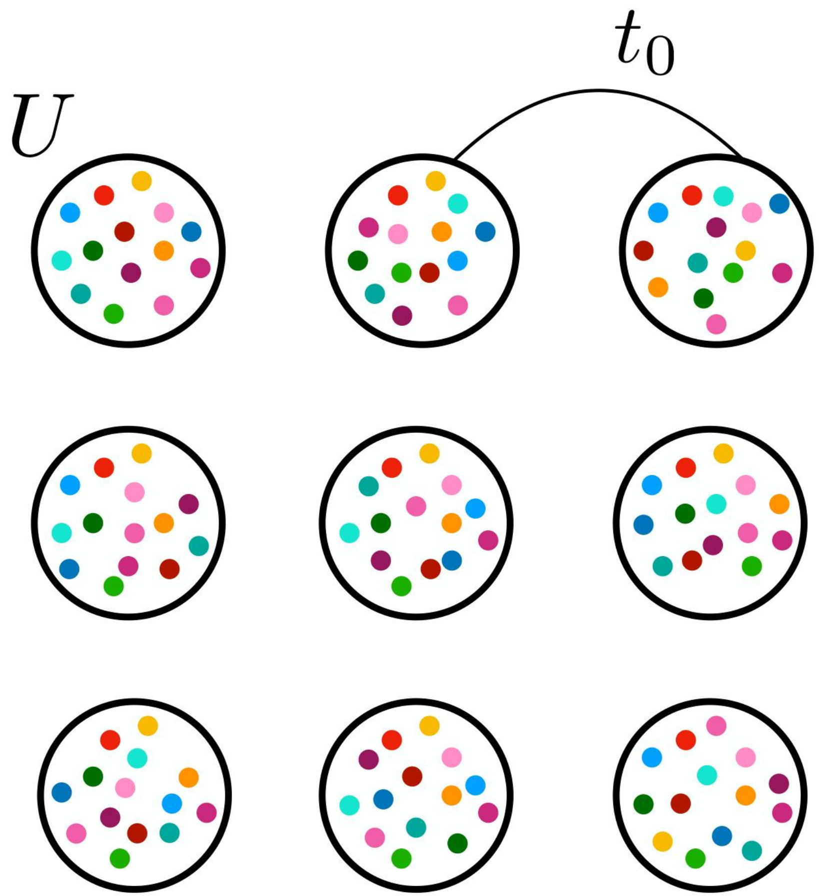
This indicates an exponentially small $\sim e^{-\mathcal{S}_0 N}$ energy level spacing in low-lying states

In contrast, a Fermi liquid implies $\mathcal{S}_0 = 0$ because the level spacing is polynomial in $1/N$.

$$E[\{n_\alpha\}] = E_0 + \sum_\alpha \delta n_\alpha \varepsilon_\alpha + \sum_{\alpha\beta} f_{\alpha\beta}\delta n_\alpha \delta n_\beta$$

# Lattice SYK Model

# Lattice SYK Model




SYK island+hopping

To obtain transport, we have to put the model on a lattice

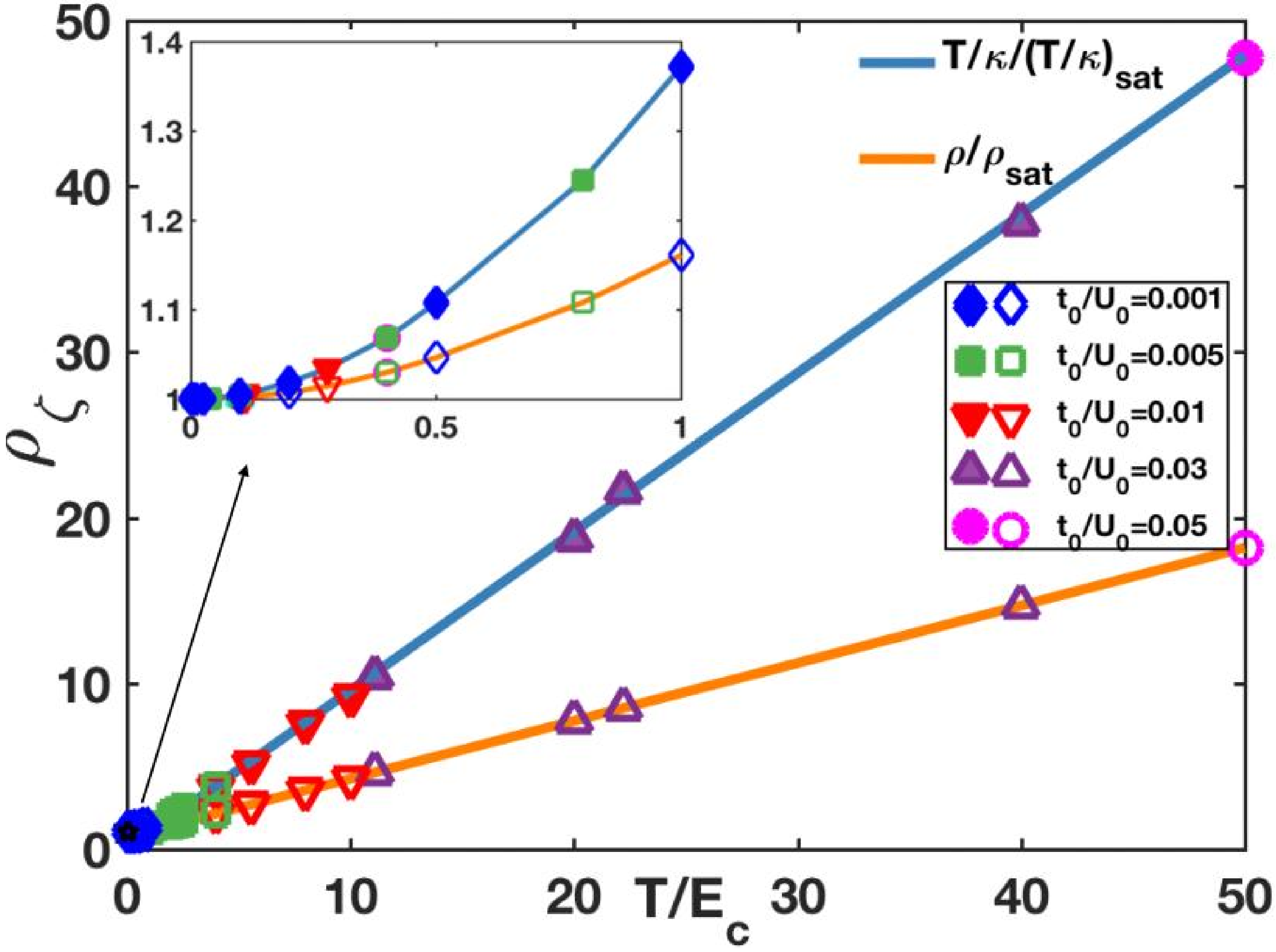
$$
\begin{aligned}
H &= H_{\text{hopping}} + H_{\text{SYK}} \\
H_{\text{hopping}} &= \frac{1}{\sqrt{zN}} \sum_{<xx'>} \sum_{ab} t^{ab}_{xx'} c^{\dagger}_{ax} c_{bx'} \\
H_{\text{SYK}} &= \frac{1}{(2N)^{3/2}} \sum_{x} U_{abcd,x} c^{\dagger}_{ax} c^{\dagger}_{bx} c_{cx} c_{dx}
\end{aligned}
$$

