University of Chinese Academy of Sciences

Kavli Institute for Theoretical Sciences
University of Chinese Academy of Sciences (UCAS)

Pair Density Wave in the Presence of a Nested Fermi Surface

Kavli ITS, UCAS

Jintao Jin (金锦涛)

2023.03.15

Normal state | Superconducting (SC) state

general fermi surface —BCS theory→ s-wave SC

nested fermi surface

pair density wave

?

experiments in AV_3Sb_5

A phenomenological model

<u>Jin-Tao Jin</u>, Kun Jiang*, Hong Yao† and Yi Zhou‡, PRL, **129**, 167001 (2022).

Outline

1. Pair Density Wave (PDW)

Definition and general properties

PDW in the presence of a nested fermi surface (FS)

2. Experimental observables in kagome SC AV_3Sb_5 (A=K, Rb, Cs)

Normal state

Superconducting state

3. A related phenomenological model

Model Hamiltonian and ground state

Density of states

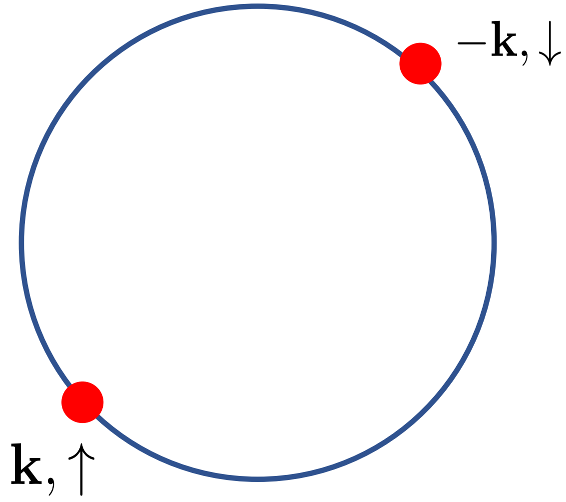
Quasiparticle interference

4. Summary and outlook

Pair Density Wave

➢ BCS effective Hamiltonian

$$H = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \frac{1}{N} \sum_{\mathbf{k}\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow}$$



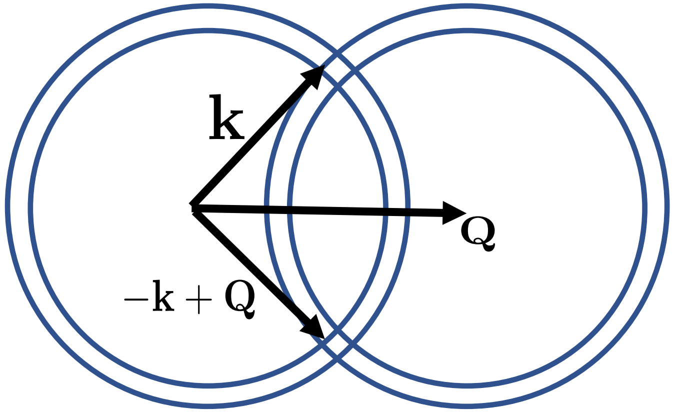
- Mean-field $\langle c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} \rangle$

$$H = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} \left(\Delta_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} + \Delta^*_{\mathbf{k}} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \right) + \sum_{\mathbf{k}} \Delta_{\mathbf{k}} \langle c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} \rangle$$

- Gap function $\Delta_{\mathbf{k}} = -\frac{1}{N} \sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \langle c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} \rangle$

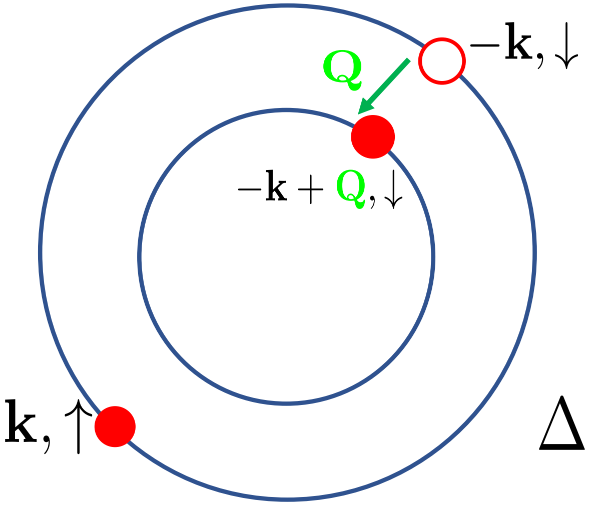
➢ Why zero momentum pairing?

$$\xi_{\mathbf{k}} = \xi_{-\mathbf{k}}$$



Finite momentum pairing?

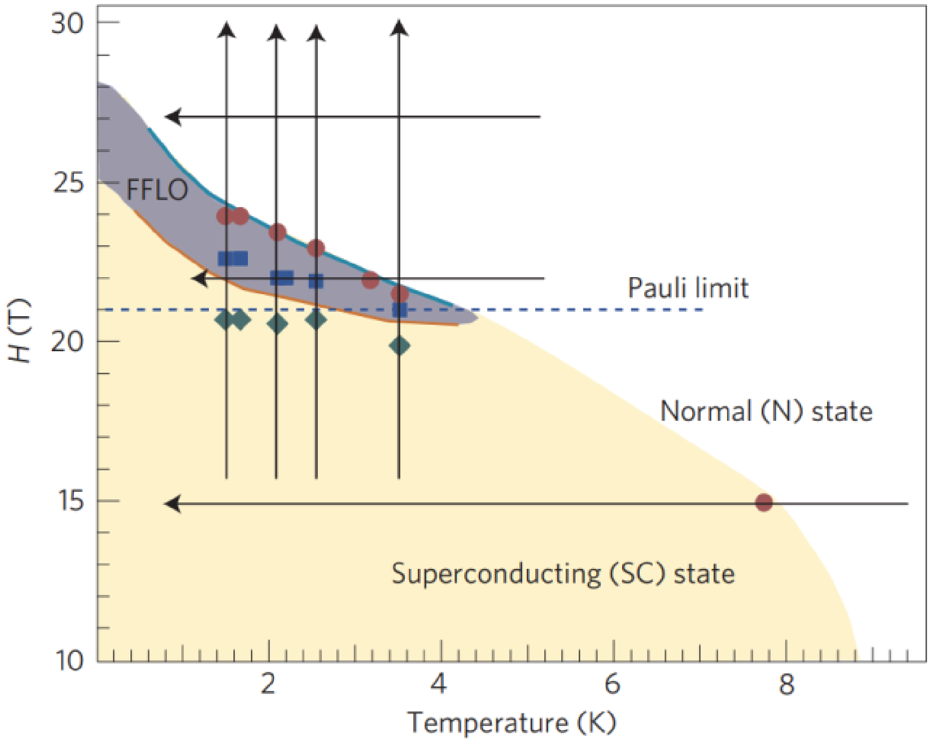
FFLO state in the presence of extra magnetic field



$$\Delta_{\mathbf{Q}} = -g\langle c_{\mathbf{k},\uparrow} c_{-\mathbf{k}+\mathbf{Q},\downarrow}\rangle \neq 0$$

FF: $\Delta(\mathbf{r}) \propto e^{i\mathbf{Q}\cdot\mathbf{r}}$

LO: $\Delta(\mathbf{r}) \propto \cos(\mathbf{Q}\cdot\mathbf{r}+\phi)$



κ-(BEDT-TTF)$_2$Cu(NCS)$_2$

Nature Physics, **10**(12):928–932, 2014.

