Bosonization study of the higher harmonic spectral properties of 1D extended Hubbard model.

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Sept 27th, 2023

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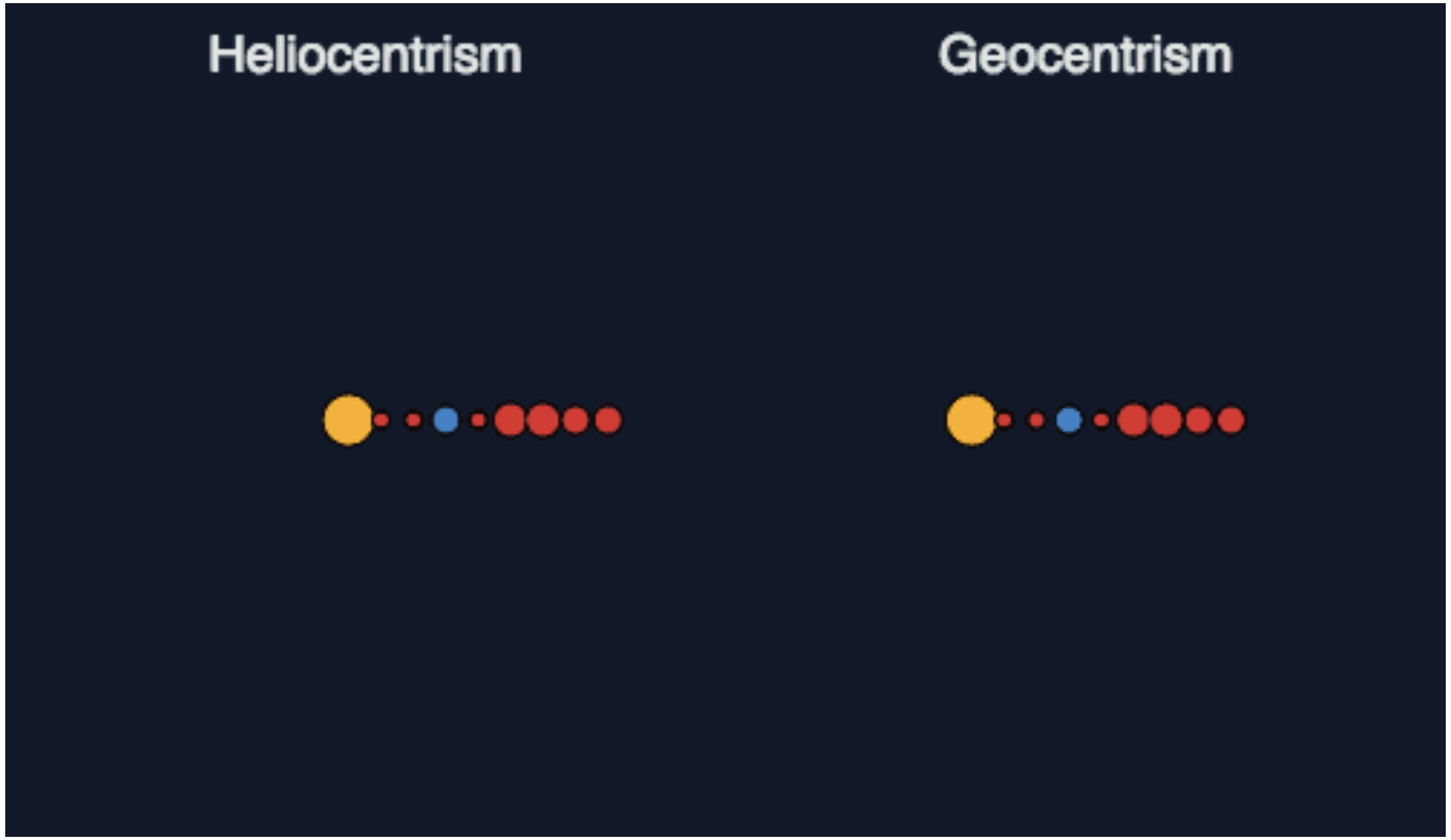
1. Introduction



1. Introduction

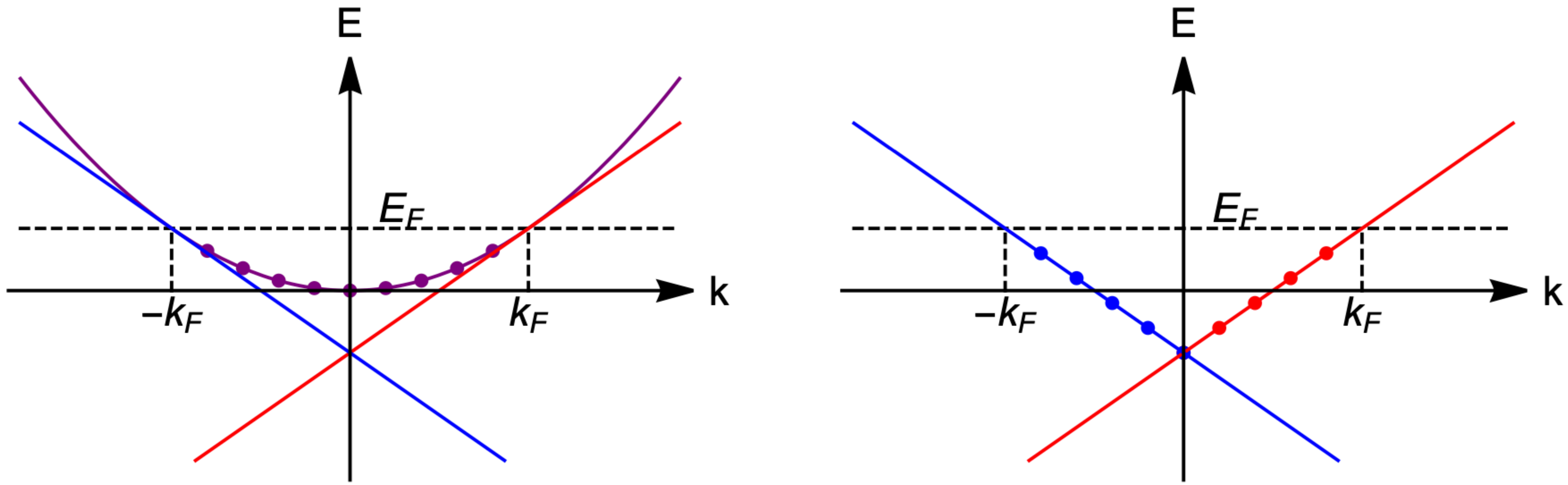
Bosonization. Many giants, 70s to 80s

Fermions:
anti-symmetric under permutation;
represented by Grassmann numbers;
Pauli exclusion principle;
Fermi surface; ...

Bosons:
symmetric under permutation;
represented by complex numbers;
Collective modes;
Bose-Einstein condensation;...

Jordan-Wigner transformation (1928);
sine-Gordon = massive Thirring, Coleman (1975)

2. Bosonization I: exact bosonization from T-L model



Arouca, Cappelli and Hansson, SciPost (2022)

Tomonaga-Luttinger model

infinitely filled states below E_F

$$\epsilon(k) = v_F(rk - k_F)$$

$r = +$: right mover

$r = -$: left mover

$$H_0 = \sum_{k,r,s} v_F(rk - k_F)\psi^\dagger_{r,s}(k)\psi_{r,s}(k)$$

$s = +$: spin up;

$s = -$: spin down

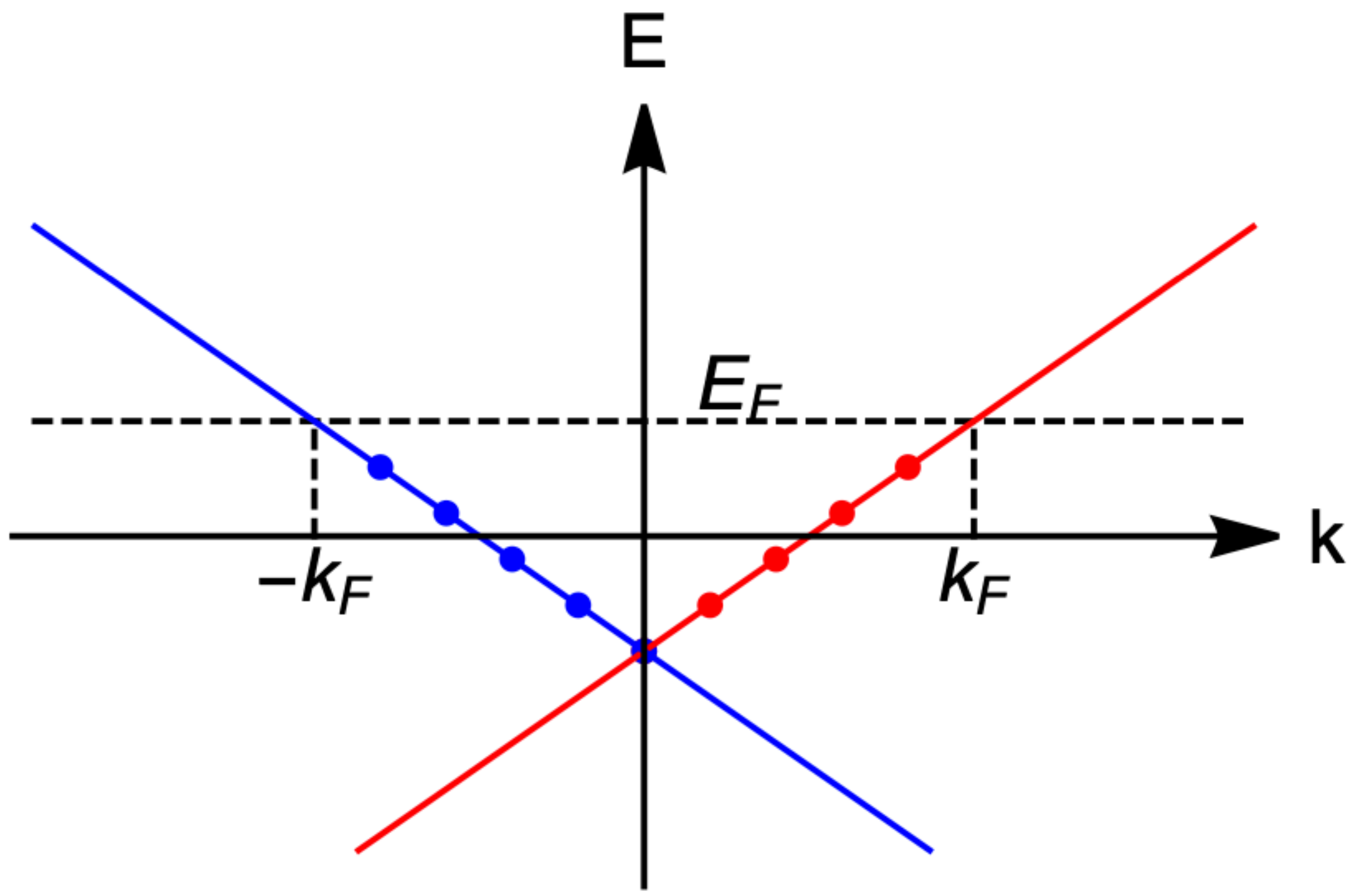
$$H_I = g_{1,ss'} \sum_r \sum_{k,p,q} \psi^\dagger_{r,s}(k)\psi^\dagger_{-r,s'}(p)\psi_{r,s'}(p+q)\psi_{-r,s}(k-q)$$