$$
\overline{|t^{ab}_{xx'}|^2} = t_0^2 \,, \quad \overline{|U_{abcd,x}|^2} = U^2
$$


Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL 119, 216601 (2017);
Pengfei Zhang, PRB 96, 205138 (2017);
Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX 8, 031024 (2018);
Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX 8, 021049 (2018)
HG, Yingfei Gu, Subir Sachdev, PRB 100, 045140(2019)

# Lattice SYK Model: Transport

- Hopping term is relevant: A new energy scale $E_c \sim t_0^2/U$:
- $T < E_c$: $\rho(T)$ grows quadratically (heavy fermi liquid)
- $T > E_c$: $\rho(T)$ grows linearly (SYK incoherent




Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL 119, 216601 (2017)

# Lattice SYK Model

- What worked?
  - We obtained a crossover from FL to NFL
  - In the NFL regime we get linear–in–T resistivity
- What can be improved?
  - The NFL does not persist to T=0
  - The fermion is not a MFL ($\Delta_c = \frac{1}{4}$)
  - The linearity in T is also related to $\Delta_c = \frac{1}{4}$
  - The model is quite far away from Hubbard model (no Mott–ness)

# Random t–J model

# A model with Mott–nes

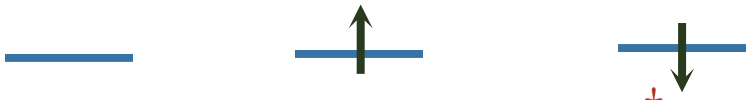
- We try to take Mott–ness into account exactly, by studying the low–energy limit of the Hubbard model: the t–J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy. Each

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^{\dagger}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = 1 - p$$



$|0\rangle$ $\quad c_{\uparrow}^{\dagger}|0\rangle$ $\quad c_{\downarrow}^{\dagger}|0\rangle$

# SYK down to T=0: fractionalization

To retain SYK solubility, we make t and J random, and take the appropriate large M limit.

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the '*superspin*' space of a boson $b$ (the holon) and a fermion $f_\alpha$ (the spinon):

$$\begin{aligned}
|0\rangle \Rightarrow b^\dagger\,|v\rangle \quad & , \quad c_\alpha^\dagger\,|0\rangle \Rightarrow f_\alpha^\dagger\,|v\rangle \\
c_\alpha &= f_\alpha b^\dagger \\
\vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\
f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \\
\text{U(1) gauge invariance,} \quad & b \to b e^{i\phi}\,, \quad f_\alpha \to f_\alpha e^{i\phi}
\end{aligned}$$

The physical electron ($c_\alpha$) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.

# t-J model in large z and M limit

$$H = \sum_{\langle ij\rangle,\ell,\alpha} t_{ij}\left(c^\dagger_{i\ell\alpha}c_{j\ell\alpha} + \text{H.c.}\right) + \sum_{\langle ij\rangle} J_{ij}\vec{S}_i\cdot\vec{S}_j$$

$\alpha = 1, 2, \ldots, M$: Spin index,
$l = 1, 2, \ldots, M'$: Orbital index
Physically $M = 2, M' = 1$

$M, M' \to \infty$
with $k = \frac{M'}{M}$ fixed

SU$(M'|M)$ approach

$$c_{i\ell\alpha} = b^\dagger_{i\ell} f_{i\alpha}$$

$$S_i^a = \sum_{\alpha\beta} f^\dagger_{i\alpha} T^a_{\alpha\beta} f_{i\beta}$$

U(1) gauge symmetry

$b_{i\ell} \to b_{i\ell}e^{i\phi_i(\tau)}$, $f_{i\alpha} \to f_{i\alpha}e^{i\phi_i(\tau)}$

$$\sum_{\alpha=1}^{M} f^\dagger_{i\alpha}f_{i\alpha} + \sum_{\ell=1}^{M'} b^\dagger_{i\ell}b_{i\ell} = \frac{M}{2}$$

Doping condition

$$\sum_{\ell=1}^{M'}\left\langle b^\dagger_{i\ell}b_{i\ell}\right\rangle = M'p$$

# SYK–like saddle point

$$
\begin{aligned}
G_b(i\omega_n) &= \frac{1}{i\omega_n + \mu_b - \Sigma_b(i\omega_n)} \\
\Sigma_b(\tau) &= -t^2 G_f(\tau) G_f(-\tau) G_b(\tau) \\
G_f(i\omega_n) &= \frac{1}{i\omega_n + \mu_f - \Sigma_f(i\omega_n)} \\
\Sigma_f(\tau) &= -J^2 G_f(\tau)^2 G_f(-\tau) + k t^2 G_f(\tau) G_b(\tau) G_b(-\tau) \\
G_c(\tau) &= -G_f(\tau) G_b(-\tau)
\end{aligned}
$$

$\mu_f, \mu_b$ are chosen to satisfy $\langle f^\dagger f \rangle = \frac{1}{2} - kp \quad , \quad \langle b^\dagger b \rangle = p$

Darshan G. Joshi, Chenyuan Li, Grigory Tarnopolsky, Antoine Georges, and Subir Sachdev. PRX 10, 2020.
HG, Yingfei Gu and Subir Sachdev, Annals of Physics 418, 168202 (2020)

# The role of UV physics

$$G(\tau_1,\tau_2) \to G(f(\tau_1), f(\tau_2)) f'(\tau_1)^{\Delta} f'(\tau_2)^{\Delta}$$
$$\Sigma(\tau_1,\tau_2) \to \Sigma(f(\tau_1), f(\tau_2)) f'(\tau_1)^{1-\Delta} f'(\tau_2)^{1-\Delta}$$