P. Fulde, R.Ferrell, Phys. Rev. **135**, A550-63 (1964).
A. Larkin, Y.Ovchinnikov, JETP **20**, 762 (1965.)

Pair density wave （PDW）

$$\Delta_{\mathbf{Q}} = -g\langle c_{\mathbf{k},\uparrow} c_{-\mathbf{k}+\mathbf{Q},\downarrow}\rangle \neq 0$$ with no extra field

- Break translational symmetry
- Amperean pairing in cuprates $|\mathbf{Q}| \simeq 2k_F$

Patrick A. Lee. PRX, 4:031017 (2014).

➢ **Experimental evidences of possible PDW SC**

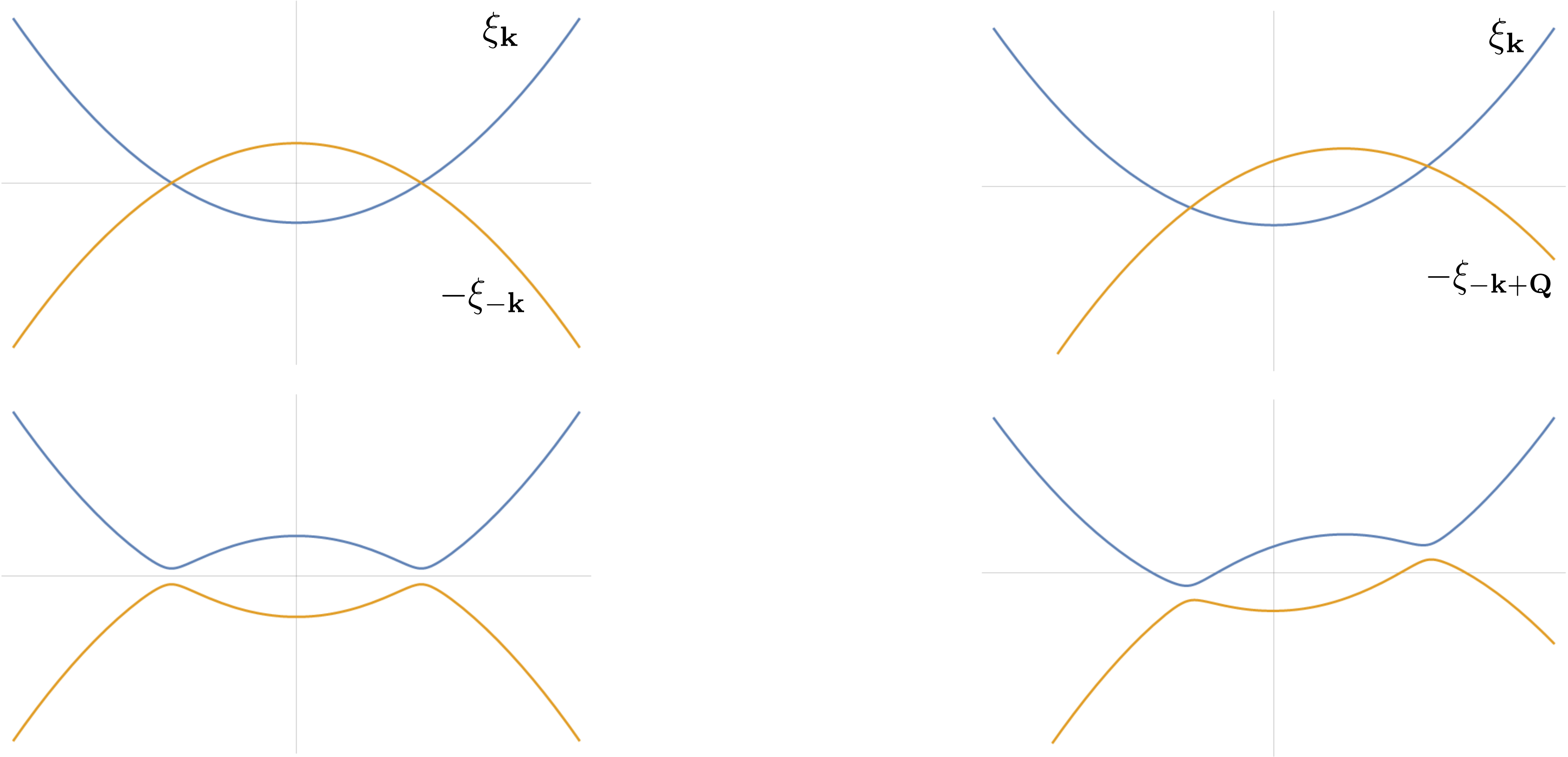
Cuprates

$Bi_2Sr_2CaCu_2O_8$ Science, **364**, 976, (2019)

$Bi_2Sr_2CuO_{6+\delta}$ PRX, **11,** 011007 (2021)

CsV_3Sb_5 Nature **599**, 222–228 (2021)

➢ Gapless fermi surfaces of Bogoliubov quasiparticles



$$E^{\pm}(\mathbf{k}) = \pm\sqrt{\xi_{\mathbf{k}}^2 + |\Delta|^2}$$

$$E^{\pm}(\mathbf{k}) = \frac{\xi_{\mathbf{k}} - \xi_{-\mathbf{k}+\mathbf{Q}}}{2} \pm \sqrt{\left(\frac{\xi_{\mathbf{k}} + \xi_{-\mathbf{k}+\mathbf{Q}}}{2}\right)^2 + |\Delta_{\mathbf{Q}}|^2}$$

➢ Descendant orders (secondary orders in PDW state)

- **Charge density wave (CDW) order**

$$\rho_{\mathbf{P}_i-\mathbf{P}_j} \propto (\Delta_{\mathbf{P}_i}\Delta^*_{\mathbf{P}_j} + \Delta_{-\mathbf{P}_j}\Delta^*_{-\mathbf{P}_i})$$