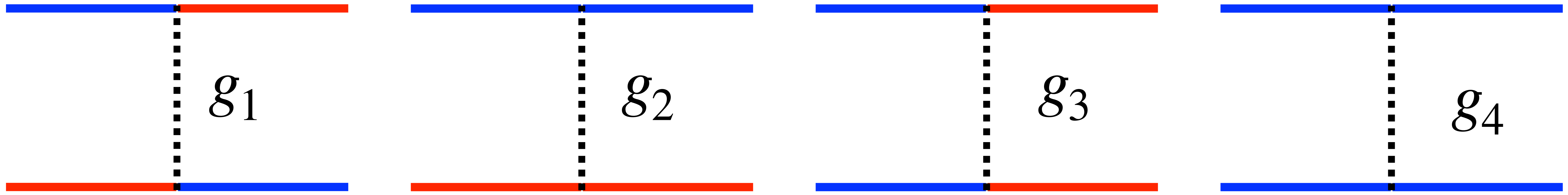
$$+ g_{2,ss'} \sum_{r,q} \rho_{r,s}(q)\rho_{-r,s'}(-q) + g_{4,ss'} \sum_{r,q} \rho_{r,s}(q)\rho_{r,s'}(-q)$$

density operator:

$$\rho_{r,s}(q) = \sum_k \psi^\dagger_{r,s}(k)\psi_{r,s}(k+q)$$

2. Bosonization I: exact bosonization from T-L model





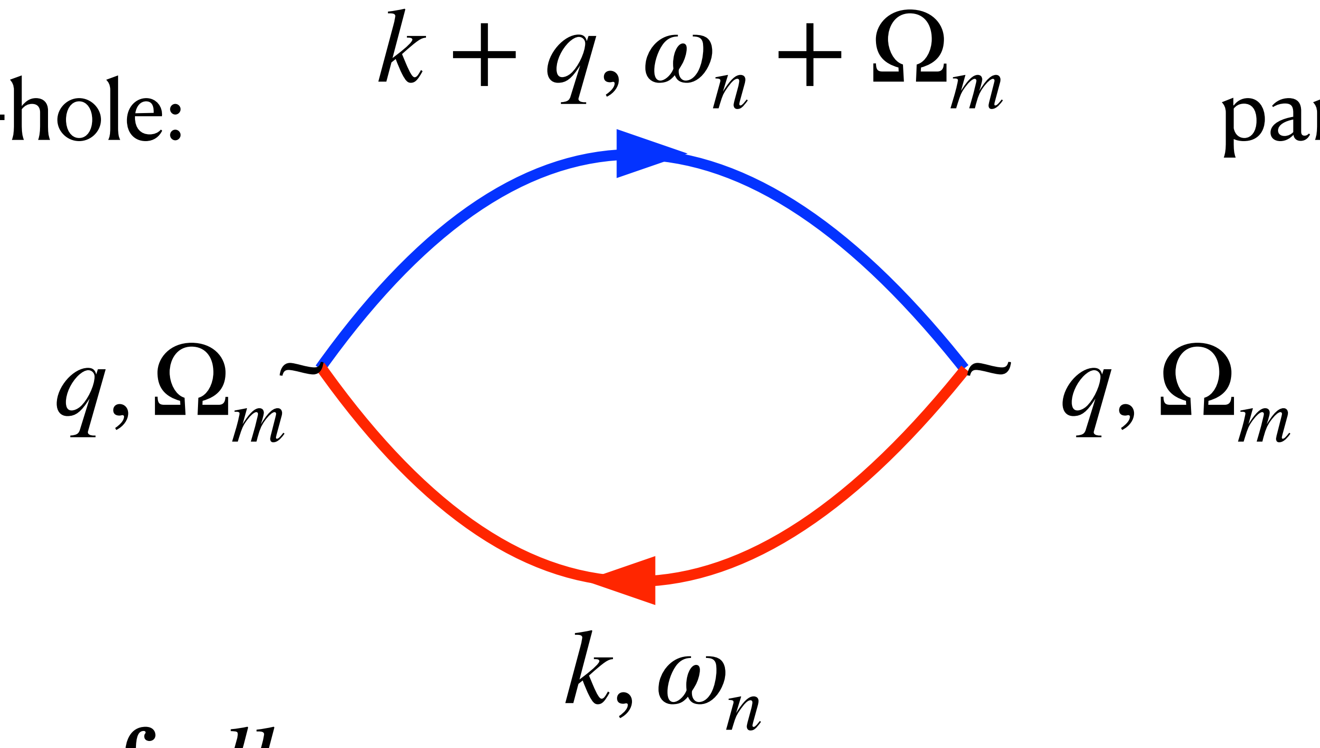
back scattering

Umklapp

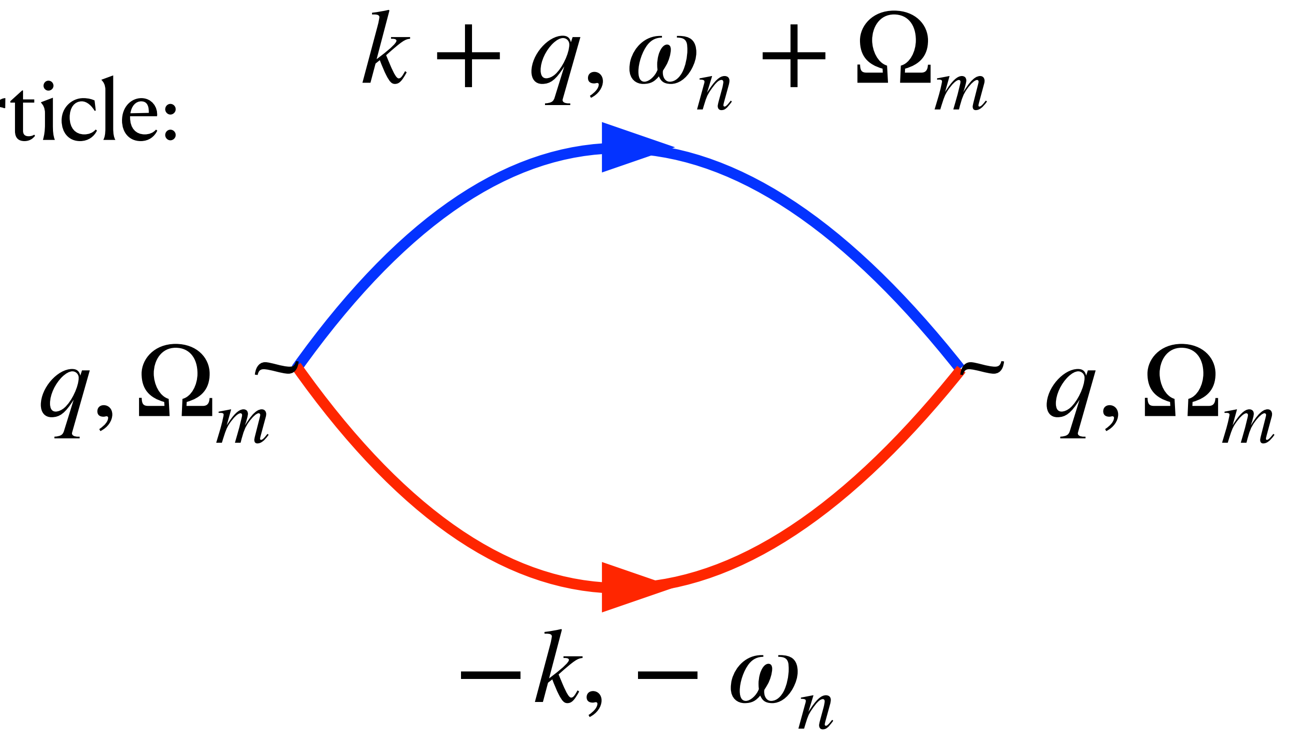
German word for abrupt turn around

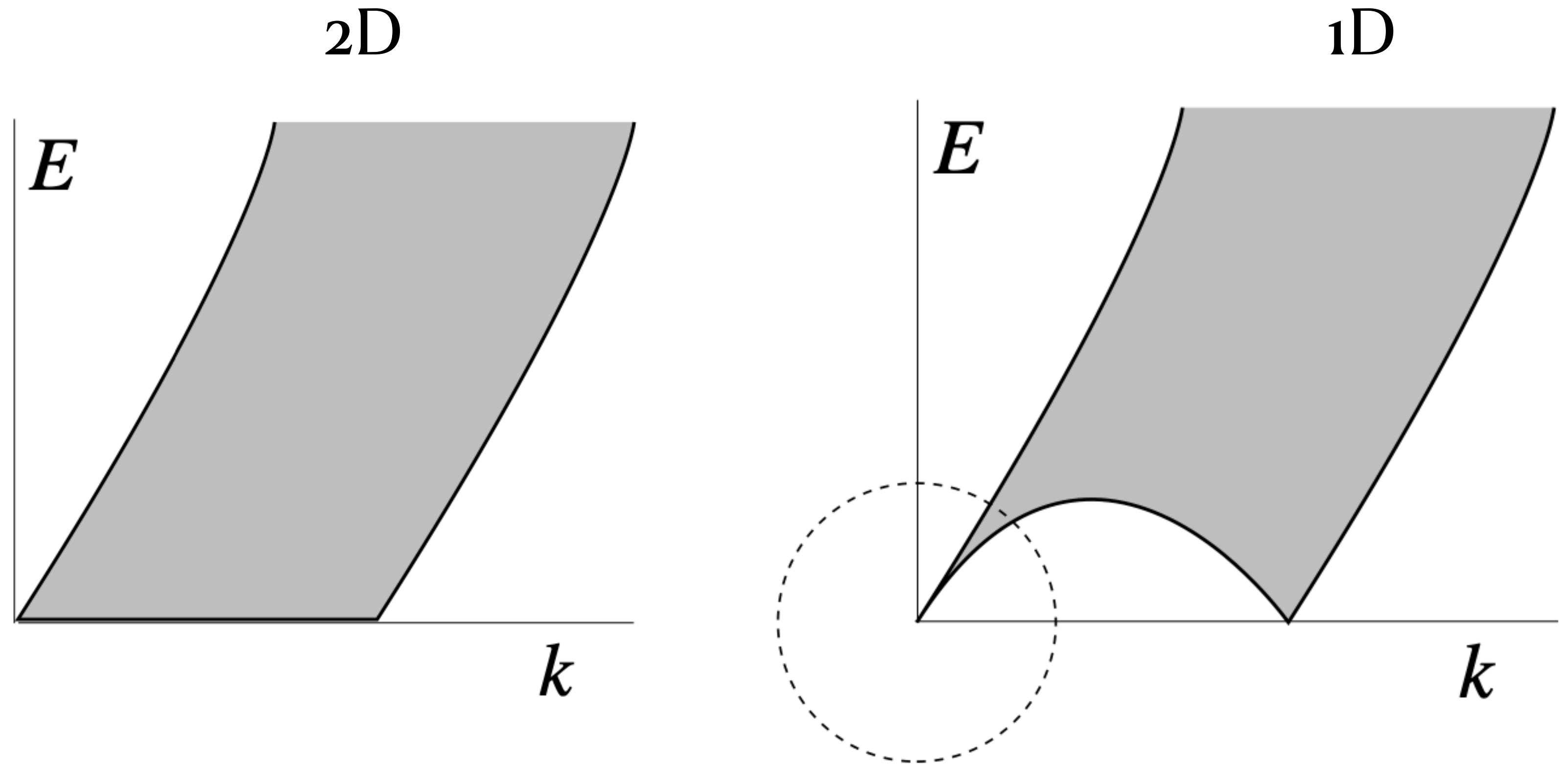
Why is 1D so special?

particle-hole:



particle-particle:





Sénéchal, (1999)

$$\mp T \sum_n \int \frac{dk}{2\pi} G_-(\mp\omega_n, \mp k) G_+(\omega_n + \Omega_m, k+q)$$

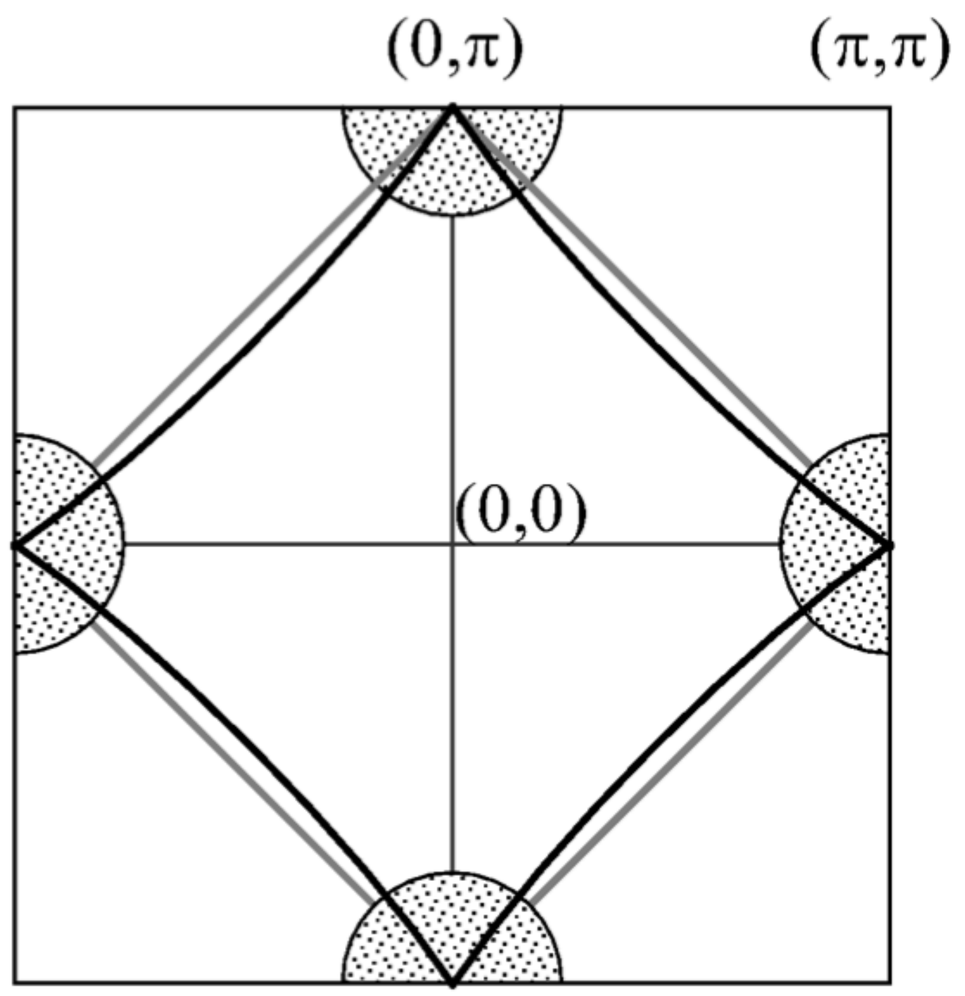
$$= -T \sum_n \int \frac{dk}{2\pi} \frac{1}{(i\omega_n + v_F k)[i\omega_n + i\Omega_m - v_F(k+q)]} \sim \boxed{\log \frac{\Lambda}{\max(T, v_F q, \Omega_m)}}$$