- The emergent time-reparameterization symmetry is <u>spontaneously</u> broken in the IR and <u>explicitly broken</u> in the UV

$$G(z) = C\frac{e^{-i(\pi\Delta+\theta)\mathrm{sgn}\ \mathrm{Im} z}}{z^{1-2\Delta_a}}$$

$$G(\tau) = -\frac{C\Gamma(2\Delta)}{\pi|\tau|^{2\Delta}}\sin(\theta+\pi\Delta\mathrm{sgn}\ \tau),$$

is only invariant under PSL(2,R)

$$f(\tau) = \frac{a\tau+b}{c\tau+d}, \quad ad-bc=1$$

$$G^{-1}(i\omega_n) = \cancel{i\omega_n + \mu} - \Sigma(i\omega_n)$$

$$\Sigma(\tau) = -J^2 G(\tau)^2 G(-\tau)$$

The ignored UV terms break it explicitly

# Nearly a CFT

Shift self energies $\Sigma \to \Sigma - \sigma$, $\sigma(\tau_1, \tau_2) = (\partial_\tau - \mu)\delta(\tau_1 - \tau_2)$

$$S = S_{\mathrm{CFT}} + S_{\mathrm{UV}}$$

$$S_{\mathrm{CFT}} = -\ln\det(\Sigma(\tau,\tau')) - \int \mathrm{d}\tau \mathrm{d}\tau' \left[\Sigma(\tau',\tau)G(\tau,\tau') + \frac{J^2}{4} G(\tau,\tau')^2 G(\tau',\tau)^2\right]$$

respects all the IR symmetries

$$S_{\mathrm{UV}} = \int \mathrm{d}\tau \mathrm{d}\tau' \sigma(\tau,\tau') G(\tau',\tau)$$

# Irrelevant corrections from UV: RG picture

Separate timescales:

$$|\tau| \ll J^{-1}, \quad \text{UV}$$

$$J^{-1} \ll |\tau| \lesssim \beta, \quad \text{IR}$$

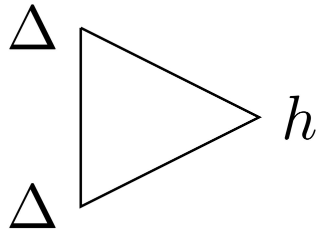
A.Kitaev and S. J. Suh,
JHEP 05, 183 (2018)

Yingfei Gu, A. Kitaev,
S. Sachdev, and G. Tarnopolsky
JHEP 02, 157 (2020)

In the IR, the UV sources renormalizes to a combination of irrelevant operators in the SYK CFT

$$S_{\text{UV}} \to \int \mathrm{d}\tau \sum_I J^{1-h_I} \alpha_I \mathcal{O}_{h_I}$$

Appears in the OPE of $cc^\dagger$




$$G(\tau) = G_{\text{conformal}}(\tau) \left( 1 + \sum_I \frac{\alpha_I}{|J\tau|^{h_I - 1}} \right)$$

# Operator spectrum

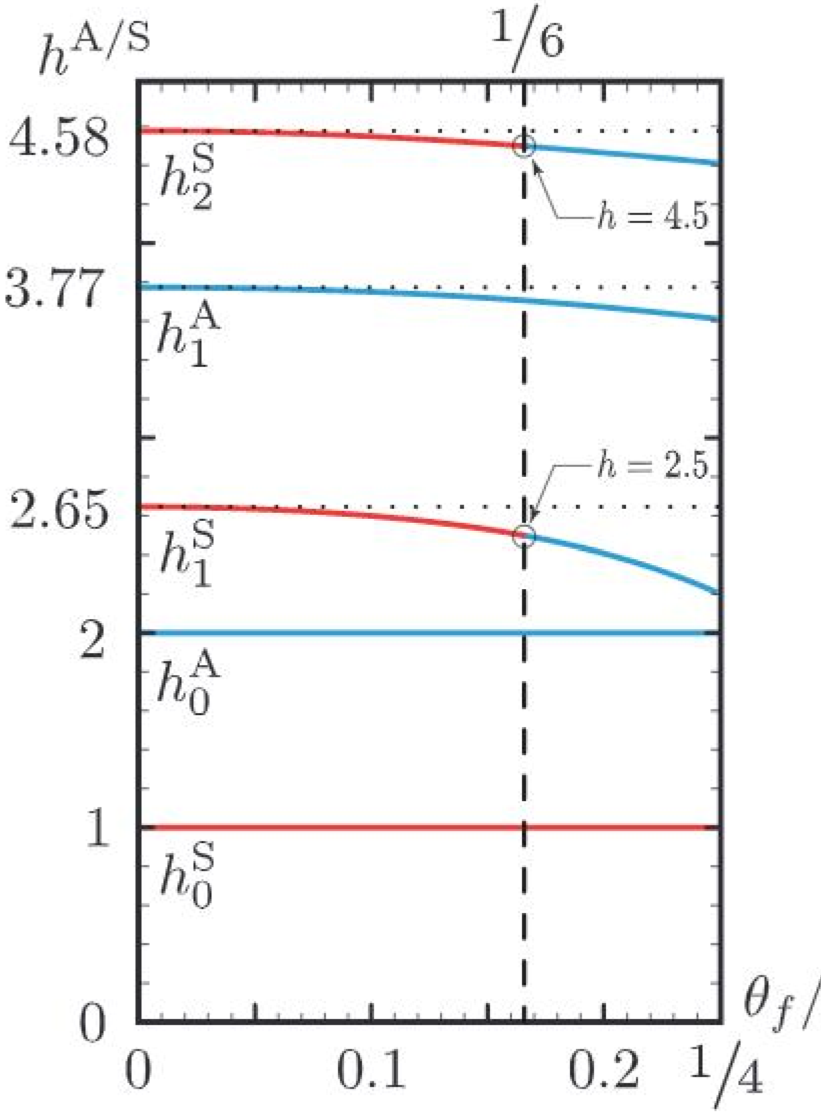
$h=1$: One operator $\mathcal{Q}$, the charge

Renormalize charges $\delta G(\tau) = \dfrac{\partial G(\theta;\tau)}{\partial \theta_a}\delta\theta$

$h=2$: One operator $\mathcal{O}_2$: energy/Schwarzian

$$\frac{\delta G(\tau)}{G(\tau)} = -\alpha_G \frac{2-3\sin(2\theta)\,\mathrm{sgn}\,\tau}{|J\tau|}$$

$\alpha_G$ depends on UV detail,
cannot be obtained analytically



# Schwarzian action and Specific heat

The $h = 2$ operator is the Schwarzian derivative

$$\mathcal{O}_2(\tau) = \mathrm{Sch}(e^{if(\tau)}, \tau) \qquad \mathrm{Sch}(f(z), z) = \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$$