- **Charge-4e SC order (uniform)**

$$\Delta_{4e} \propto \Delta_{\mathbf{P}}\Delta_{-\mathbf{P}}$$

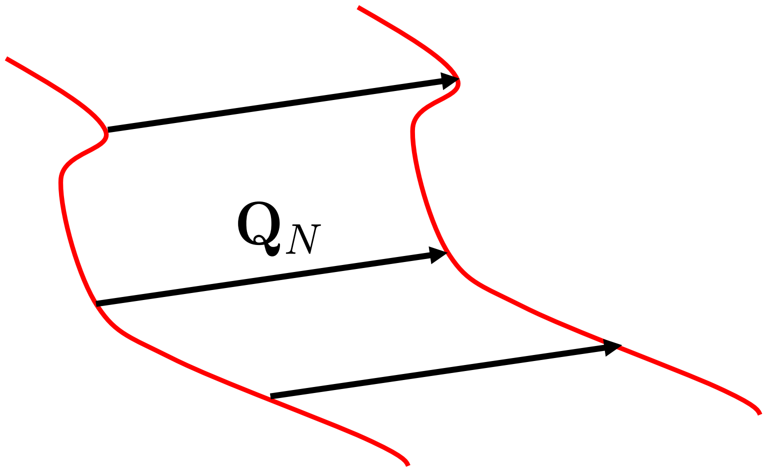
- **Ising nematic order**

$$\epsilon_{x^2-y^2} \propto (|\Delta_{\mathbf{P}_x}|^2 + |\Delta_{-\mathbf{P}_x}|^2 - |\Delta_{\mathbf{P}_y}|^2 - |\Delta_{-\mathbf{P}_y}|^2)$$

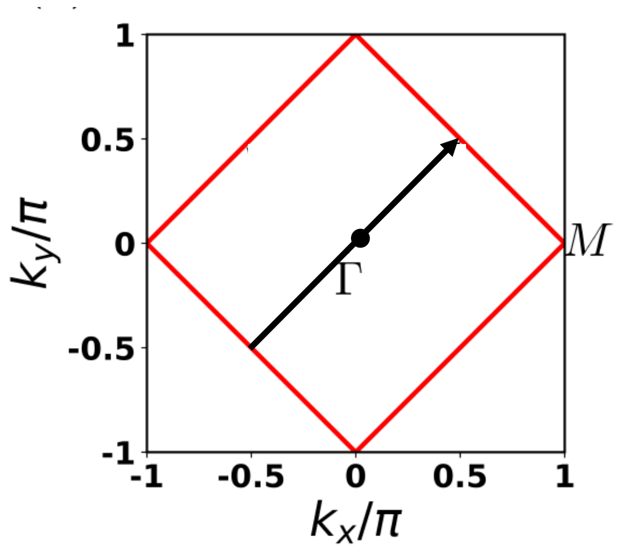
Annu. Rev. Condens. Matter Phys. 11, 231 (2020)

Nested fermi surface

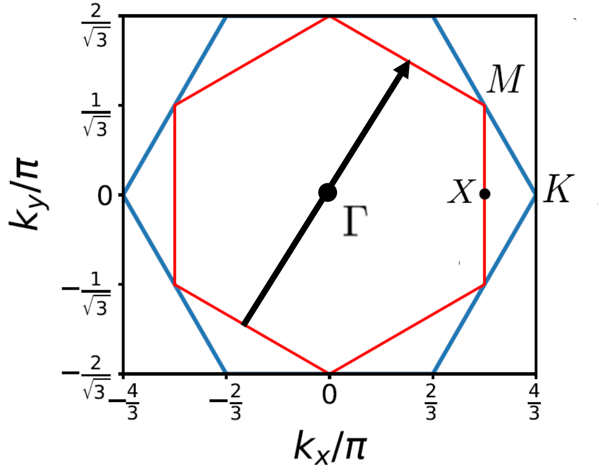
$$\xi_{\mathbf{k}} = -\xi_{\mathbf{k}+\mathbf{Q}_N}$$



Parallel fermi surface



tetragonal lattice

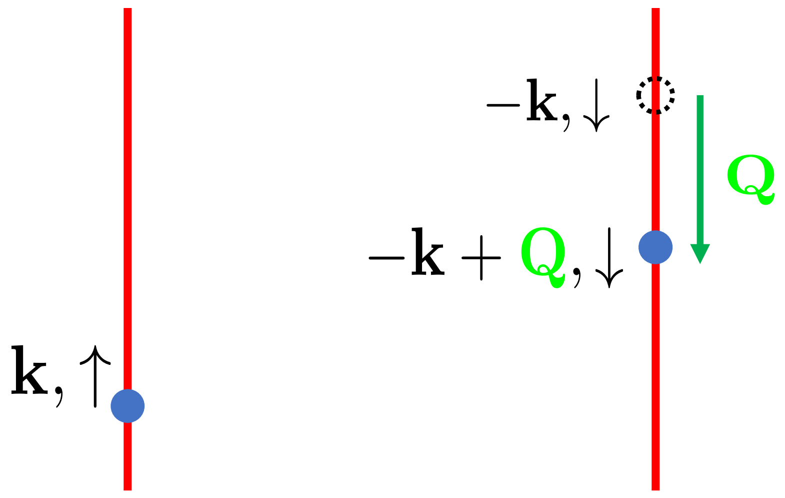


hexagonal lattice

➢ PDW pairing on parallel fermi surface segments

$$\xi_{\mathbf{k}} = -\xi_{\mathbf{k}+\mathbf{Q}_N}$$

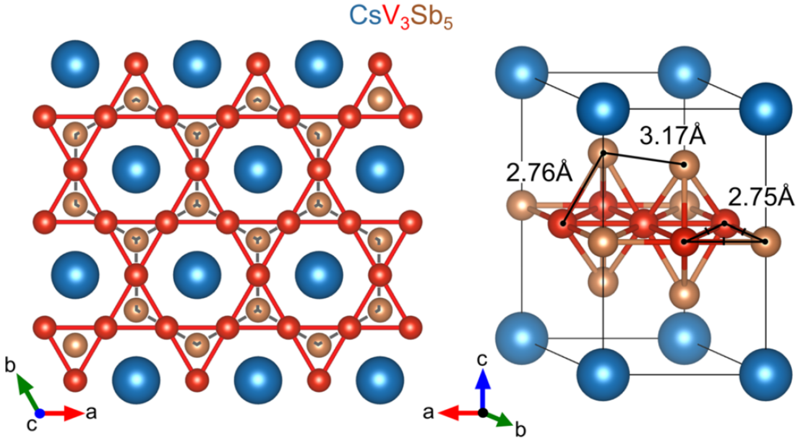
$$\xi_{\mathbf{k}} = \xi_{-\mathbf{k}}$$



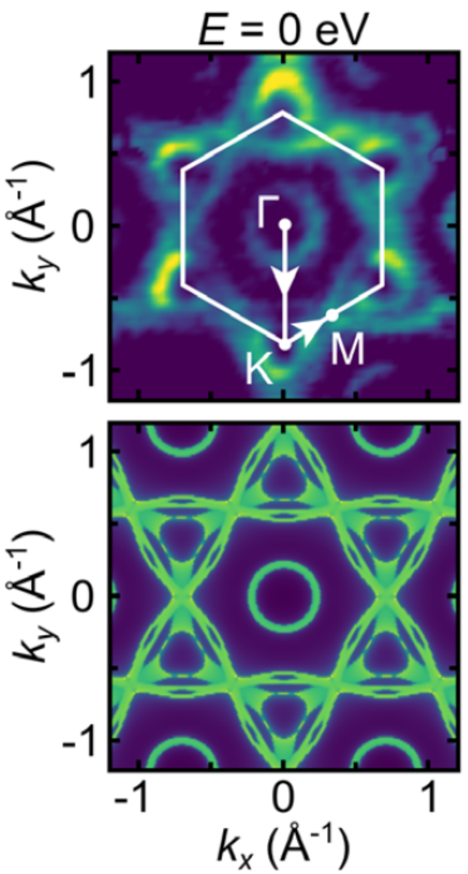
The nesting feature allows full pairing in the region near the FS!