Zheleznyak, Yakovenko and Dzyaloshinskii (1997)

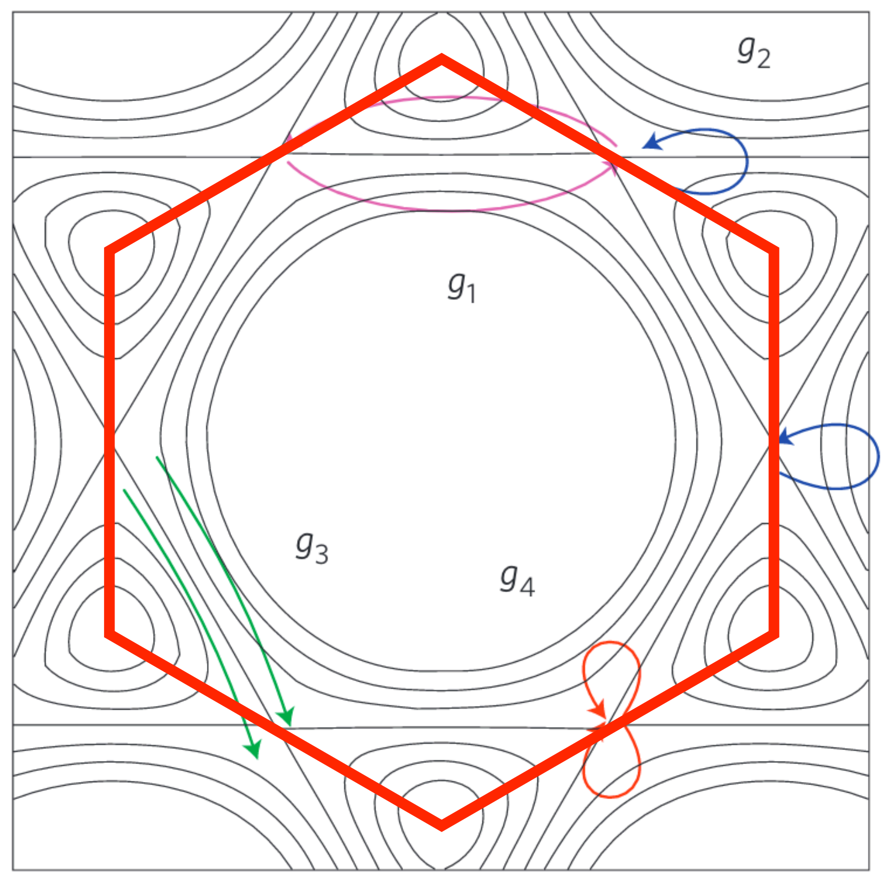
2. Bosonization I: exact bosonization from T-L model

- Particle-hole and particle-particle susceptibility contains the same log: typical in 1D
- In higher dimensions, usually only particle-particle bubble has log scaling, which is the Cooper logarithm relevant for weak coupling uniform superconductivity.
- Exception, Fermi surface nesting which leads to particle-hole logarithm and hence C/SDW.

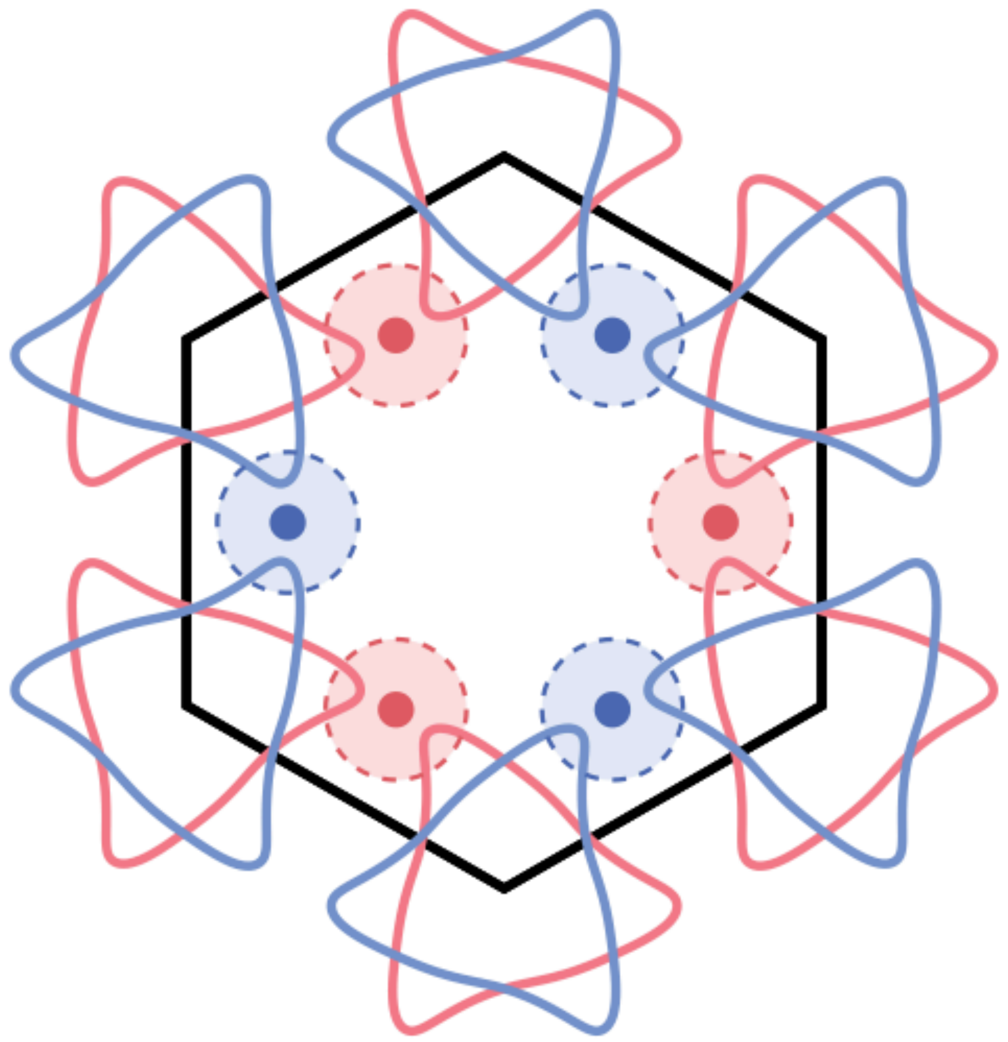
Application to 2D: FS nesting + Van Hove singularity (+ sublattice interference effects)



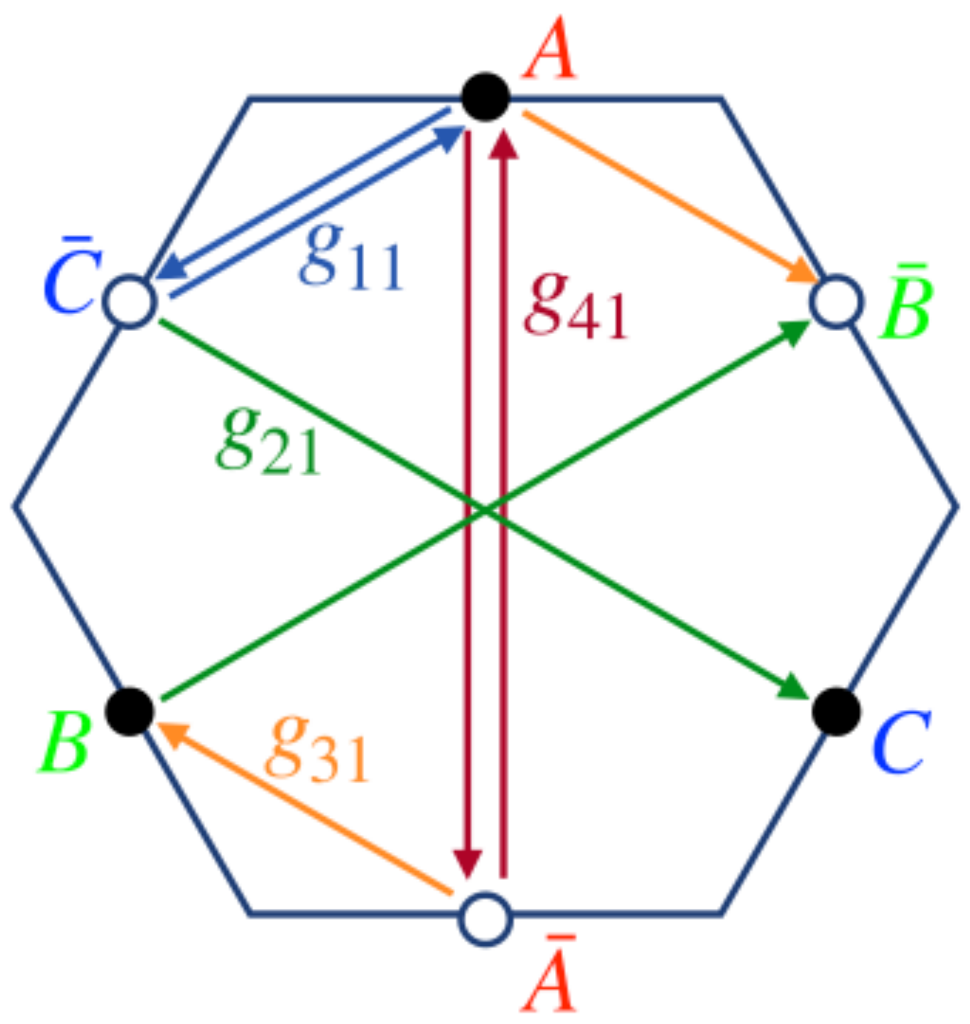
Furukawa, Rice and Salmhofer (1998)



Nandkishore, Levitov and Chubukov (2012)



Isobe, Yuan and Fu (2018)



Wu, Thomale and Raghu (2023)