It measures the cost of time-reparameterization

This is the unique object with $h = 2$ and invariant under $\mathrm{PSL}(2, \mathbb{R})$

$$S_{\mathrm{UV}} \supset -\frac{\alpha_S}{J}\int \mathrm{d}\tau\, \mathrm{Sch}(e^{if(\tau)}, \tau) \quad \text{is minimized at } f(\tau) = 2\pi\tau/\beta$$

$$= -2\pi^2 \frac{\alpha_S}{\beta J}$$

Specific heat $C = N\gamma T$, $\gamma = 4\pi^2 \frac{\alpha_S}{J}$

$\alpha_G \propto \alpha_S$

# Consequences of the Schwarzian term

- The system has an emergent time-reparameterization symmetry, whose soft mode is the dual of graviton in NAdS2
  - The electron spectral function is consistent with marginal Fermi liquid
    $$A_c(\omega) = A_0 + A_1|\omega|$$
  - The symmetry ensures exact linear-in-T resistivity and Planckian dissipation
    $$\rho(T) = \rho_0(1 + \alpha_G T + \dots)$$
  - The system shows maximal chaos
    $$\frac{1}{N^2}\sum_{ab}\langle c_a^\dagger(t)c_b(0)c_a(t)c_b^\dagger(0)\rangle = C_0 + C_1\frac{\exp(\lambda_L t)}{N} \quad \lambda_L = 2\pi T$$

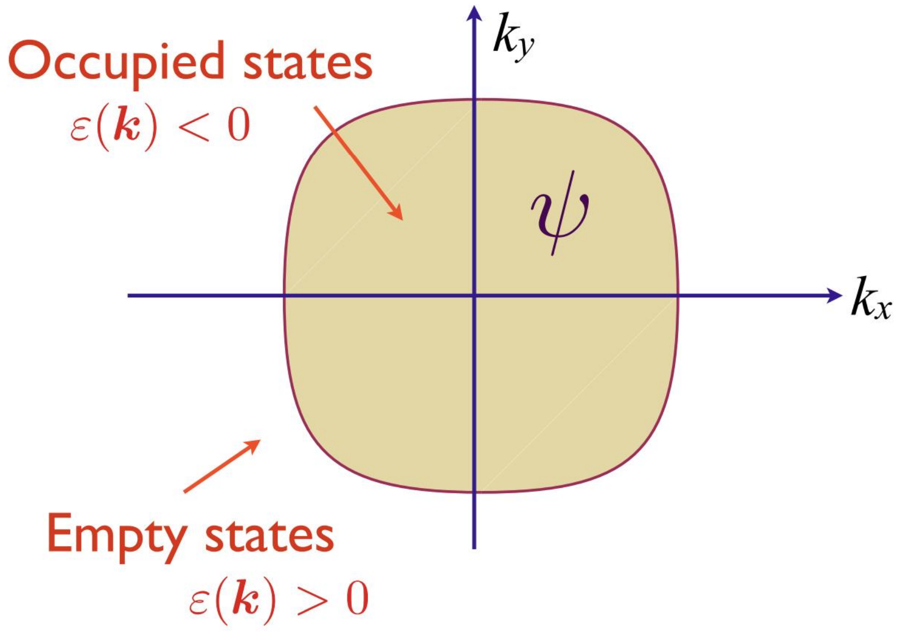
solarseven via Getty Images

HG, Yingfei Gu and Subir Sachdev, Annals of Physics 418, 168202 (2020)
Maria Tikhanovskaya, HG, Subir Sachdev, Grigory Tarnopolsky, Physical Review B 103(7), 0
Maria Tikhanovskaya, HG, Subir Sachdev, Grigory Tarnopolsky, Physical Review B 103(7), 0

# 2+1D: Yukawa–SYK Theory

Can we have a theory with real Fermi surface?

# Critical Fermi Surface




- Ising-Nematic order
- Ferrormagnetic order
- Transverse component of abelian or non-abelian gauge field

$$S_\psi = \int \mathrm{d}\tau \sum_k \psi_k^\dagger (\partial_\tau + \varepsilon_k) \psi_k$$

$$S_\phi = \frac{1}{2} \int \mathrm{d}\tau \sum_q \phi_{-q} (\partial_\tau^2 + \omega_q^2) \phi_q$$

# Yukawa coupling in critical Fermi surface

- Strong fermion–boson interaction => fermion–boson drag

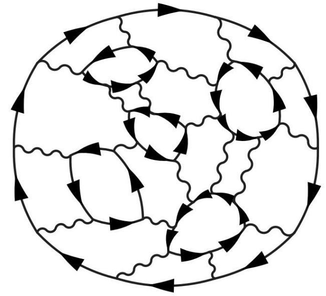
$$S_g = g \int \mathrm{d}\tau \mathrm{d}^2 x \psi^\dagger(x,\tau)\psi(x,\tau)\phi(x,\tau)$$

Comparison: For electron-phonon scattering in a Fermi liquid, the Bloch's law states that $\rho(T) \propto T^5$. However, this result ignores conservation of momentum and the feedback of phonons to electrons (phonon drag). This approximation is valid for phonons because of weak electron-phonon coupling.

In a non-Fermi liquid, the fermion-boson coupling is so strong such that the fermion and boson are individually destroyed, and we have to consider the case of extreme drag.

# Combining things together?

- A critical FS in 2D is notoriously hard to tame:
  - Large-N theory with N flavors of fermion and one boson runs IR divergence. Sung-Sik Lee



  - Tuning boson dispersion $q^{1+\epsilon}|\phi(q)|^2$ loses locality in real space.

David F. Mross, John McGreevy, Hon Liu, and T. Senthil

  - Dimensional regularization of FS loses locality in momentum space and runs into UV/IR mixing. Ipsita Mandal and Sung-Sik Lee
  - Codimension regularization of FS breaks charge conservation

Denis Dalidovich and Sung-Sik Lee

# Yukawa–SYK theory

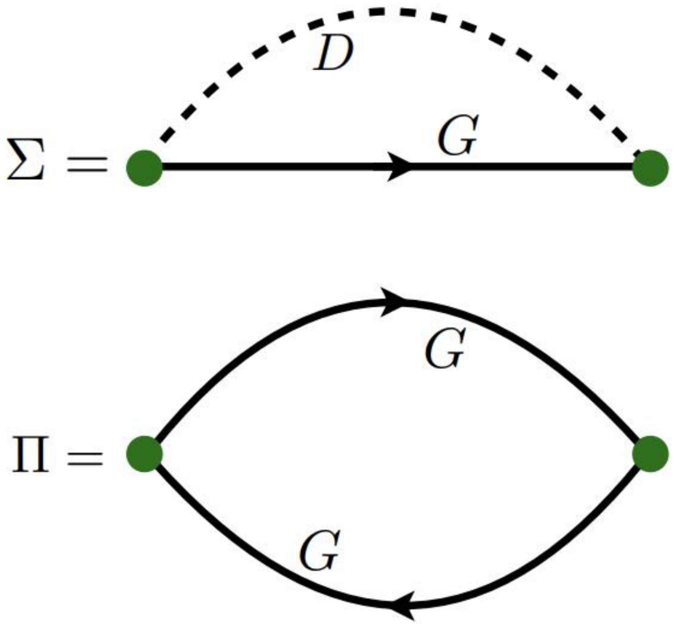
Ilya Esterlis, J. Schmalian, PRB 100, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR 2, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv
Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB 103, 2351