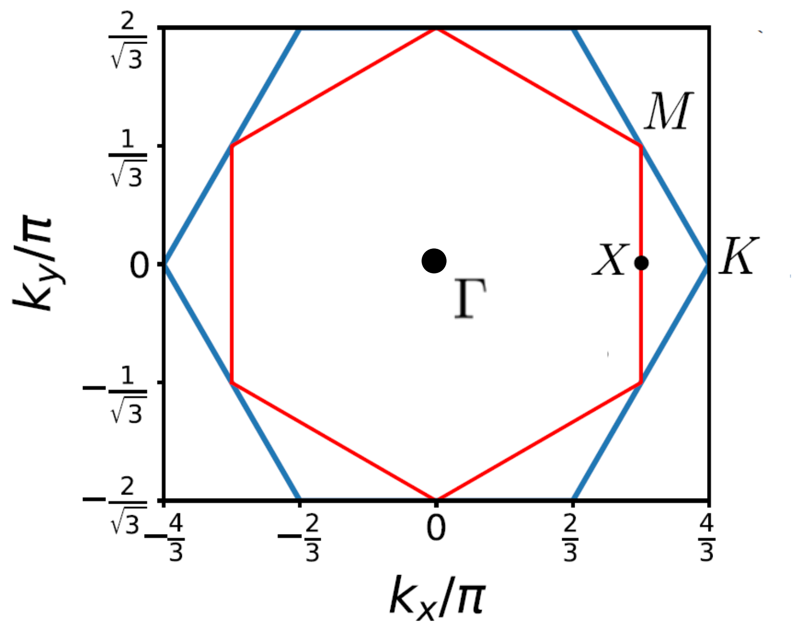
Kagome Materials AV_3Sb_5

Normal state



- **Cs:** Triangular lattice
- **V:** Kagome lattice
- **Sb:** Honeycomb lattice





Hexagonal first Brillouin Zone (BZ)

Nearly nested fermi surface

PRL, **125**, 247002 (2020)

Superconducting state

S-Wave Superconductivity in Kagome Metal CsV_3Sb_5 Revealed by $^{121/123}$Sb NQR and ^{51}V NMR Measurements

Chin. Phys. Lett. **38**, 077402 (2021)

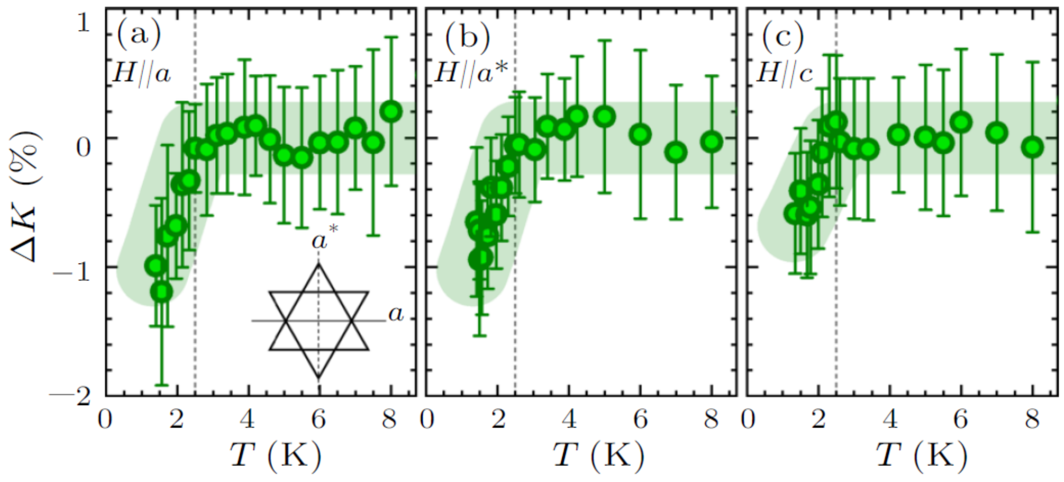


Fig. 3. Temperature dependence of ΔK of ^{121}Sb with (a) $H \parallel a$, (b) $H \parallel a^*$, and (c) $H \parallel c$, where a and a^* are orthogonal directions in the basal plane. The vertical dashed lines indicate the position of T_c.

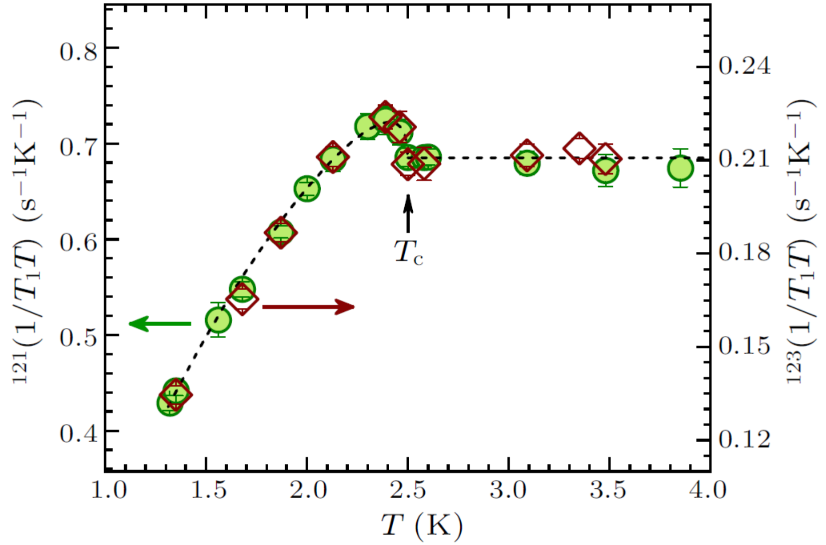


Fig. 4. Temperature dependence of $^{121}(1/T_1T)$ (left axis) and $^{123}(1/T_1T)$ (right axis). A Hebel–Slichter coherence peak appears just below T_c. The curve and line are guides to the eyes.