Algebraic structure of the (spineless) density operator

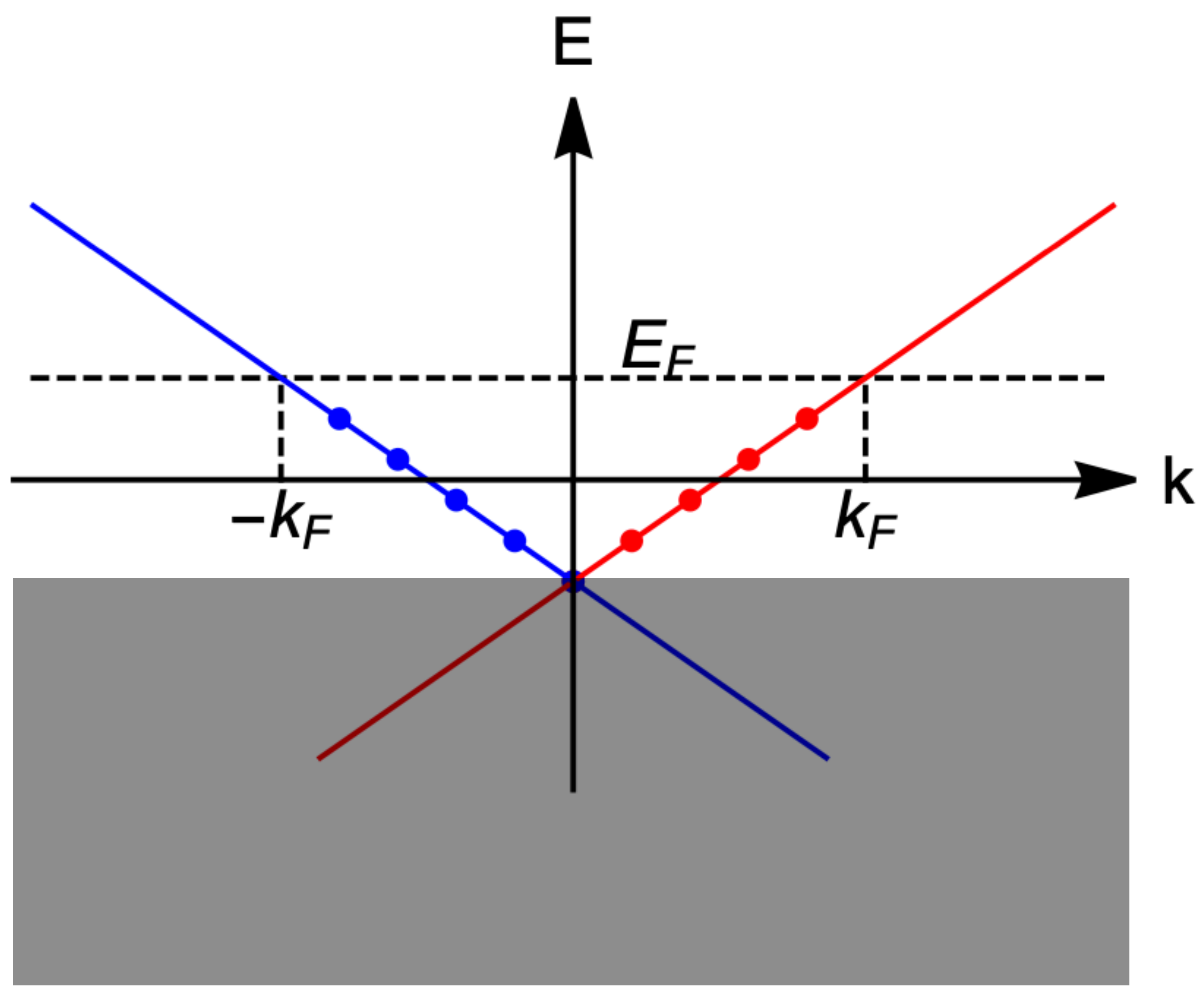
$$\rho_{r,s}(q) = \sum_k \psi^\dagger_{r,s}(k)\psi_{r,s}(k+q)$$ ill-defined for $q = 0$, because of infinitely filled states below Fermi level

$$\rho_{r,s}(q) = \sum_k \psi^\dagger_{r,s}(k)\psi_{r,s}(k+q) - \delta_{q,0}\langle\psi^\dagger_{r,s}(k)\psi_{r,s}(k)\rangle$$ Normal ordering: reference subtraction

Alternatively,

$$\rho_{+,s}(q) = \sum_{k>0} \psi^\dagger_{+,s}(k)\psi_{+,s}(k+q)$$

$$\rho_{-,s}(q) = \sum_{k<0} \psi^\dagger_{-,s}(k)\psi_{-,s}(k+q)$$



2. Bosonization I: exact bosonization from T-L model

Algebraic structure of the (spineless) density operator

$$[\rho_+(q), \rho_+(-q)] = \sum_{k,k'>0} [\psi_k^\dagger \psi_{k+q}, \psi_{k'}^\dagger \psi_{k'-q}]$$

$$= \sum_{k,k'>0} (\psi_k^\dagger \psi_{k'-q} \delta_{k+q,k'} - \psi_{k'}^\dagger \psi_{k+q} \delta_{k,k'-q})$$

$$= \sum_{k>0} \Theta(k+q)(\psi_k^\dagger \psi_k - \psi_{k+q}^\dagger \psi_{k+q}) = \frac{L}{2\pi} q$$

$$[\rho_-(q), \rho_-(-q)] = -\frac{L}{2\pi} q$$

$$[\rho_+(q), \rho_+(-q')] = +\delta_{q,q'} \frac{L}{2\pi} q$$

$$[\rho_-(q), \rho_-(-q')] = -\delta_{q,q'} \frac{L}{2\pi} q$$

$$[\rho_+(q), \rho_-(-q')] = 0$$

$$H_0 = v_F \sum_{k>0} k(\psi_+^\dagger(k)\psi_+(k) - \psi_-^\dagger(k)\psi_-(k))$$

$$[H_0, \rho_+(q)] = -v_F q \rho_+(q)$$

$$[H_0, \rho_-(q)] = +v_F q \rho_-(q)$$

Free Hamiltonian is bilinear in density operators: $H_0 = \dfrac{2\pi v_F}{L} \sum_{q>0} (\rho_+(q)\rho_+(-q) + \rho_-(q)\rho_-(-q))$

2. Bosonization I: exact bosonization from T-L model

Hamiltonian in the bosonized form (spin-charge separation)

$$H_0 = \frac{2\pi v_F}{L} \sum_{q>0,r,s} \rho_{r,s}(q)\rho_{r,s}(-q)$$

$$H_I = \sum_{q,r,s} [(g_{2\parallel,q} - g_{1\parallel,q})\delta_{s,s'} + g_{2\perp,q}\delta_{s,-s'}]\rho_{r,s}(q)\rho_{-r,s'}(-q)$$

$$+ \sum_{q,r,s} (g_{4\parallel,q}\delta_{s,s'} + g_{4\perp,q}\delta_{s,-s'})\rho_{r,s}(q)\rho_{r,s'}(-q)$$

Charge

$$H_c = \sum_{q,r} [(\pi v_F + g_{4\parallel,q} + g_{4\perp,q})\rho_r^c(q)\rho_r^c(-q) + (g_{2\parallel,q} - g_{1\parallel,q} + g_{2\perp,q})\rho_r^c(q)\rho_{-r}^c(-q)]$$

$$H_s = \sum_{q,r} [(\pi v_F + g_{4\parallel,q} - g_{4\perp,q})\rho_r^s(q)\rho_r^s(-q) + (g_{2\parallel,q} - g_{1\parallel,q} - g_{2\perp,q})\rho_r^s(q)\rho_{-r}^s(-q)]$$

Spin

$$H_0 + H_I = H_c + H_s$$

$$\rho_r^c = \frac{\rho_{r,+}(q) + \rho_{r,-}(q)}{\sqrt{2}} \qquad \rho_r^s = \frac{\rho_{r,+}(q) - \rho_{r,-}(q)}{\sqrt{2}}$$

Introducing:

$$\partial_x \phi_\nu(x)/\pi = \rho_+^\nu(x) + \rho_-^\nu(x), \quad \nu = c, s$$

$$\Pi_\nu(x) = \rho_+^\nu(x) - \rho_-^\nu(x), \qquad [\phi_\nu(x), \Pi_\nu(y)] = i\delta(x-y)$$

$$H_\nu = \int_{-L/2}^{L/2} dx \left[\frac{\pi v_\nu K_\nu}{2}(\Pi_\nu(x))^2 + \frac{v_\nu}{2\pi K_\nu}(\partial_x \Phi_\nu(x))^2\right]$$

Spin and charge sectors have different velocities v_ν and Luttinger parameters K_ν

2. Bosonization I: exact bosonization from T-L model

In momentum space

$$H_\nu = \frac{1}{L}\sum_k \left(\frac{\pi v_\nu K_\nu}{2}\Pi_\nu(k)\Pi_\nu(-k) + \frac{v_\nu}{2\pi K_\nu}k^2\phi_\nu(k)\phi_\nu(-k) \right)$$

Bogoliubov transformation:

$$\gamma_k = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{|k|}{\pi K_\nu}}\phi_\nu(k) + i\sqrt{\frac{\pi K_\nu}{|k|}}\Pi_\nu(k) \right) \quad \gamma_k^\dagger = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{|k|}{\pi K_\nu}}\phi_\nu(-k) - i\sqrt{\frac{\pi K_\nu}{|k|}}\Pi_\nu(-k) \right)$$

$$\phi_\nu(k) = \sqrt{\frac{\pi K_\nu}{2|k|}}(\gamma_k + \gamma_{-k}^\dagger),\ \Pi_\nu(k) = -i\sqrt{\frac{|k|}{2\pi K_\nu}}(\gamma_k - \gamma_{-k}^\dagger)$$

Diagonalized Hamiltonian:

$$H_\nu = v_\nu \sum_k |k|\gamma_k^\dagger \gamma_k$$

2. Bosonization I: exact bosonization from T-L model

From Hamiltonian to Lagrangian:

$$H_\nu = \int_{-L/2}^{L/2} dx \left[\frac{\pi v_\nu K_\nu}{2} (\Pi_\nu(x))^2 + \frac{v_\nu}{2\pi K_\nu} (\partial_x \Phi_\nu(x))^2 \right]$$

$$H(x,p) = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

$$[x,p] = i\hbar$$

$$[\phi_\nu(x), \Pi_\nu(y)] = i\delta(x-y)$$

$$L(x,\dot{x}) = p\dot{x} - H(x,p) = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$$

$$p = m\dot{x}$$

$$\mathscr{L}[\phi_\nu] = \frac{1}{2\pi K_\nu} \left(\frac{1}{v_\nu} (\partial_\tau \phi_\nu)^2 + v_\nu (\partial_x \phi_\nu)^2 \right)$$

Free massless boson field (conformal field theory in 1+1 D):

$$S = \int d^2x (\nabla \phi)^2$$

2. Bosonization I: exact bosonization from T-L model

What we've done so far is to express the theory in terms of boson field. A complete theory should also establish the relation between fermion and boson field.

$$\psi_{r,s} = \lim_{\alpha\to 0} \frac{U_{r,s}}{\sqrt{2\pi\alpha}} \exp\left(-\frac{i}{\sqrt{2}} \left[r(\phi_c(x,\tau) + s\phi_s(x,\tau)) + (\theta_c(x,\tau) + s\theta_s(x,\tau)) \right] \right)$$

$$\theta_\nu(x) = \pi \int^x dx' \Pi_\nu(x')$$

Heidenreich, Seiler and Uhlenbrock, 1980

Haldane, 1981

Now we can discuss backscattering $g_{1,\perp} \sum_r \psi^\dagger_{r,+}\psi^\dagger_{-r,-}\psi_{r,-}\psi_{-r,+}$

$$\sum_r \psi^\dagger_{r,+}\psi^\dagger_{-r,-}\psi_{r,-}\psi_{-r,+} = e^{\frac{i}{\sqrt{2}}(\phi_c+\phi_s+\theta_c+\theta_s)} e^{\frac{i}{\sqrt{2}}(-\phi_c+\phi_s+\theta_c-\theta_s)} e^{-\frac{i}{\sqrt{2}}(\phi_c-\phi_s+\theta_c-\theta_s)} e^{-\frac{i}{\sqrt{2}}(-\phi_c-\phi_s+\theta_c+\theta_s)} + \text{h.c.}$$

$$= 2\cos(\sqrt{8}\phi_s)$$

Only $\phi_s(x,\tau)$ remains.