- Introduce N flavors of fermions and bosons, and examine an ensemble of theories with different Yukawa couplings. In the large N limit, every member of the ensemble is expected to have the same critical properties, and so it is easier to study the average theory.

$$S_g = \sum_{ijl} \frac{g_{ijl}}{N} \int \mathrm{d}^2 x \mathrm{d}\tau \psi_i^\dagger(x,\tau)\psi_j(x,\tau)\phi_l(x,\tau)$$

$$\overline{g_{ijl}} = 0, \ \overline{g_{ijl}^2} = g^2$$

$$\text{Large } N\text{: Eliashberg Equations}$$



# Three theories we can solve

- 1.A critical Fermi surface without spatial disorder: $S = S_\psi + S_\phi + S_g$

  A non-Fermi liquid but not a strange metal
- 2.A critical Fermi surface with potential disorder: $S = S_\psi + S_\phi + S_g + S_v$

  A marginal Fermi liquid but not a strange metal
- 3.A critical Fermi surface with interaction disorder:

  A marginal Fermi liquid and a strange metal $S = S_\psi + S_\phi + S_g + S_{g'} (+S_v)$

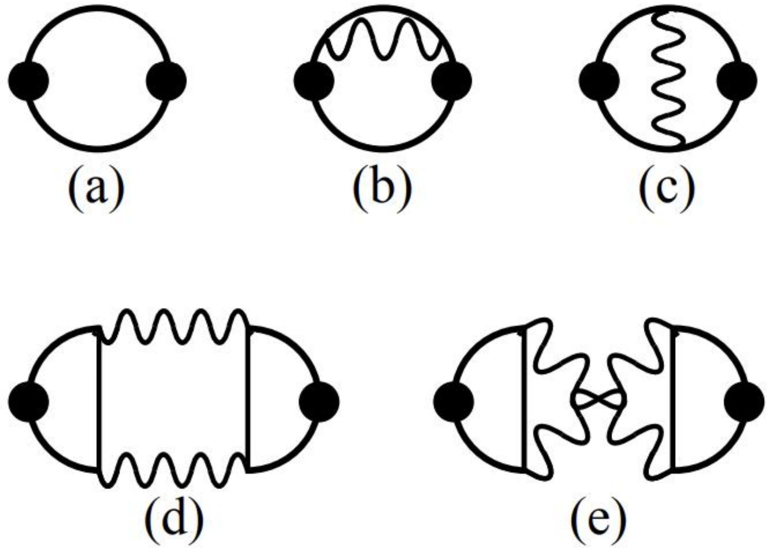
Ilya Esterlis, HG, Aavishkar Patel, Subir Sachdev PRB 103, 235129
HG, Aavishkar A Patel, Ilya Esterlis, Subir Sachdev, PRB 106, 11515
Aavishkar A Patel, HG, Ilya Esterlis, Subir Sachdev, Science (2023)

# Translational invariant: a non–Fermi liquid but not a strange metal

- Transport:

Eliashberg Theory:

$$\Sigma(i\omega,k)=\frac{-ig^2}{2\sqrt{3}\pi v_F\gamma^{1/3}}\mathrm{sgn}(\omega)|\omega|^{2/3}\quad \Pi(i\Omega,q)=-\gamma\frac{|\Omega|}{q}\,,\quad \gamma=\frac{\mathcal{N}g^2}{v_F}$$




+ all ladders and bubbles…..

Conservation of momentum implies the d.c. conductivity is infinite

$$\mathrm{Re}\,\sigma(\omega)=D\delta(\omega)+\ldots$$

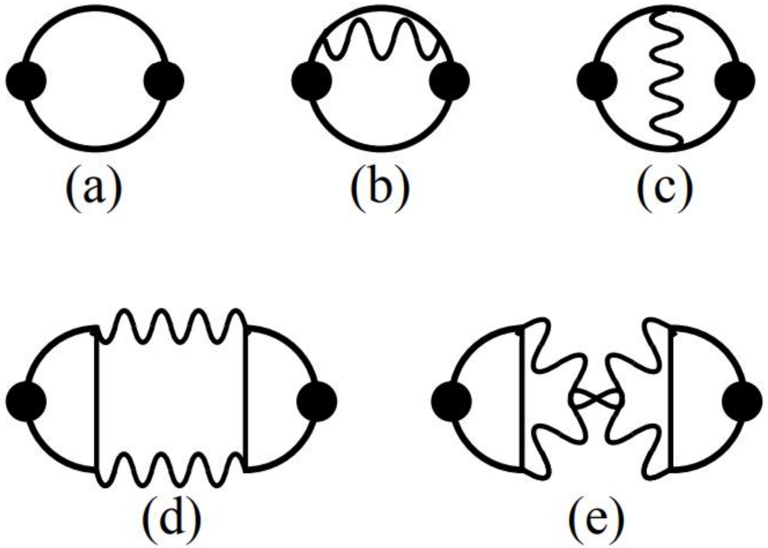
S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S. S. PRB **94**, 045133 (2016)

$$\sigma(\omega)\sim\frac{1}{-i\omega}+\underset{\cancel{|\omega|^{-2/3}}}{|\omega|^0}+\ldots$$

Zhengyan Darius Shi, Hart Goldman, Dominic V. Else, T. Senthil arXiv:2204.07585
arXiv:2208.04328
Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990
HG, AP, IE, SS: arXiv:
2207.08841

# A critical fermi surface without spatial disorder

- Transport:




+ all ladders and bubbles…..

Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, P. A. Lee, PRB **50**, 17917 (1994) examined these graphs and concluded that the d.c. resistivity $\rho(T) \sim T^{4/3}$ (analog of Bloch's law) and $\sigma(\omega \gg T) \sim \omega^{-2/3}$.