Knight shift: spin-singlet pairing

Hebel–Slichter coherence peak below T_c: no sign-change of gap

- Tunneling diode oscillator: magnetic penetration depth

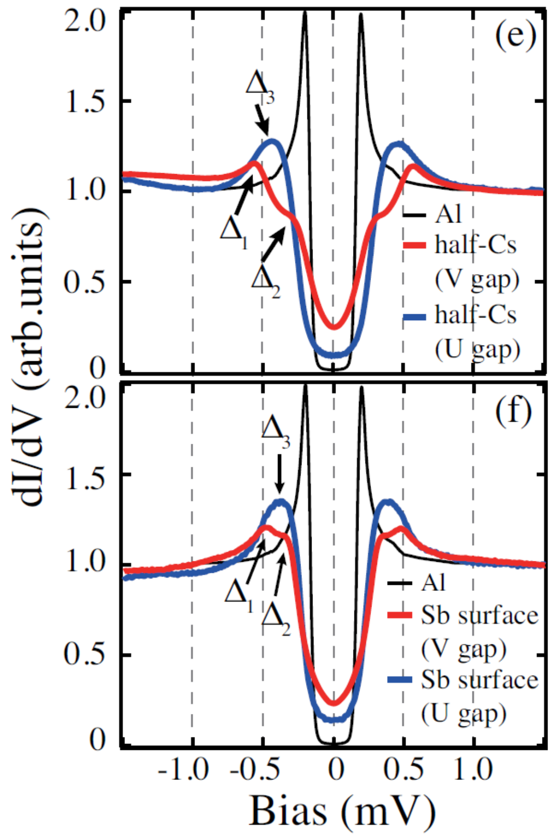
Nodeless superconductivity in the kagome metal CsV_3Sb_5

Sci. China Phys. Mech. Astron. **64**, 107462 (2021)

$$\Delta\lambda(T) \sim T^{-\frac{1}{2}} \exp\left(-\frac{\Delta(0)}{k_B T}\right)$$

- Multiband s-wave

- STM: density of states (DOS)



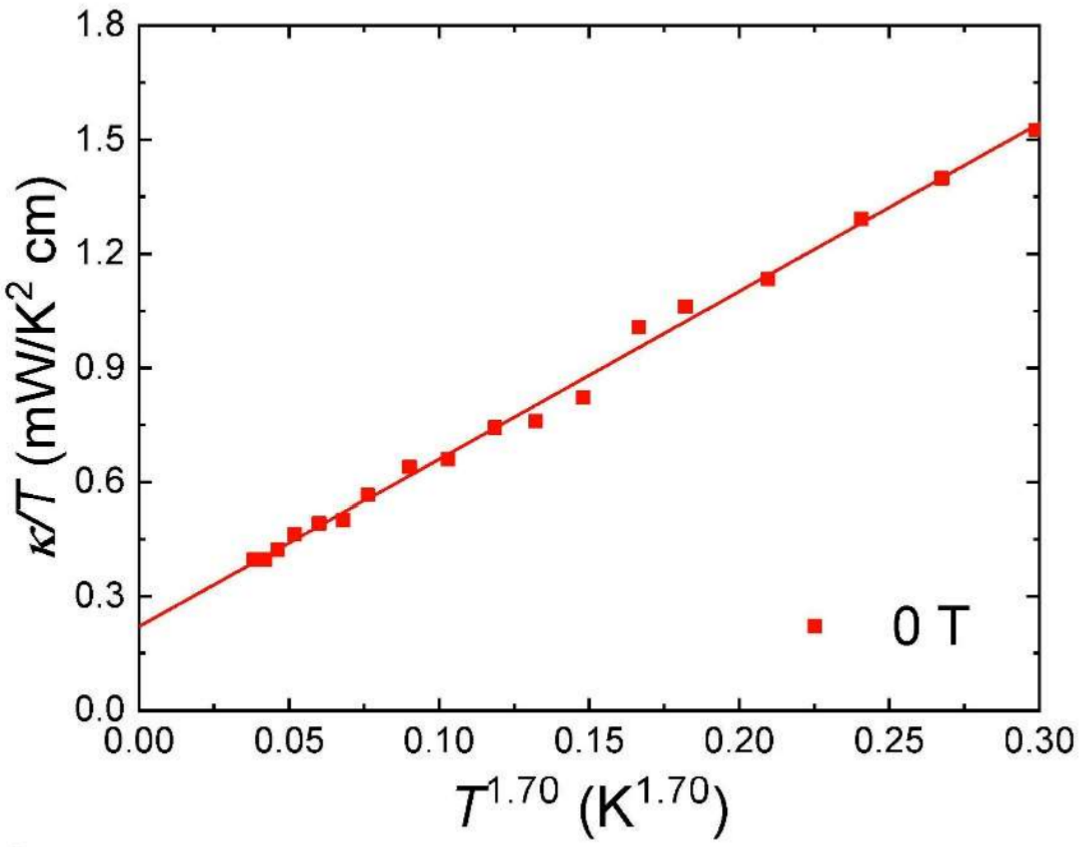
Multiband Superconductivity with Sign-Preserving Order Parameter in Kagome Superconductor CsV_3Sb_5

PRL **127**, 187004 (2021)

- Thermal conductivity: residual DOS in the SC state

Nodal superconductivity and superconducting domes in the topological Kagome metal CsV_3Sb_5