2. Bosonization I: exact bosonization from T-L model

$$S = \frac{1}{2\pi K_s}\int dx d\tau \left(\frac{1}{v_s}(\partial_\tau \phi_s)^2 + v_s(\partial_x \phi_s)^2 \right) + \frac{2g_{1,\perp}}{(2\pi\alpha)^2}\int dx d\tau \cos(\sqrt{8}\phi_s)$$

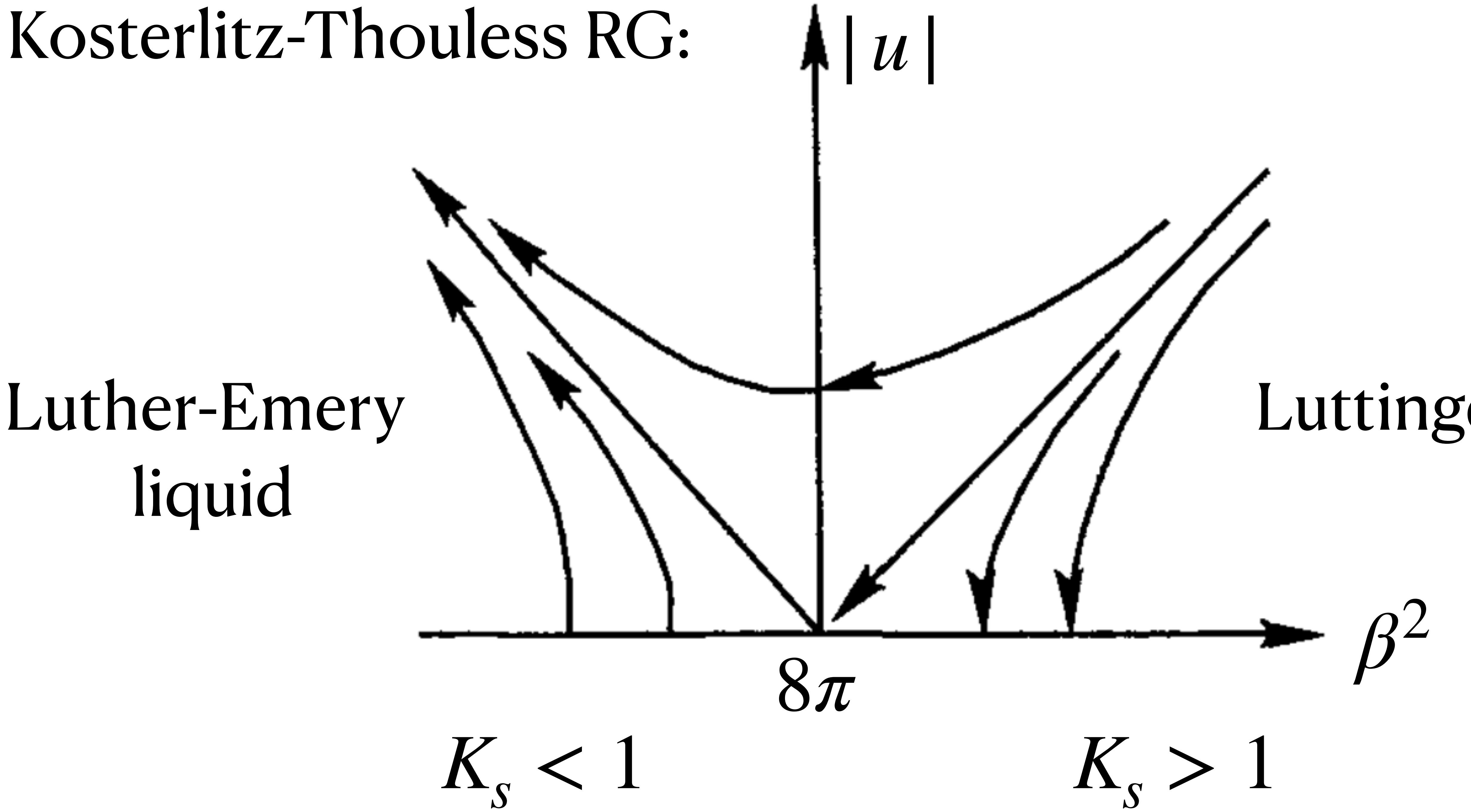
$$= \frac{1}{2}\int d^2x(\nabla\phi)^2 + \frac{u}{\alpha^2}\int d^2x \cos\beta\phi$$

Sine-Gordon model

$$u = \frac{g_{1,\perp}}{2\pi^2 v_s}$$

$$\beta = \sqrt{8\pi K_s}$$

Kosterlitz-Thouless RG:



Similar analysis applies for charge sector when g_3 is included.

3. Bosonization II: Dzyaloshinskii-Larkin loop-cancellation theorem.

Introducing bosons via Hubbard-Stratonovich transformation:

$$\mathcal{L}(x,\tau) = \bar{\psi}(\partial_\tau - \frac{\partial_x^2}{2m} - \mu)\psi + \frac{1}{2}\int_{x'} \bar{\psi}(x)\bar{\psi}(x')V(x-x')\psi(x')\psi(x)$$

H.S. tranformation

$$\mathcal{L}(x,\tau) = \bar{\psi}(\partial_\tau - \frac{\partial_x^2}{2m} - \mu)\psi + \frac{1}{2}\int_{x'} \phi(x)V^{-1}(x-x')\phi(x') - i\phi(x)\bar{\psi}\psi$$

Linearizing the dispersion

Integrating out fermions

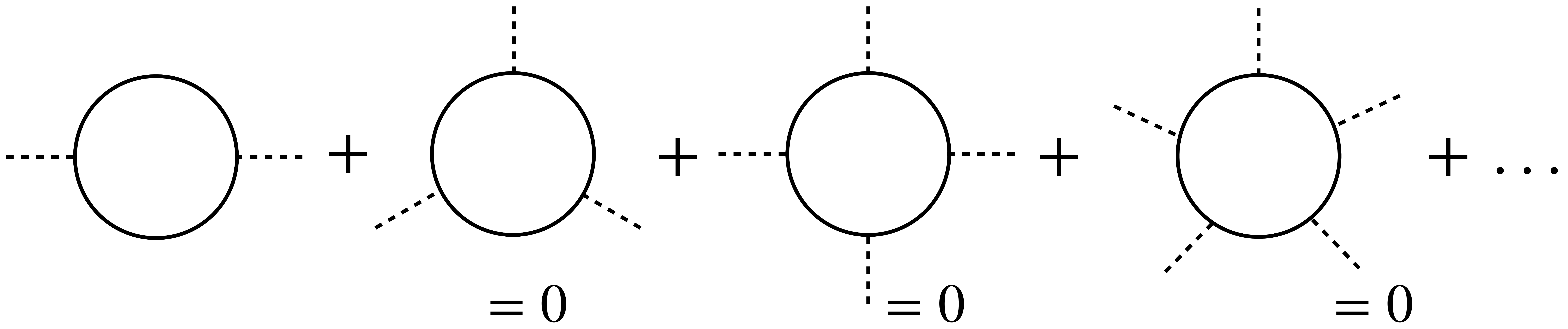
$$\mathcal{L}(x,\tau) = \frac{1}{2}\int_{x'} \phi(x)V^{-1}(x-x')\phi(x') + \mathrm{tr}\ln\begin{pmatrix} \partial_\tau - iv_F\partial_x - i\phi & 0 \\ 0 & \partial_\tau + iv_F\partial_x - i\phi \end{pmatrix}$$

3. Bosonization II: Dzyaloshinskii-Larkin loop-cancellation theorem.

$$\mathrm{tr}\ln\begin{pmatrix}\partial_\tau - iv_F\partial_x - i\phi & 0 \\ 0 & \partial_\tau + iv_F\partial_x - i\phi\end{pmatrix} = \mathrm{tr}\ln(\partial_\tau - iv_F\partial_x - i\phi) + \mathrm{tr}\ln(\partial_\tau + iv_F\partial_x - i\phi)$$

Loop expansion near the mean field solution:

$$\mathrm{tr}\ln(\partial_\tau - iv_F\partial_x - i\phi) = \mathrm{tr}\ln(\partial_\tau - iv_F\partial_x) - \sum_{n=1}^{\infty}\frac{1}{n}\mathrm{tr}(G_+^0 i\phi)^n \qquad G_\pm^0 = (\partial_\tau \mp iv_F\partial_x)^{-1}$$



Exact when with fermion linear dispersion. Bosons are bilinear and free

Dzyaloshinskii and Larkin (1973)

3. Bosonization II: Dzyaloshinskii-Larkin loop-cancellation theorem.

Full Fermion Green's functions:

$$G(x,\tau) = \frac{\int \mathcal{D}[\bar\psi,\psi]\mathcal{D}[\phi]\bar\psi(x,\tau)\psi(0)\exp[-S[\bar\psi,\psi,\phi]]}{\int \mathcal{D}[\bar\psi,\psi]\mathcal{D}[\phi]\exp[-S[\bar\psi,\psi,\phi]]}$$

$$S[\bar\psi,\psi,\phi] = \int \frac{1}{2}\phi V^{-1}\phi + \bar\psi[\partial_\tau + \xi - i\phi]\psi$$

$$= \frac{\int \mathcal{D}[\phi]e^{-\int \frac{1}{2}\phi V^{-1}\phi}\int \mathcal{D}[\bar\psi,\psi]\bar\psi(x,\tau)\psi(0)e^{-\int \bar\psi[\partial_\tau+\xi-i\phi]\psi}}{\int \mathcal{D}[\phi]e^{-\int \frac{1}{2}\phi V^{-1}\phi}\int \mathcal{D}[\bar\psi,\psi]e^{-\int \bar\psi[\partial_\tau+\xi-i\phi]\psi}}$$

$$\int \mathcal{D}[\bar\psi,\psi]e^{-\int \bar\psi[\partial_\tau+\xi-i\phi]\psi} = e^{Tr\ln[\partial_\tau+\xi-i\phi]}$$

$$= \frac{\int \mathcal{D}[\phi]e^{-S[\phi]}\frac{\int \mathcal{D}[\bar\psi,\psi]\bar\psi(x,\tau)\psi(0)e^{-\int \bar\psi[\partial_\tau+\xi-i\phi]\psi}}{\mathcal{D}[\bar\psi,\psi]e^{-\int \bar\psi[\partial_\tau+\xi-i\phi]\psi}}}{\int \mathcal{D}[\phi]e^{-S[\phi]}}$$

$$S[\phi] = \int \frac{1}{2}\phi V^{-1}\phi - \mathrm{Tr}\ln[\partial_\tau + \xi - i\phi]$$

$$= \frac{1}{Z}\int \mathcal{D}[\phi]e^{-S[\phi]}\mathcal{G}[x,\tau,\phi]$$

$$\mathcal{G}[x,\tau,\phi] = \frac{\int \mathcal{D}[\bar\psi,\psi]\bar\psi(x,\tau)\psi(0)e^{-\int \bar\psi[\partial_\tau+\xi-i\phi]\psi}}{\mathcal{D}[\bar\psi,\psi]e^{-\int \bar\psi[\partial_\tau+\xi-i\phi]\psi}}$$

D K K Lee and Y Chen , 1988 I V Yurkevich, 2022

3. Bosonization II: Dzyaloshinskii-Larkin loop-cancellation theorem.

If we know $\mathscr{G}[x,\tau,\phi]$, then the fermion properties are determined by the boson field ϕ

$$\mathscr{G}[x,\tau;x',\tau',\phi] = \frac{\int \mathscr{D}[\bar{\psi},\psi]\bar{\psi}(x,\tau)\psi(x',\tau')e^{-\int \bar{\psi}[\partial_\tau+\xi-i\phi]\psi}}{\mathscr{D}[\bar{\psi},\psi]e^{-\int \bar{\psi}[\partial_\tau+\xi-i\phi]\psi}}$$

$$(\partial_\tau - iv_F\partial_x - i\phi(x,\tau))\mathscr{G}_+[x,\tau;x',\tau',\phi] = \delta(x-x')\delta(\tau-\tau')$$

$$(\partial_\tau + iv_F\partial_x - i\phi(x,\tau))\mathscr{G}_-[x,\tau;x',\tau',\phi] = \delta(x-x')\delta(\tau-\tau')$$

The solutions are written as the following ansatz

$$\mathscr{G}_+[x,\tau;x',\tau',\phi] = G_+^0(x-x',\tau-\tau')e^{if(x,\tau)-if(x',\tau')} \qquad \mathscr{G}_-[x,\tau;x',\tau',\phi] = G_-^0(x-x',\tau-\tau')e^{if^*(x,\tau)-if^*(x',\tau')}$$

Substituting back to the equations leads to

$$G_\pm^0 = (\partial_\tau \mp iv_F\partial_x)^{-1}$$

$$(\partial_\tau - iv_F\partial_x)f(x,\tau) = \phi(x,\tau)$$

Schwinger, 1962

3. Bosonization II: Dzyaloshinskii-Larkin loop-cancellation theorem.

Bosonization of this procedure can be generalized to include interactions with retardation effect

PHYSICAL REVIEW B **69**, 165108 (2004)

Functional integral bosonization for an impurity in a Luttinger liquid

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(Received 17 July 2003; revised manuscript received 18 November 2003; published 7 April 2004)

PHYSICAL REVIEW B **83**, 041106(R) (2011)

Impurity scattering in a Luttinger liquid with electron-phonon coupling

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(Received 24 December 2010; published 26 January 2011)

Direct generalization to higher dimension

PHYSICAL REVIEW B VOLUME 52, NUMBER 15 15 OCTOBER 1995-I

Bosonization of interacting fermions in arbitrary dimension beyond the Gaussian approximation

Peter Kopietz, Joachim Hermisson, and Kurt Schönhammer
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(Received 24 February 1995)

Bosonization of coupled electron-phonon systems

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Received: 27 July 1995 / Revised version: 25 October 1995

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Nonlinear bosonization of Fermi surfaces: The method of coadjoint orbits

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4. Bosonization III: Haldane's phenomenological approach

Linear dispersion is only a good approximation in low energy limit. Nonlinearity is always present.