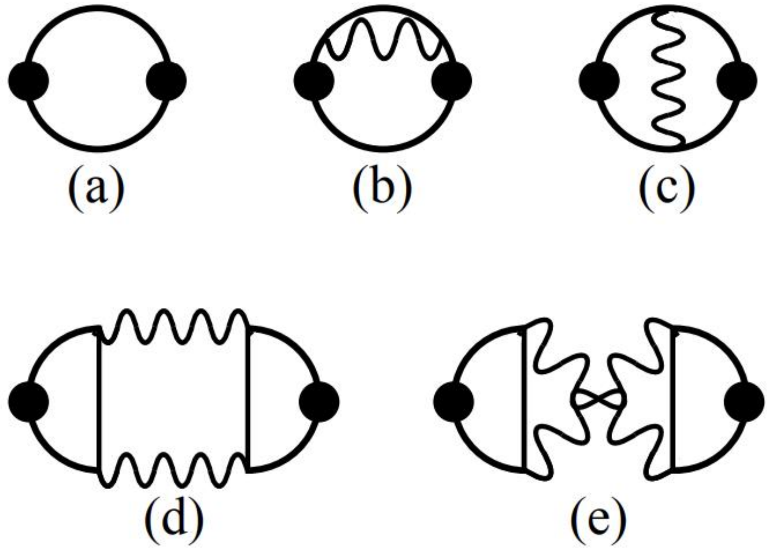
These conclusions are not consistent with conservation of total momentum *i.e.* 'boson drag'.

# Translational invariant: a non-Fermi liquid but not a strange metal

- Transport:

Eliashberg Theory:

$$\Sigma(i\omega,k) = \frac{-ig^2}{2\sqrt{3}\pi v_F \gamma^{1/3}}\mathrm{sgn}(\omega)|\omega|^{2/3} \quad \Pi(i\Omega,q) = -\gamma\frac{|\Omega|}{q}, \quad \gamma = \frac{\mathcal{N}g^2}{v_F}$$



+ all ladders and bubbles…..

Conservation of momentum implies the d.c. conductivity is infinite

$$\mathrm{Re}\,\sigma(\omega) = D\delta(\omega) + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S. S. PRB **94**, 045133 (2016)

$$\sigma(\omega) \sim \frac{1}{-i\omega} + \underset{\cancel{|\omega|^{-2/3}}}{|\omega|^0} + \ldots$$

Zhengyan Darius Shi, Hart Goldman, Dominic V. Else, T. Senthil arXiv:2204.07585
arXiv:2208.04328
Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990
HG, AP, IE, SS: arXiv:2207.08841

# A would-be $|\omega|^{-2/3}$ conductivity (Bloch's law)

- Let's first discuss how to get the wrong answer $|\omega|^{-2/3}$
- We can imagine scattering on the FS as a random walk
- The Fermi surface dynamics is diffusive

$$\partial_t n(\theta) = D_2 \partial_\theta^2 n(\theta)$$

- The diffusion constant consists of the rate of random walk (single particle scattering rate) and the typical angular step
  - Self energy $\Sigma_R''(\omega) \propto |\omega|^{2/3}$
  - Angular step from typical boson momentum $\delta\theta \sim \frac{q}{k_F} \propto |\omega|^{1/3}$

$$D_2 \sim \Sigma_R''(\omega)(\delta\theta)^2 \propto |\omega|^{4/3}$$

- The conductivity

$$\sigma(\omega) \sim \frac{1}{-i\omega + D_2\partial_\theta^2} \sim \frac{1}{-i\omega + |\omega|^{4/3}} \sim \frac{1}{-i\omega} + |\omega|^{-2/3}$$

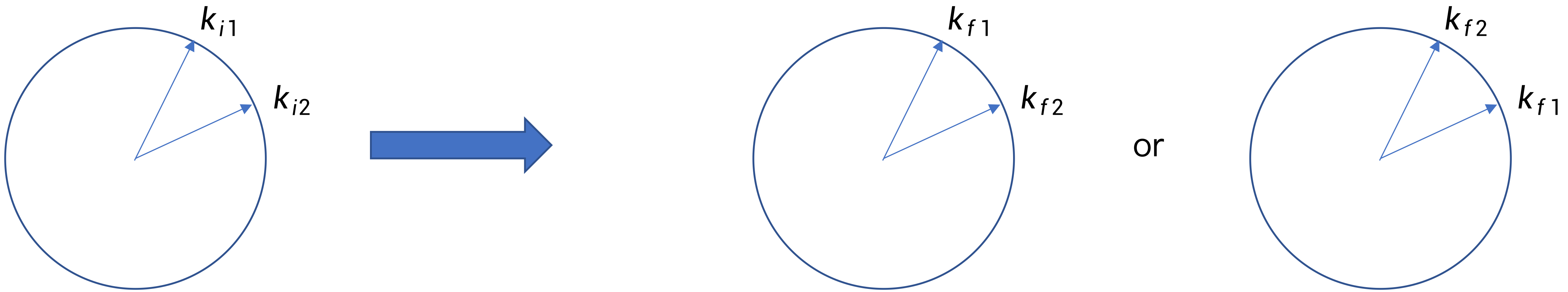
# A would–be $|\omega|^{-2/3}$ conductivity (Bloch's law)

- Let's first discuss how to get the wrong answer $|\omega|^{-2/3}$
- We can imagine scattering on the FS as a random walk

But, due to momentum conservation, the random walk events are correlated

# Generic scattering $k_{i1} + k_{i2} \neq 0$

- Only two scattering configurations
  - Forward scattering: $k_{f1} = k_{i1}, k_{f2} = k_{i2}$
  - Particle exchange: $k_{f1} = k_{i2}, k_{f2} = k_{i1}$
- Neither relaxes current effectively

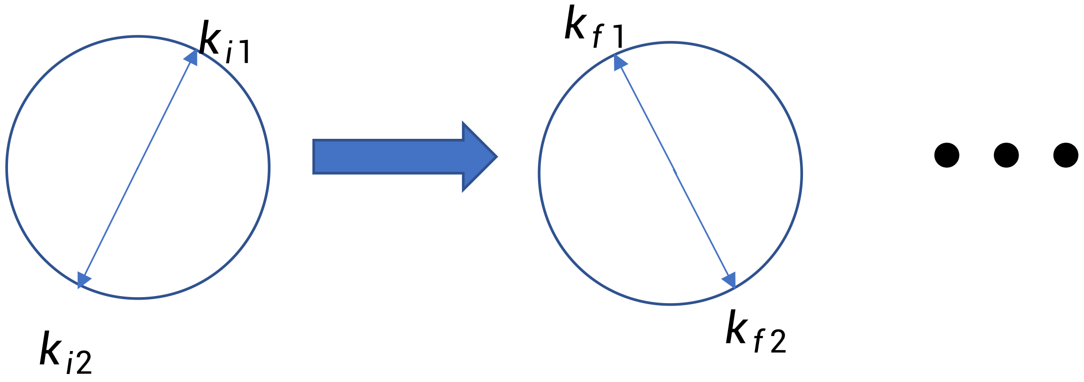


# Head–on Scattering $k_{i1} + k_{i2} = 0$

- Any head–on initial pair can scatter to any head–on pair
- However, head–on configurations have even parity, and they can't relax current which is of odd parity
- Similar reasonings apply to any convex, inversion–symmetric FS