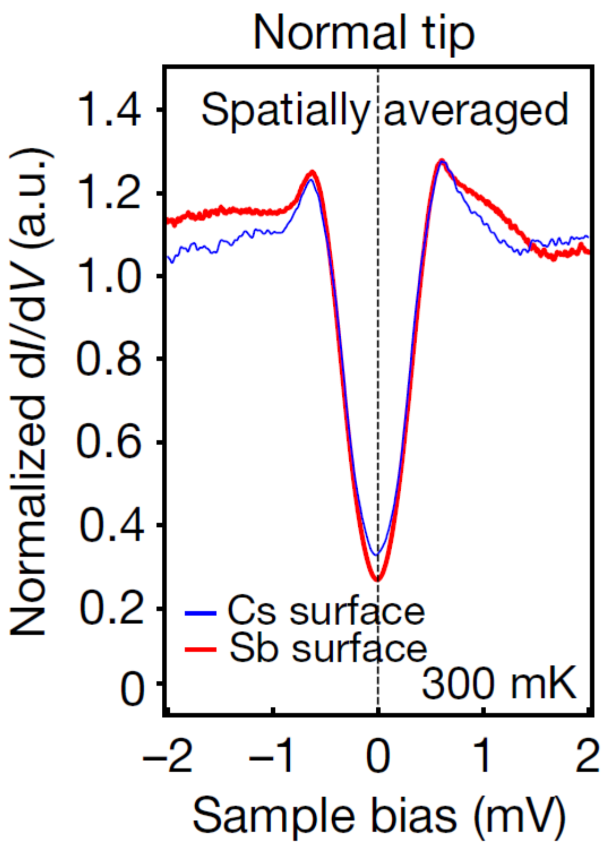
arXiv:2102.08356

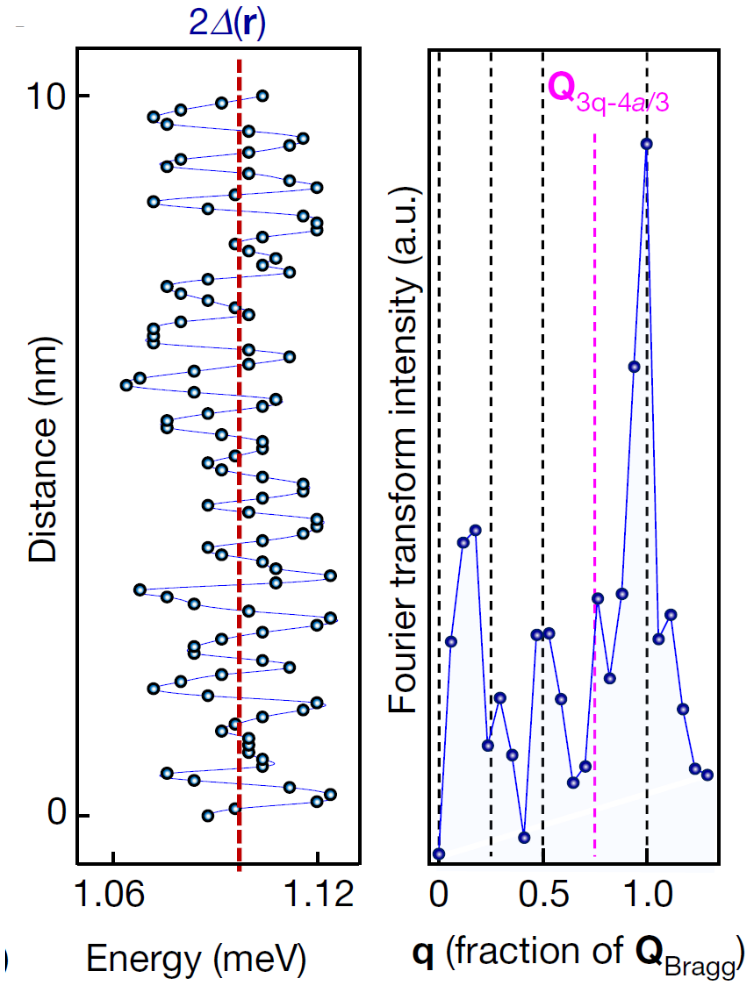


- PDW: STM measurements

Roton pair density wave in a strong-coupling kagome superconductor

Nature **599**, 222–228 (2021)





$$\mathbf{Q} = \pm\frac{3}{4}\mathbf{G}_\alpha$$

The AV_3Sb_5 is shown to be a spin-singlet SC hosting s-wave features

A residual thermal transport at T =0 and “multigap” V-shaped DOS with residual zero-energy contributions conflict with the conventional s-wave nature

A PDW state ordering has been observed in STM measurements

$$\mathbf{Q} = \pm\frac{3}{4}\mathbf{G}_\alpha$$

Kagome superconductors AV_3Sb_5 (A=K, Rb, Cs), *National Science Review*, nwac199 (2022)

Phenomenological Model

Model Hamiltonian

$$H = \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k}} c^{\dagger}_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} + \sum_{\mathbf{k},\alpha} [\Delta_{\mathbf{Q}_\alpha}(\mathbf{k}) c^{\dagger}_{\mathbf{k},\uparrow} c^{\dagger}_{-\mathbf{k}+\mathbf{Q}_\alpha,\downarrow} + \Delta_{-\mathbf{Q}_\alpha}(\mathbf{k}) c^{\dagger}_{\mathbf{k},\uparrow} c^{\dagger}_{-\mathbf{k}-\mathbf{Q}_\alpha,\downarrow} + \mathrm{H.c.}]$$

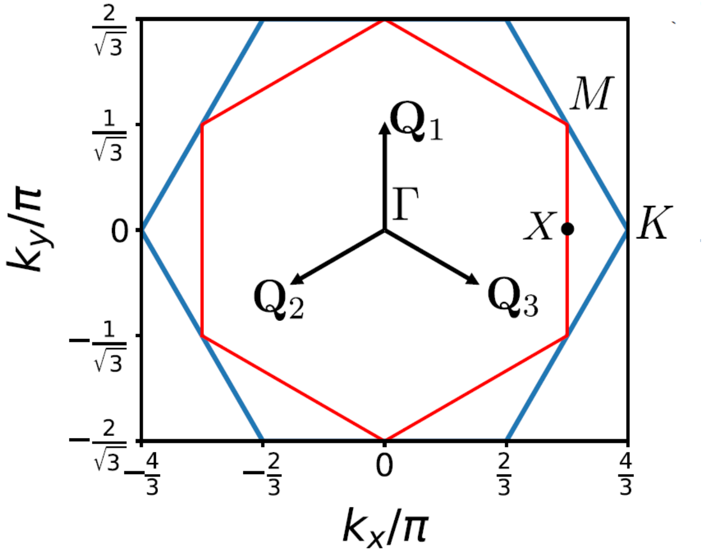
$$\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu = -2\{\cos(k_x) + \cos[\tfrac{1}{2}k_x + \sqrt{3}/2)k_y] + \cos[\tfrac{1}{2}k_x - (\sqrt{3}/2)k_y] + 1\}$$

$$\Delta_{\pm\mathbf{Q}_\alpha}(\mathbf{k}) = \Delta_{\pm\mathbf{Q}_\alpha} \exp[-(|\xi_{\mathbf{k}}| + |\xi_{-\mathbf{k}\pm\mathbf{Q}_\alpha}|)/(2\Lambda)]$$

Λ: energy cutoff

$$\Delta \ll \Lambda \ll t = 1$$

$$\Delta_{\pm\mathbf{Q}_\alpha} = \Delta e^{i\theta_\alpha} e^{\pm i(\phi_\alpha/2)}$$



$$\mathbf{Q}_\alpha = \frac{1}{4}\mathbf{G}_\alpha$$

- **Real space pairing function**

$$\Delta(\mathbf{r}) \propto \sum_\alpha e^{i\theta_\alpha} \cos\left(\mathbf{Q}_\alpha \cdot \mathbf{r} + \frac{\phi_\alpha}{2}\right)$$

$\phi_\alpha \neq 0$: Inversion symmetry breaking

$\theta_\alpha \neq 0, \pi$: Time reversal symmetry breaking

$$H = \frac{1}{16}\sum_{\mathbf{k}} H_{\mathbf{k}} + \sum_{\mathbf{k}} \xi_{\mathbf{k}}$$

$$= \frac{1}{16}\sum_{\mathbf{k}} (\hat{C}^{\dagger}_{\mathbf{k},\uparrow}, \hat{C}_{-\mathbf{k},\downarrow})\hat{\mathcal{H}}_{\mathbf{k}} \begin{pmatrix} \hat{C}_{\mathbf{k},\uparrow} \\ \hat{C}^{\dagger}_{-\mathbf{k},\downarrow} \end{pmatrix} + \sum_{\mathbf{k}} \xi_{\mathbf{k}}$$

$$\hat{\mathcal{H}}_{\mathbf{k}} = \begin{pmatrix} \hat{D}(\mathbf{k}) & \hat{\Delta}(\mathbf{k}) \\ \hat{\Delta}^{\dagger}(\mathbf{k}) & -\hat{D}(-\mathbf{k}) \end{pmatrix}.$$