Effective Harmonic-Fluid Approach to Low-Energy Properties of One-Dimensional Quantum Fluids

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(Received 29 December 1980)

A universal description of the low-energy properties of one-dimensional quantum fluids, based on a harmonic theory of long-wavelength density fluctuations with use of renormalized parameters, is outlined. The structure of long-distance correlations of a spinless fluid is obtained, showing the essential similarity of one-dimensional Bose and Fermi fluids. The results are illustrated by application to the one-dimensional Bose fluid with δ-function interaction.

PACS numbers: 67.40.Db, 05.30.-d

Non-linear dispersion → interacting bosons & the presence of higher order harmonics: $3k_F$, $5k_F$, ... branches

4. Bosonization III: Haldane's phenomenological approach

Two basic facts:

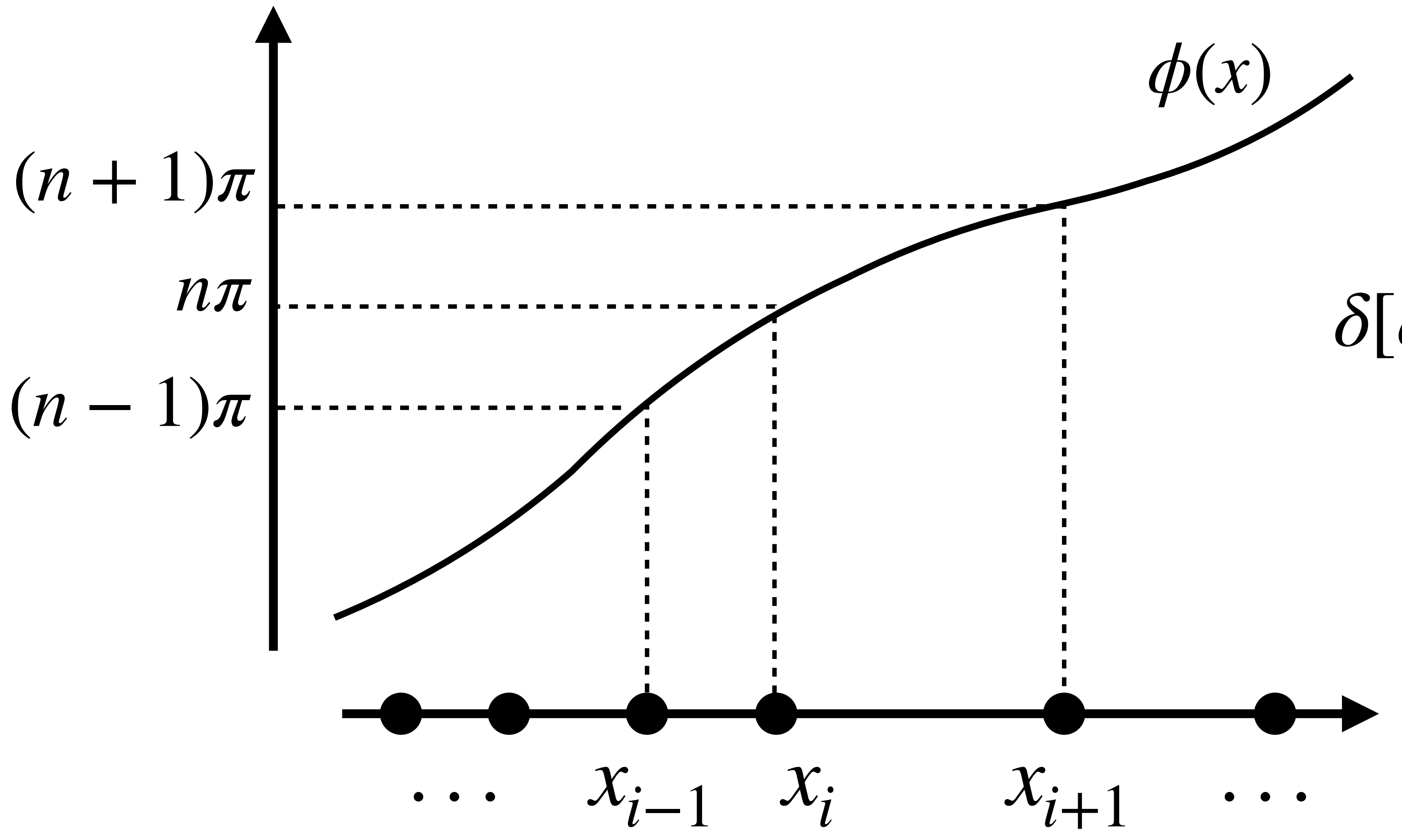
$1:\delta$-function

$$\delta[f(x)] = \sum_{x_0} \frac{\delta(x-x_0)}{|f'(x)|},\ f(x_0)=0$$

$2:$ Poisson summation

$$f(t) \leftrightarrow \tilde{f}(s) = \int dt e^{i2\pi st} f(t):$$

$$\sum_{n\in\mathbb{Z}} f(n) = \sum_{m\in\mathbb{Z}} \tilde{f}(m)$$



$$\delta[\phi(x) - n\pi] = \frac{\delta(x-x_i)}{\partial_x\phi(x)}$$

$$\rho(x) = \sum_i \delta(x-x_i)$$

$$\rho(x) = \sum_n \frac{\partial_x\phi(x)}{\pi}\delta[\phi(x)/\pi - n]$$

$$f(t) = \delta[t-\phi(x)/\pi] \leftrightarrow \tilde{f}(s) = e^{i2s\phi(x)}$$

$$\rho(x) = \frac{\partial_x\phi(x)}{\pi}\sum_{m\in\mathbb{Z}} e^{i2m\phi(x)} \qquad \phi(x) = \pi\rho_0 x + \Phi(x)$$

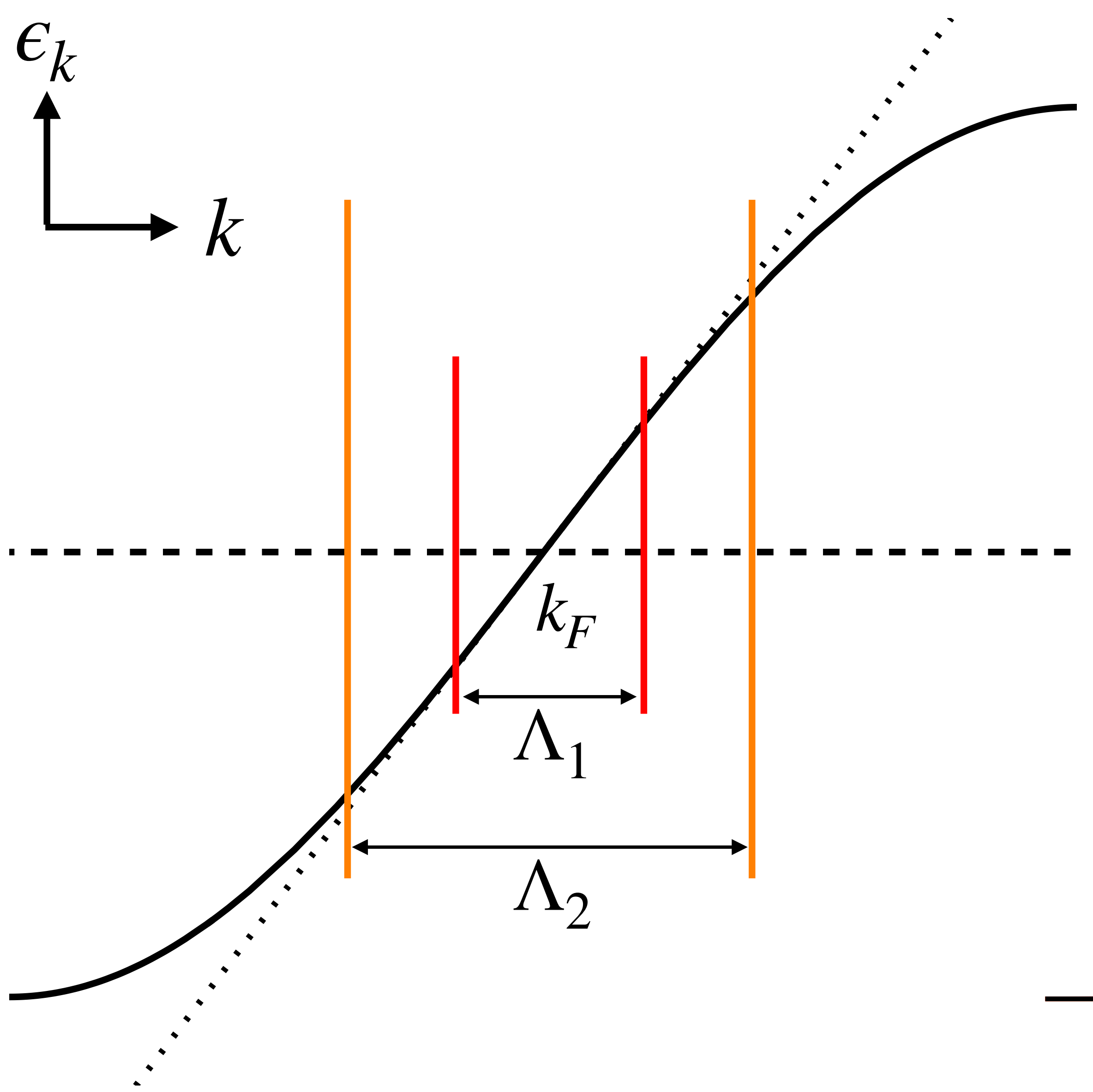
4. Bosonization III: Haldane's phenomenological approach

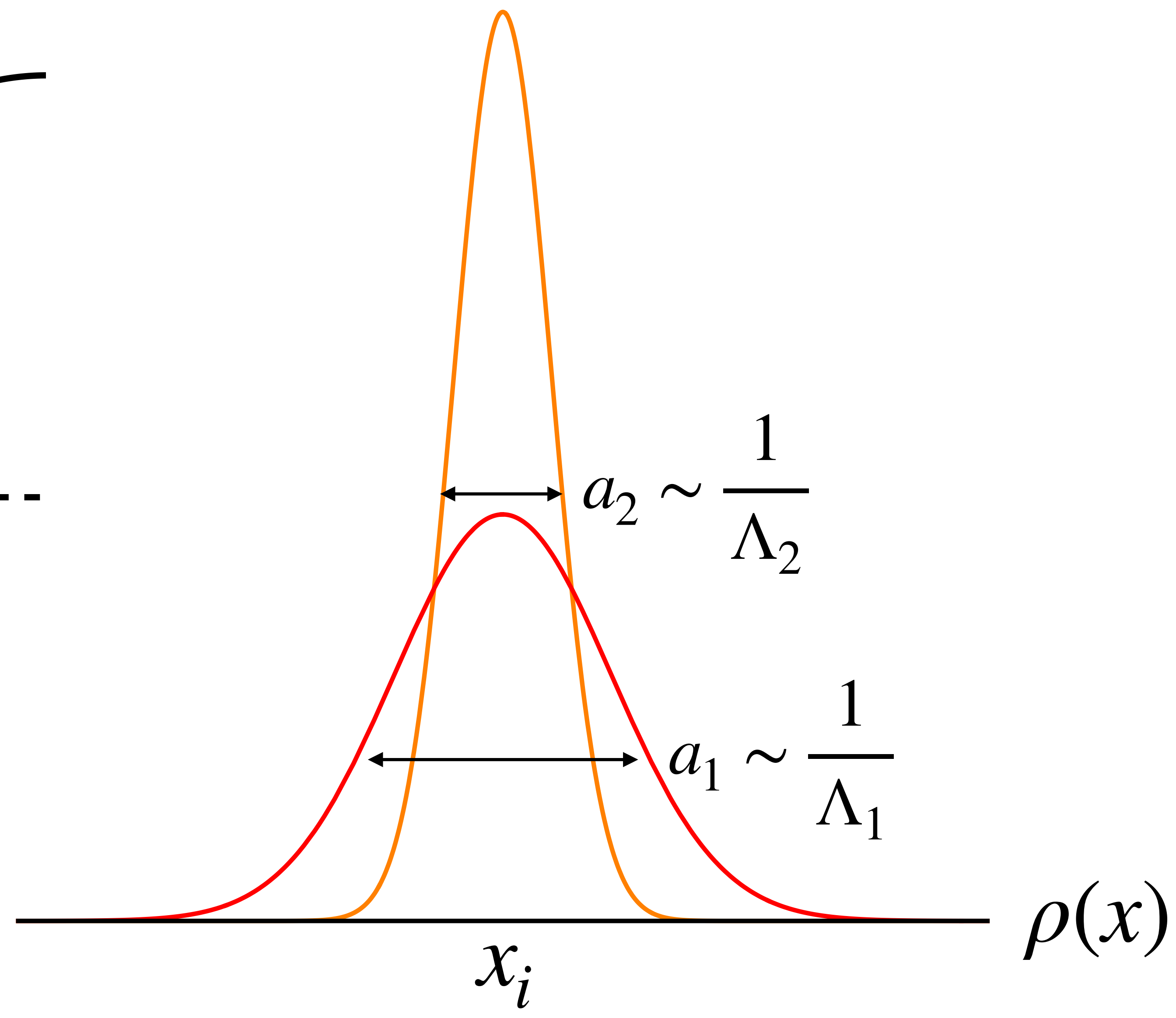
Since $\rho(x) = \Psi^\dagger(x)\Psi(x)$, each operator $\Psi(x)$ can be understood as square root of $\rho(x)$, accompanied by a phase factor $e^{i\theta(x)}$. We have $[\partial_x\phi(x), \theta(x')] = i\pi\delta(x - x')$.