P. J. Ledwith, HG and L. Levitov, Annals of Physics 411, 167913 (2019)

D. L. Maslov, V. I. Yudson, and A. V. Chubukov, Phys. Rev. Lett. 106, 106403 (2011)



# Relaxation of Odd-parity modes

- Transport property determined by odd parity distortions of FS
- Scattering=> Correlated random walk on the FS which conserves center of mass
- The projected dynamics on the FS is a (correlated) super-diffusion:

$$\partial_t n(\theta) = D_4 \partial_\theta^4 n(\theta)$$

P. J. Ledwith, HG , and L. Levitov, Annals of Physics 411, 167913 (2019)

- Estimate of diffusion constant $D_4$:

$$\Sigma(i\omega, k) = \frac{-ig^2}{2\sqrt{3}\pi v_F \gamma^{1/3}} \mathrm{sgn}(\omega)|\omega|^{2/3} \qquad \Pi(i\Omega, q) = -\gamma\frac{|\Omega|}{q}, \quad \gamma = \frac{\mathcal{N}g^2}{v_F}$$

  - $D_4 = \Sigma_R^{''}(\omega)\,(\delta\theta)^4$
  - Self energy $\Sigma_R^{''}(\omega) \propto |\omega|^{2/3}$
  - Angular step from typical boson momentum $\delta\theta \sim \frac{q}{k_F} \propto |\omega|^{1/3}$
  - $D_4 \propto \omega^2$, and this gives $\sigma(\omega) \sim \frac{1}{-i\omega+\omega^2} \sim \frac{1}{-i\omega} + |\omega|^0$

# Roles of disorder

- We can put in disorder in the conventional way: a random potential

$$S_v = \int \mathrm{d}\tau \mathrm{d}^2 x \; v(x) \psi^\dagger(x,\tau)\psi(x,\tau)$$

$$\overline{v(x)} = 0, \qquad \overline{v(x)v(x')} = \delta^2(x - x')$$

# A critical Fermi surface with potential disorder

- Action $S = S_\psi + S_\phi + S_g + S_v$

$$S_v = \sum_{ij} \int \mathrm{d}^2 x \mathrm{d}\tau \frac{v_{ij}(x)}{\sqrt{N}} \psi_i^\dagger(x,\tau)\psi_j(x,\tau)$$

$$\overline{v_{ij}(x)} = 0, \ \overline{v_{ij}^*(x) v_{lm}(x')} = v^2 \delta_{il}\delta_{jm}\delta(x-x')$$

- Marginal Fermi liquid $\Sigma = \Sigma_v + \Sigma_g$

Dynamical exponent z=2

$$\Sigma_v(i\omega) = -i\frac{\Gamma}{2}\mathrm{sgn}\omega \quad \Sigma_g(i\omega) = \frac{-ig^2\omega}{2\pi^2\Gamma} \ln\left(\frac{e\Gamma^2}{|\omega|\gamma' v_F^2}\right) \qquad \Pi(i\Omega) = -\gamma'|\Omega|$$

$$\Gamma = 2\pi v^2 \mathcal{N} \qquad \gamma' = \frac{\mathcal{N} g^2}{\Gamma}$$

$\mathcal{N}$: Fermionic Density of States

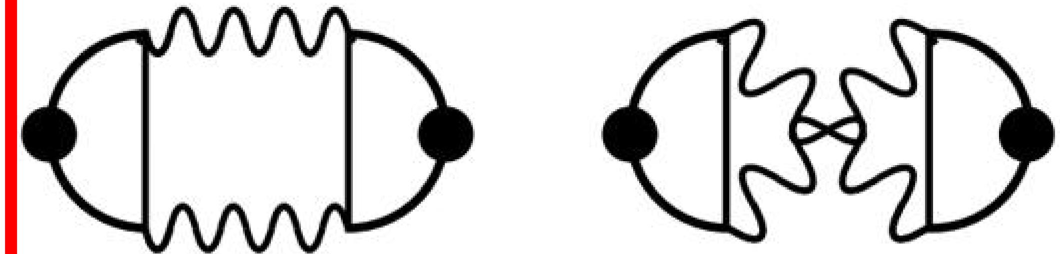
# Potential disorder: A marginal FL but not a strange metal

- Potential disorder induces a marginal Fermi liquid self energy $\Sigma_g \sim \omega \ln\left(\frac{1}{\omega}\right)$
- However this contribution doesn't enter into transport, due to cancellation with Maki–Thompson diagram
- Aslamazov–Larkin diagrams are small on their own

$$\sigma(\omega) \sim \frac{1}{-i\tilde{\omega} + \tau_{tr}^{-1}(\omega)}$$

$$\tilde{\omega} = \omega\left(1 - \frac{g^2}{2\pi^2\mathcal{N}^2 v_F^4}\left[\frac{v_F\Lambda}{4} + \frac{\Gamma}{4\pi}\ln\left(\frac{\Gamma}{e\Lambda v_F}\right)\right]\right)$$