$$\hat{C}^{\dagger}_{\mathbf{k},\sigma} = (c^{\dagger}_{\mathbf{k},\sigma}, c^{\dagger}_{\mathbf{k}+\mathbf{Q}_1,\sigma}, c^{\dagger}_{\mathbf{k}-\mathbf{Q}_1,\sigma}, c^{\dagger}_{\mathbf{k}+\mathbf{Q}_2,\sigma}, c^{\dagger}_{\mathbf{k}-\mathbf{Q}_2,\sigma}, c^{\dagger}_{\mathbf{k}+\mathbf{Q}_3,\sigma}, c^{\dagger}_{\mathbf{k}-\mathbf{Q}_3,\sigma}, c^{\dagger}_{\mathbf{k}+2\mathbf{Q}_1,\sigma}, c^{\dagger}_{\mathbf{k}+2\mathbf{Q}_2,\sigma}, c^{\dagger}_{\mathbf{k}+2\mathbf{Q}_3,\sigma}, c^{\dagger}_{\mathbf{k}+\mathbf{Q}_1-\mathbf{Q}_2,\sigma}, c^{\dagger}_{\mathbf{k}-\mathbf{Q}_1+\mathbf{Q}_2,\sigma}, c^{\dagger}_{\mathbf{k}+\mathbf{Q}_2-\mathbf{Q}_3,\sigma}, c^{\dagger}_{\mathbf{k}-\mathbf{Q}_2+\mathbf{Q}_3,\sigma}, c^{\dagger}_{\mathbf{k}+\mathbf{Q}_3-\mathbf{Q}_1,\sigma}, c^{\dagger}_{\mathbf{k}-\mathbf{Q}_3+\mathbf{Q}_1,\sigma}),$$

- Diagonalize the 32 by 32 matrix gives rise to the energy spectrum

$$H_{\mathbf{k}} = \sum_{i=1}^{16} E(\mathbf{k})^{+}_{i}\gamma^{\dagger}_{\mathbf{k},\uparrow,i}\gamma_{\mathbf{k},\uparrow,i} + E(\mathbf{k})^{-}_{i}(-\gamma^{\dagger}_{-\mathbf{k},\downarrow,i}\gamma_{-\mathbf{k},\downarrow,i} + 1)$$

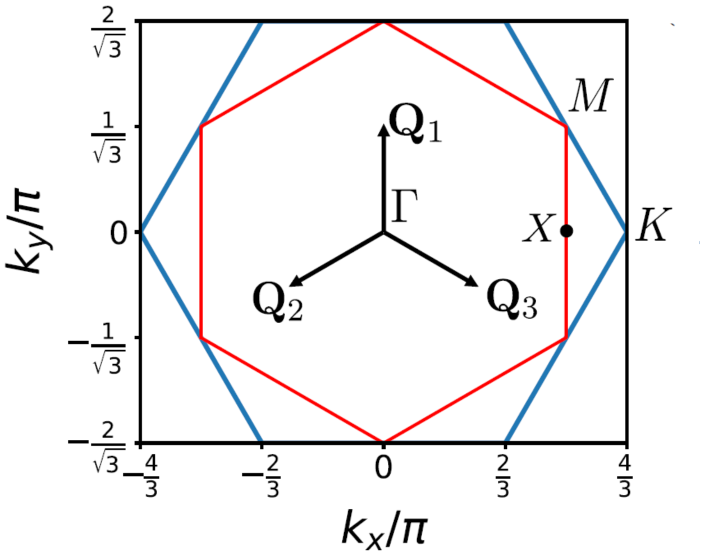
- Particle-hole symmetry: $E(\mathbf{k})^{+}_{i} = -E(-\mathbf{k})^{-}_{i}$
- 4 by 4 folding of the BZ: $E(\mathbf{k})^{\pm}_{i} = E(\mathbf{k} \pm \mathbf{Q}_{\alpha})^{\pm}_{i}$

Ground state

- Condensation energy

$$E_c[\phi_\alpha,\theta_\alpha] = (1/N)\sum_{\mathbf{k}}\left[(1/16)\sum_{i=1}^{16}\sum_{s=\pm}\sum_{E(\mathbf{k})_i^s>0} E(\mathbf{k})_i^s - |\xi_{\mathbf{k}}|\right]$$

$$\hat{\mathcal{H}}_{\mathbf{k}} \approx \hat{\mathcal{H}}_{\mathbf{k}}^{p} \oplus \hat{\mathcal{H}}_{\mathbf{k}}^{f}$$