Square root of a δ-function is still a δ-function.

Bosons: $\Psi_B(x) \sim \sqrt{\partial_x\phi(x)}e^{i\theta(x)}\sum_{m\in\mathbb{Z}} e^{i2m\phi(x)}$ Fermions: $\Psi_F(x) \sim \sqrt{\partial_x\phi(x)}e^{i\theta(x)}\sum_{m\in\mathbb{Z}} e^{i(2m\pm1)\phi(x)}$

- Haldane's result shows all harmonics have the same weight, but in reality the $\pm k_F$ $(m = 0)$ branches are the main contribution.





Instead of using δ-function, we can use

$$\rho(x) = \sum_i \frac{1}{\sqrt{\pi a}} e^{-\frac{(x-x_i)^2}{a}}$$

We can obtain a weight factor $e^{-m^2/\Lambda}$

For small Λ, only $m = 0$ should be kept.

Similarity to quantum oscillations (Lifshitz-Kosevich)

5. Spectral function from phenomenological bosonization

RESEARCH

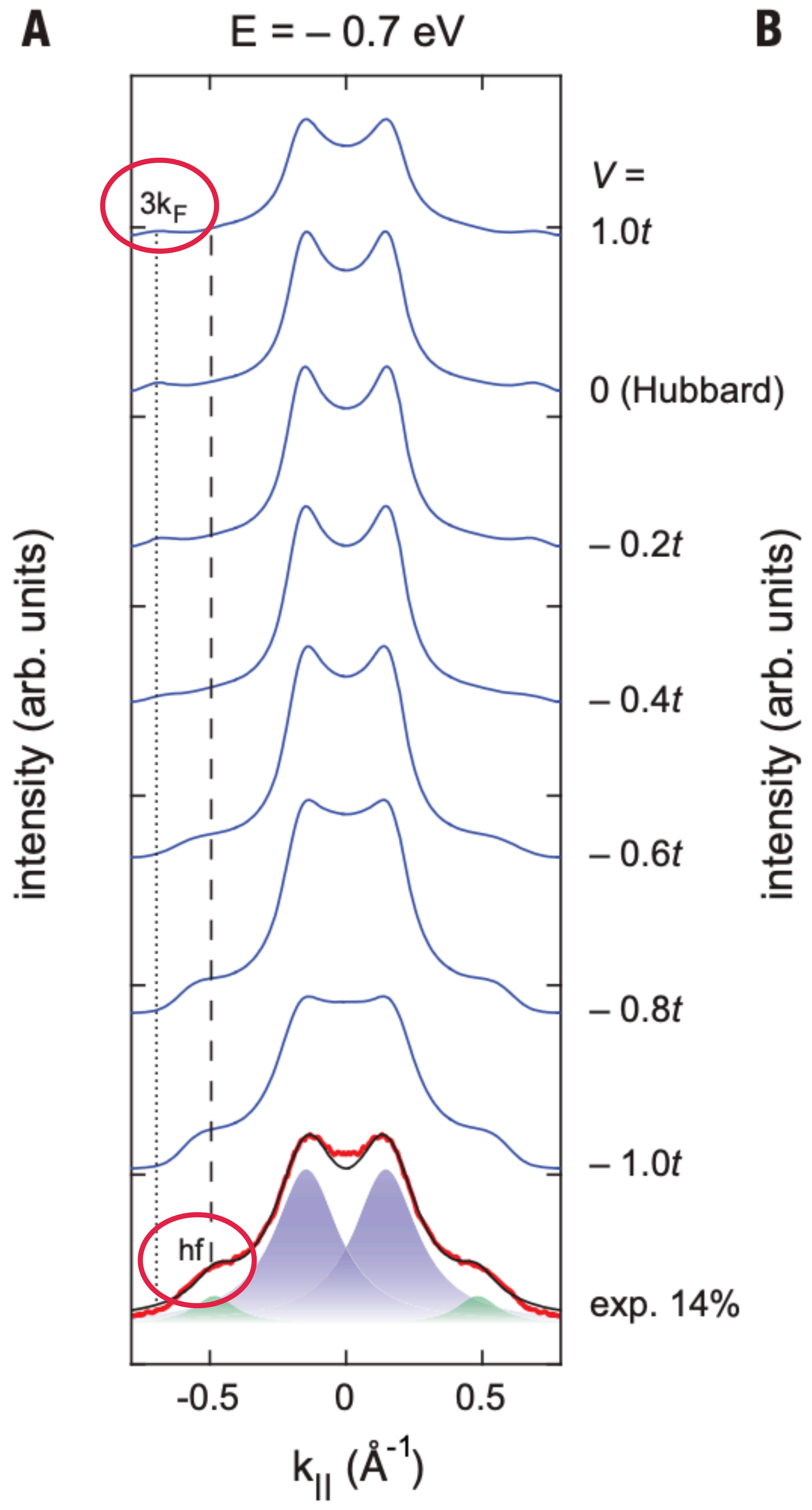
SUPERCONDUCTIVITY

Anomalously strong near-neighbor attraction in doped 1D cuprate chains

Zhuoyu Chen[1,2,3]†, Yao Wang[4]†, Slavko N. Rebec[1,2,3], Tao Jia[1,3,5], Makoto Hashimoto[6], Donghui Lu[6], Brian Moritz[1], Robert G. Moore[1,7], Thomas P. Devereaux[1,3,8]*, Zhi-Xun Shen[1,2,3,5]*

In the cuprates, one-dimensional (1D) chain compounds provide a distinctive opportunity to understand the microscopic physics, owing to the availability of reliable theories. However, progress has been limited by the challenge of controllably doping these materials. We report the synthesis and spectroscopic analysis of the 1D cuprate $Ba_{2-x}Sr_xCuO_{3+\delta}$ over a wide range of hole doping. Our angle-resolved photoemission experiments reveal the doping evolution of the holon and spinon branches. We identify a prominent folding branch whose intensity fails to match predictions of the simple Hubbard model. An additional strong near-neighbor attraction, which may arise from coupling to phonons, quantitatively explains experiments for all accessible doping levels. Considering structural and quantum chemistry similarities among cuprates, this attraction may play a similarly important role in high-temperature cuprate superconductors.

Phys. Rev. Lett. **127**, 197003; arXiv:2210.09288; …



5. Spectral function from phenomenological bosonization

As nearest neighbor interaction becomes more attractive, holon-folding branch gets enhances, while $3k_F$ branch is washed out gradually.

Is this true? If yes then why?

Spectral properties of 1D extended Hubbard model from bosonization and time-dependent variational principle: applications to 1D cuprates

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5. Spectral function from phenomenological bosonization

1D extended Hubbard model

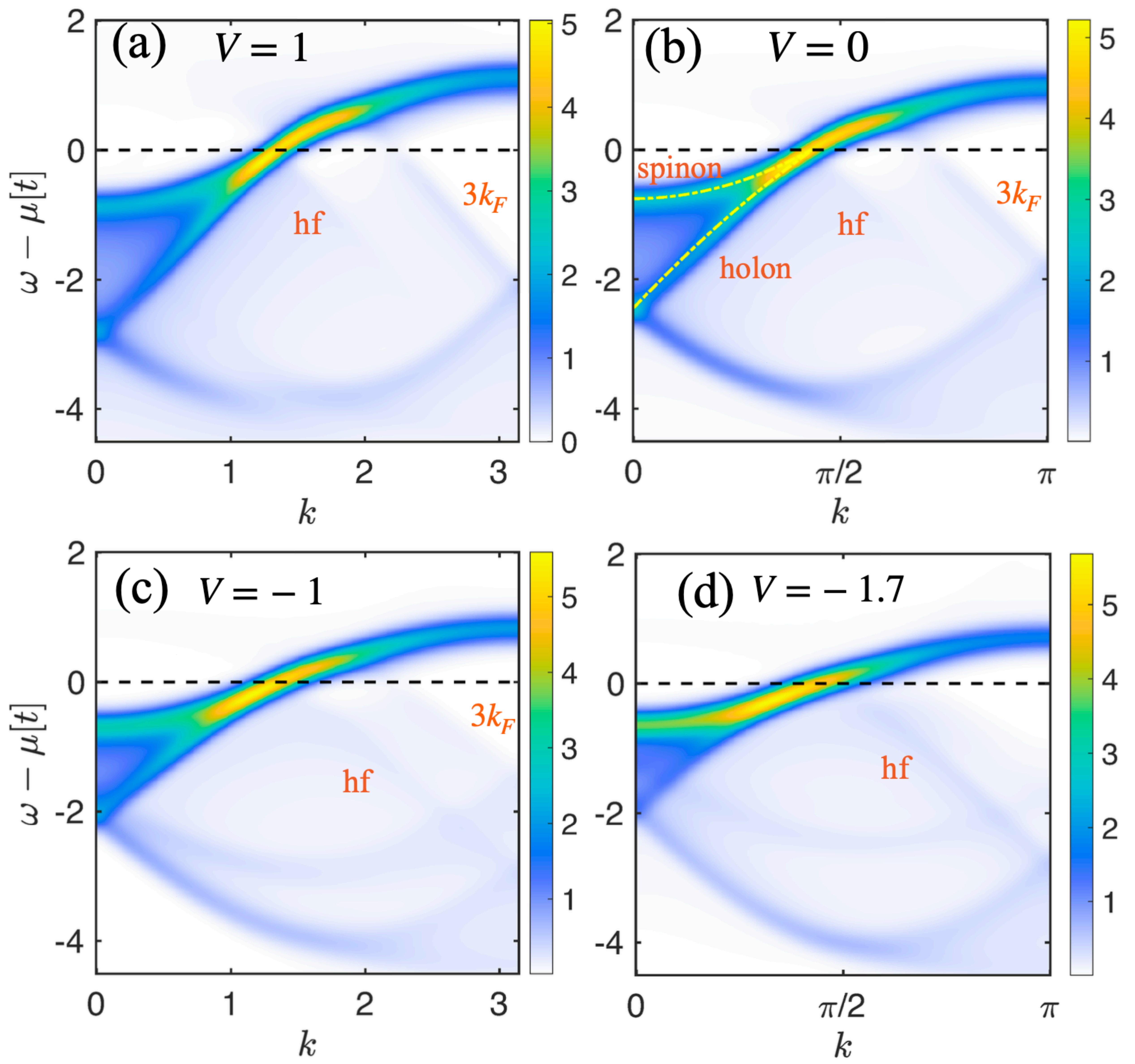
$$H = -t \sum_{\langle ij \rangle, \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} + V \sum_{\langle ij \rangle} n_i n_j$$