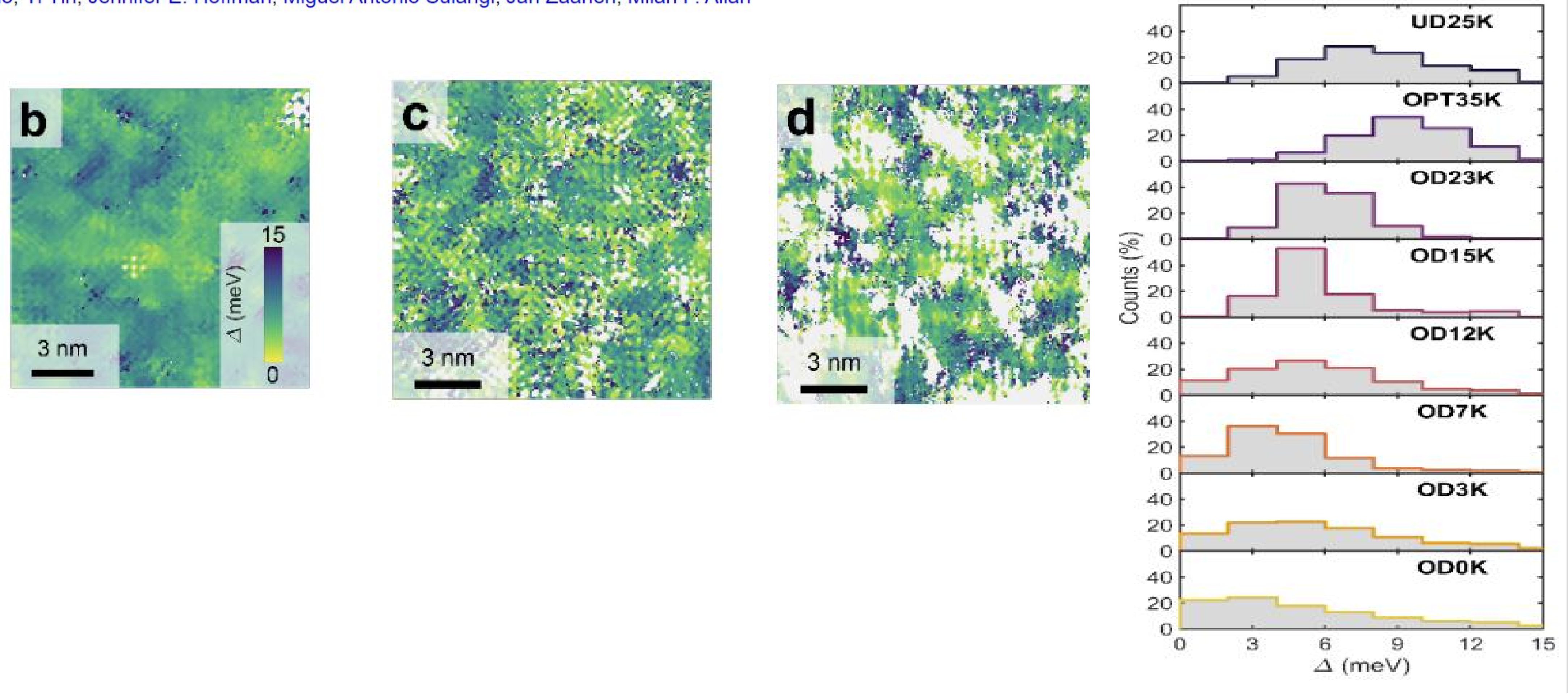
$$\tau_{tr}^{-1}(\omega) = \Gamma + A\omega^2 (+A'T^2)$$



# A new kind of disorder

**Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped (Pb,Bi)2Sr2CuO6+δ**

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge, Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos, Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden, Yingkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan



# Interaction disorder

- The disordered gap map motivates a disordered Hubbard exchange interaction $J_{ij}$
- This can be schematically written as a random mass term for the critical boson $\phi$

$$\frac{\phi(x)^2}{J+J'(x)} + \psi^\dagger(x)\psi(x)\phi(x) + v(x)\psi^\dagger(x)\psi(x)$$

- We can now rescale the boson $\phi$ to put the disorder into the coupling:

$$S_{g'} = \int \mathrm{d}\tau \mathrm{d}^2 x g'(x)\psi^\dagger(x,\tau)\psi(x,\tau)\phi(x,\tau)$$

$$\overline{g'(x)} = 0, \ \overline{g'(x)g'(x')} = (g')^2\delta(x-x')$$

# A critical Fermi surface with interaction disorder

- Action

$$S = S_\psi + S_\phi + S_g + S_{g'} + S_v$$

$$S_g = \sum_{ijl} \frac{g_{ijl}}{N} \int \mathrm{d}^2 x \mathrm{d}\tau \psi_i^\dagger(x,\tau)\psi_j(x,\tau)\phi_l(x,\tau)$$

$$\overline{g_{ijl}} = 0,\ \overline{g_{ijl}^2} = g^2$$

$$S_{g'} = \sum_{ijl} \frac{1}{N} \int \mathrm{d}\tau \mathrm{d}^2 x g'_{ijl}(x)\psi_i^\dagger(x,\tau)\psi_j(x,\tau)\phi_l(x,\tau)$$

$$\overline{g'_{ijl}(x)} = 0,\ \overline{g'_{ijl}(x)^* g'_{i'j'l'}(x')} = (g')^2\delta(x-x')\delta_{ii'}\delta_{jj'}\delta_{ll'}$$

# Interaction disorder: A Marginal Fermi liquid and a strange metal

- Linear in T resistivity: because momentum is not conserved, the vertex correction vanishes for disordered interaction $g'$
- Fermion diffusion step now becomes order one:

$$\partial_t n(\theta) = D_2 \partial_\theta^2 n(\theta) \quad D_2 \sim \Sigma_g''(\omega)(\delta\theta)^2 \propto |\omega| \qquad \delta\theta \sim \pi$$

$$\sigma(\omega) \sim \frac{1}{\tau_{tr}^{-1}(\omega) - i\omega m^*(\omega)/m}$$

$$\tau_{tr}^{-1}(\omega) \sim \Gamma + g'^2|\omega| \qquad\qquad \frac{m^*(\omega)}{m} \sim g'^2 \ln(\Lambda/|\omega|)$$

Residual resistivity detmined by potential disorder $\Gamma \propto v^2$. Linear in $T$ resistivity determined by interaction disorder $g'^2$.

# Properties of a strange metal

- Resistivity: $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$, and $\rho(T) < h/e^2$ (in 2D). Metals with $\rho(T) > h/e^2$ is a bad metal.
- A marginal Fermi liquid self energy.$\to$Specific heat: $C \sim T\ln(1/T)$ as $T \to 0$.
- Optical conductivity $\sigma(\omega) = \frac{K}{\tau(\omega)^{-1} - i\omega\frac{m^*(\omega)}{m}}$, $\frac{1}{\tau(\omega)} = \frac{k_B T}{\hbar} S\left(\frac{\hbar\omega}{k_B T}\right)$

# Summary

- In 0+1D, the SYK model is a toy model for quantum matter without quasiparticles
- Combining fractionalization and SYK model, we can have a theory of correlated metal which mimics the strange metal phenomenology
- In 2+1D, the Yukawa-SYK model provides a controlled frame work to study the critical fermi surface
- Without disorder, the critical Fermi surface is a non-Fermi liquid but not a strange metal
- With potential disorder, the critical Fermi surface becomes a marginal Fermi liquid but still not a strange metal
- With interaction disorder, the critical Fermi surface becomes a marginal Fermi liquid and a strange metal