We set $t = 1$ and $U = 8t$. V is chosen to range from $-2t$ to about $4t$

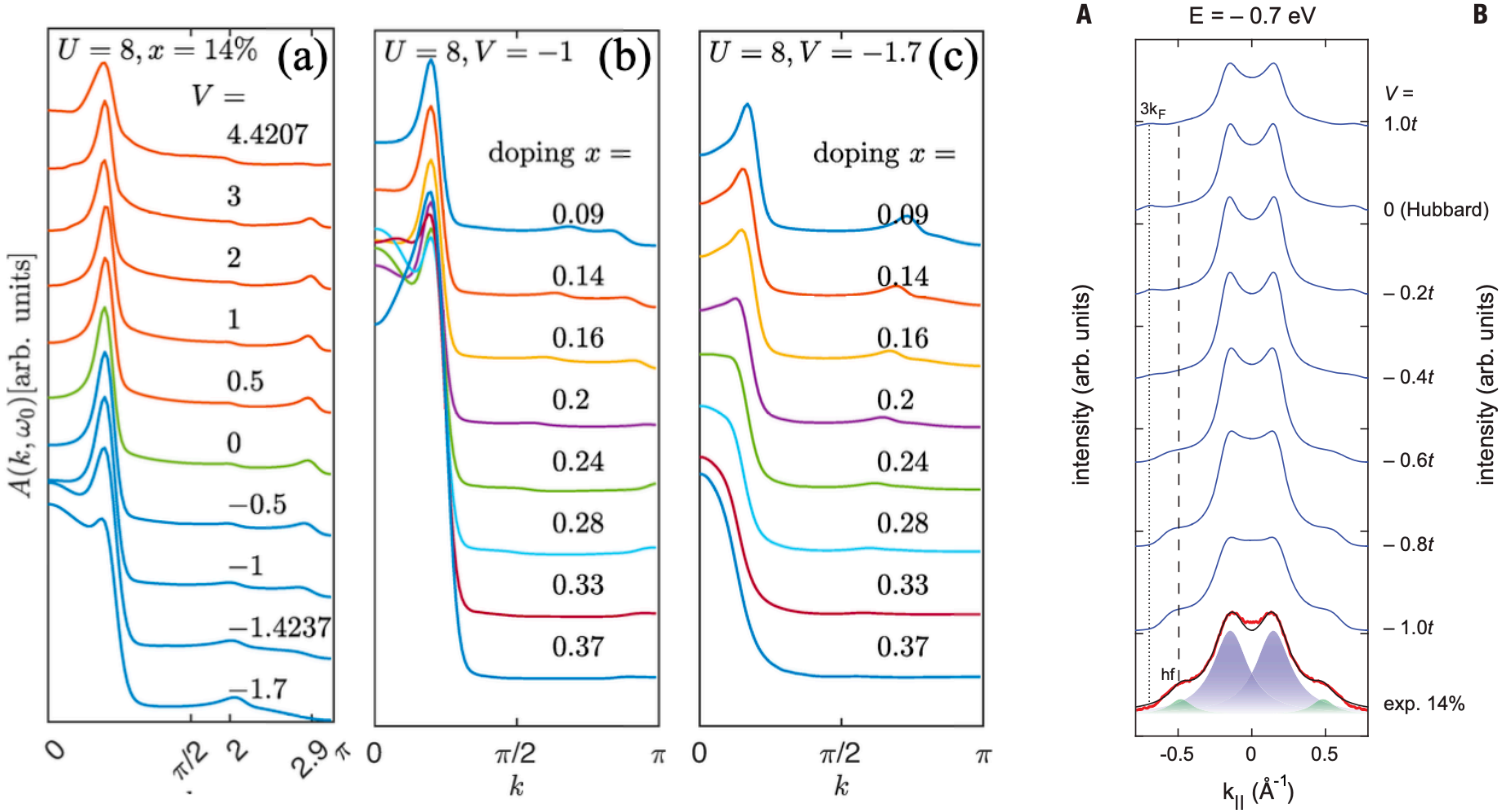
Doping factor $x = 1 - N/L$

Time-dependent variational principle calculation, excellent work done by Hao-Xin Wang

$x = 14\,\%$, $L = 100$



5. Spectral function from phenomenological bosonization



5. Spectral function from phenomenological bosonization

The spectral function $A(k, \omega)$ can be calculated from the retarded Green's function

$$G^R_{\uparrow}(x,t) \equiv -i\Theta(t)\left\langle \left\{ \Psi_{F\uparrow}(x,t), \Psi^{\dagger}_{F\uparrow}(0,0) \right\} \right\rangle = \sum_m G^R_{\uparrow,(2m+1)k_F}(x,t)$$

$$G^R_{\uparrow,(2m+1)k_F}(x,t) \sim -\Theta(t) e^{ic_m k_F x} \mathrm{Re} \prod_{\nu=\rho,\sigma} \frac{1}{[\alpha + i(u_\nu t - x)]^{c_m/2}} \left[\frac{\alpha^2}{(\alpha + iu_\nu t)^2 + x^2} \right]^{\gamma_{\nu,m}}$$

$$\gamma_{\nu,m} = \frac{1}{8}\left(c_m^2 K_\nu + \frac{1}{K_\nu} - 2c_m \right) \quad \text{with } c_m = 2m+1$$

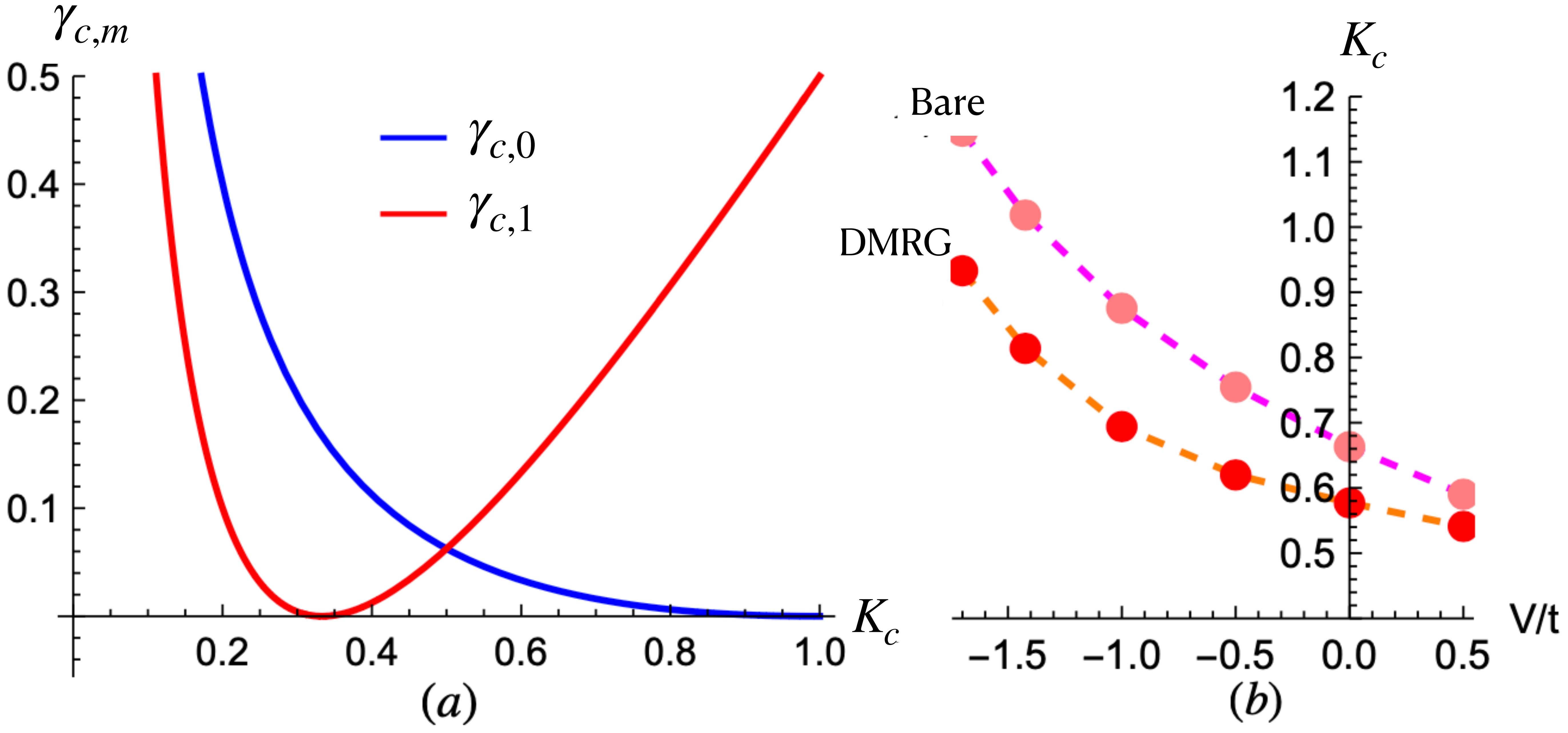
- For system with $SU(2)$ symmetry $K_s = 1$, thus only $\gamma_{c,m}$ affect the spectral properties

5. Spectral function from phenomenological bosonization

$$A(k,\omega) = -\frac{1}{\pi}\mathrm{Im}G^R(k,\omega), \text{ need Fourier transform from } G^R(x,t)$$

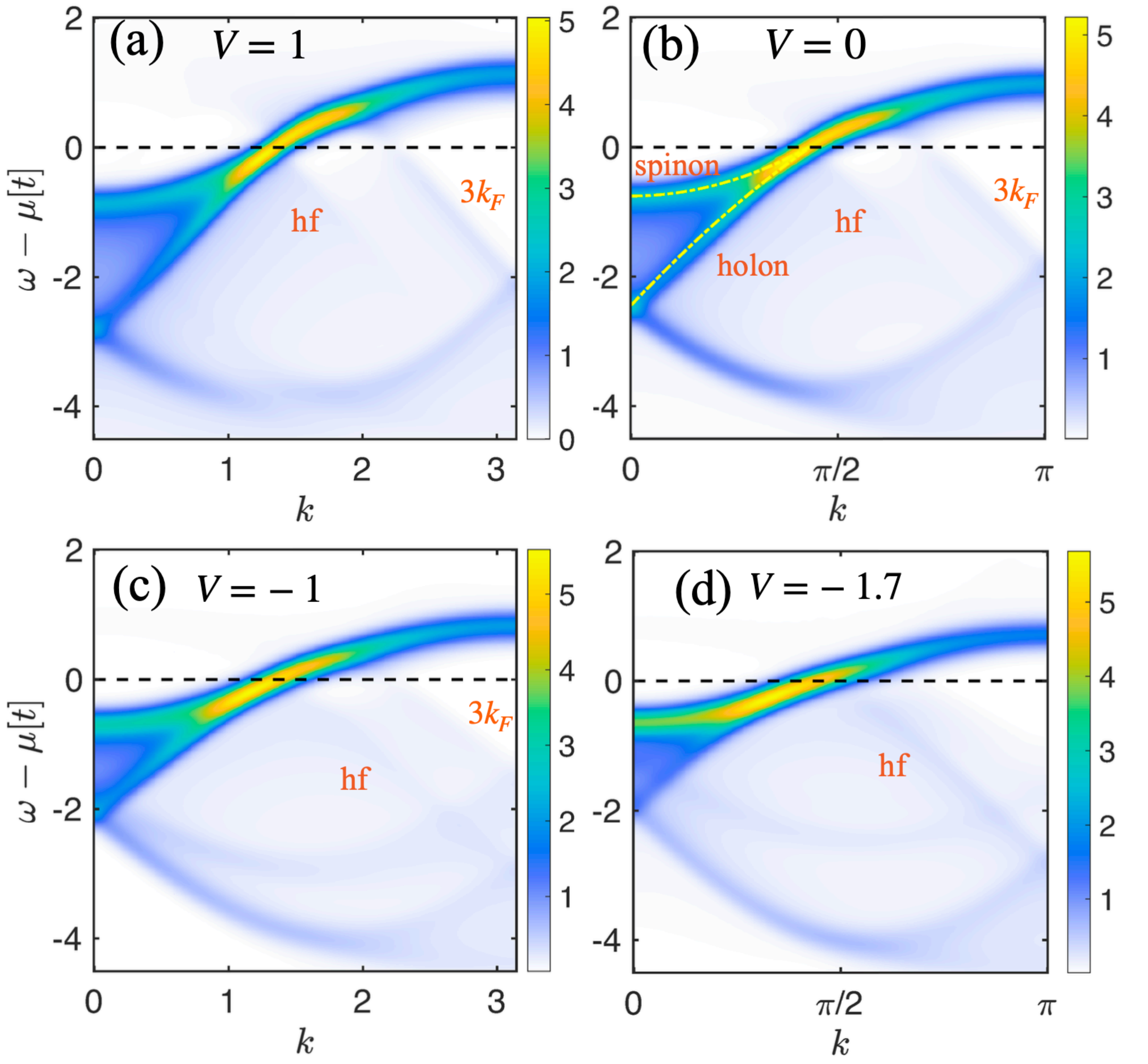
- hf : $A_0(k_F + q, \omega) \sim |\omega + v_c q|^{\gamma_{c,0}}$
- $3k_F$: $A_1(3k_F + q, \omega) \sim |\omega - v_c q|^{\gamma_{c,1}}$

A typical weight of the hf or $3k_F$ branch scales as w^γ, with w being a small derivation from the excitation center ($\omega = \pm v_c q$), and a larger γ yields a smaller weight.



As V changes from repulsion to attraction, K_ρ increases.
As a result,
$\gamma_{c,0}$ decreases -> hf gets enhanced
$\gamma_{c,1}$ increases -> $3k_F$ diminishes

6. Conclusion and outlook



Thank you!