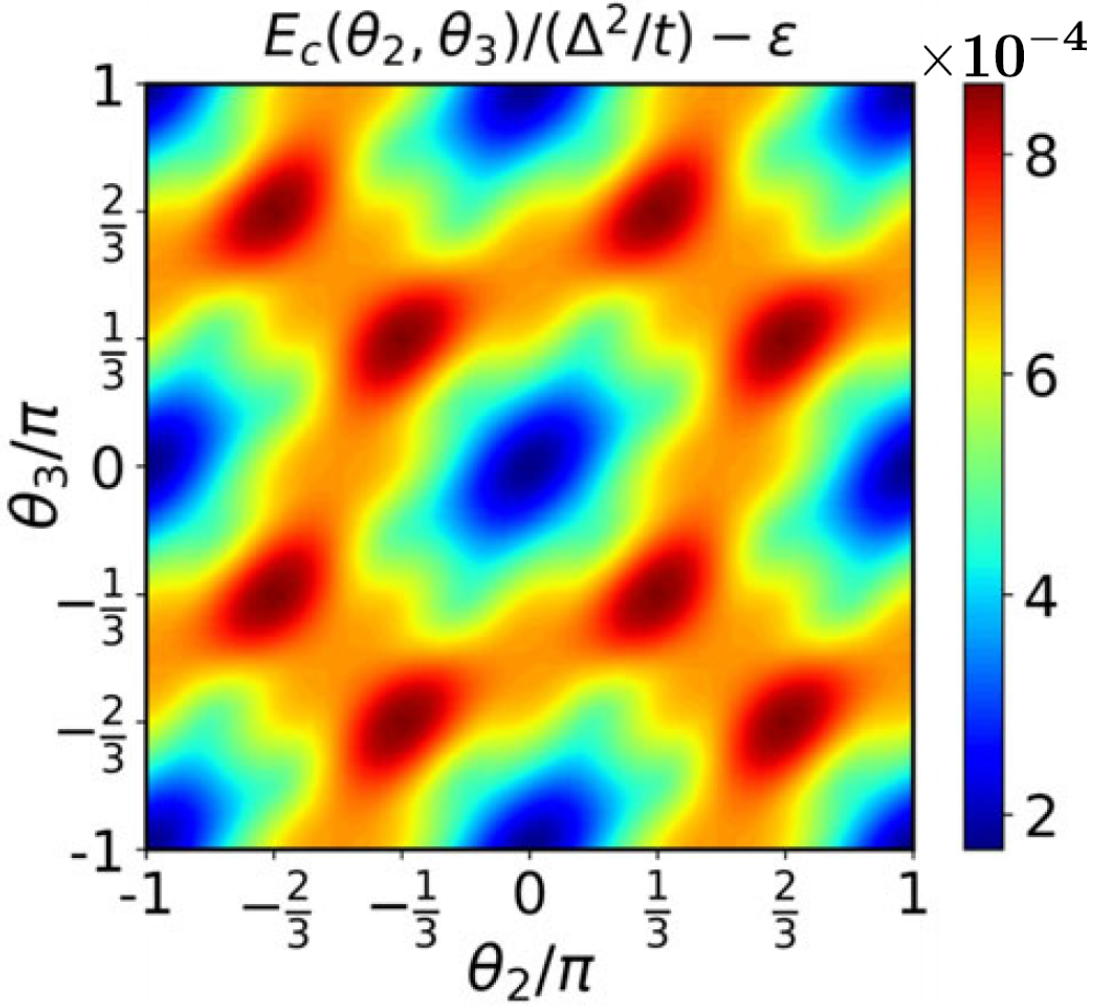


$$\hat{\mathcal{H}}_{\mathbf{k}}^{\tilde{p}} = \begin{pmatrix} 0 & 0 & \Delta_{\mathbf{Q}_1} & \Delta_{-\mathbf{Q}_1} \\ 0 & 0 & \Delta_{-\mathbf{Q}_1} & \Delta_{\mathbf{Q}_1} \\ \Delta_{\mathbf{Q}_1}^* & \Delta_{-\mathbf{Q}_1}^* & 0 & 0 \\ \Delta_{-\mathbf{Q}_1}^* & \Delta_{\mathbf{Q}_1}^* & 0 & 0 \end{pmatrix} = \Delta \begin{pmatrix} 0 & 0 & e^{i\frac{\phi_1}{2}+i\theta_1} & e^{-i\frac{\phi_1}{2}+i\theta_1} \\ 0 & 0 & e^{-i\frac{\phi_1}{2}+i\theta_1} & e^{i\frac{\phi_1}{2}+i\theta_1} \\ e^{-i\frac{\phi_1}{2}-i\theta_1} & e^{i\frac{\phi_1}{2}-i\theta_1} & 0 & 0 \\ e^{i\frac{\phi_1}{2}-i\theta_1} & e^{-i\frac{\phi_1}{2}-i\theta_1} & 0 & 0 \end{pmatrix}$$

- eigenvalues: $\pm 2\Delta\sin\left(\frac{\phi_1}{2}\right)$ and $\pm 2\Delta\cos\left(\frac{\phi_1}{2}\right)$

$$2\left(\left|\sin\left(\frac{\phi_1}{2}\right)\right| + \left|\cos\left(\frac{\phi_1}{2}\right)\right|\right) \longrightarrow \phi_\alpha = \pm\frac{\pi}{2}$$

$$c_{\mathbf{r},\sigma} \mapsto e^{i\theta_1/2} c_{\mathbf{r},\sigma} \qquad [\theta_1, \theta_2, \theta_3] \longmapsto [0, \theta_2 - \theta_1, \theta_3 - \theta_1]$$



$$\phi_\alpha = \pm \frac{\pi}{2}$$

$$\theta_2 - \theta_1 = \theta_3 - \theta_2 = \pm \frac{2\pi}{3} \mod \pi$$

Inversion and time reversal symmetry breaking

Nat. Mater. **20**, 1353 (2021)
Nature **602**, 245 (2022)
Sci. Bull. **66**, 1384 (2021)

➢ Ginzburg Landau free energy

- Gorkov Green's function

$$\mathcal{G}^{-1}(i\omega_n, \mathbf{k}) \equiv \mathcal{G}_0^{-1}(i\omega_n, \mathbf{k}) + \Sigma(\mathbf{k}) \qquad \mathcal{G}_0^{-1}(i\omega_n, \mathbf{k}) = \begin{pmatrix} G_0^{-1}(i\omega_n, \mathbf{k}) & 0 \\ 0 & -G_0^{-1}(-i\omega_n, -\mathbf{k}) \end{pmatrix}, \ \Sigma(\mathbf{k}) = \begin{pmatrix} 0 & -\hat{\Delta}(\mathbf{k}) \\ -\hat{\Delta}^\dagger(\mathbf{k}) & 0 \end{pmatrix}$$

- free energy

$$\mathcal{F}[\Delta_{\pm\mathbf{Q}_\alpha}] \equiv -\frac{1}{16\beta}\sum_{n,\mathbf{k}} \mathrm{Tr}\ln\mathcal{G}^{-1}(i\omega_n, \mathbf{k}) = \mathcal{F}^{(0)} - \frac{1}{16\beta}\sum_{n,\mathbf{k}} \mathrm{Tr}\ln\left(1 + \mathcal{G}_0(i\omega_n, \mathbf{k})\Sigma(\mathbf{k})\right) = \mathcal{F}^{(0)} + \sum_{j=1}^{\infty} \mathcal{F}^{(2j)}$$

$$\mathcal{F}^{(2j)} = \frac{1}{32j\beta}\sum_{n,\mathbf{k}} \mathrm{Tr}\left[(\mathcal{G}_0(i\omega_n, \mathbf{k})\Sigma(\mathbf{k}))^{2j}\right] = \frac{1}{16j\beta}\sum_{n,\mathbf{k}} \mathrm{Tr}\left[\left(-G_0(i\omega_n, \mathbf{k})\hat{\Delta}(\mathbf{k})G_0(-i\omega_n, -\mathbf{k})\hat{\Delta}^\dagger(\mathbf{k})\right)^j\right]$$

$$
\begin{aligned}
\mathcal{F}^{(4)} =& g_1^{(4)}\left(|\Delta_{\mathbf{Q}_1}|^4+|\Delta_{-\mathbf{Q}_1}|^4+|\Delta_{\mathbf{Q}_2}|^4+|\Delta_{-\mathbf{Q}_2}|^4+|\Delta_{\mathbf{Q}_3}|^4+|\Delta_{-\mathbf{Q}_3}|^4\right)\\
&+g_2^{(4)}\left(|\Delta_{\mathbf{Q}_1}|^2|\Delta_{-\mathbf{Q}_1}|^2+|\Delta_{\mathbf{Q}_2}|^2|\Delta_{-\mathbf{Q}_2}|^2+|\Delta_{\mathbf{Q}_3}|^2|\Delta_{-\mathbf{Q}_3}|^2\right)\\
&+g_3^{(4)}\left(|\Delta_{\mathbf{Q}_1}|^2|\Delta_{\mathbf{Q}_2}|^2+|\Delta_{\mathbf{Q}_2}|^2|\Delta_{\mathbf{Q}_3}|^2+|\Delta_{\mathbf{Q}_3}|^2|\Delta_{\mathbf{Q}_1}|^2+|\Delta_{-\mathbf{Q}_1}|^2|\Delta_{-\mathbf{Q}_2}|^2+|\Delta_{-\mathbf{Q}_2}|^2|\Delta_{-\mathbf{Q}_3}|^2+|\Delta_{-\mathbf{Q}_3}|^2|\Delta_{-\mathbf{Q}_1}|^2\right)\\
&+g_4^{(4)}\left(|\Delta_{\mathbf{Q}_1}|^2|\Delta_{-\mathbf{Q}_2}|^2+|\Delta_{\mathbf{Q}_2}|^2|\Delta_{-\mathbf{Q}_3}|^2+|\Delta_{\mathbf{Q}_3}|^2|\Delta_{-\mathbf{Q}_1}|^2+|\Delta_{-\mathbf{Q}_1}|^2|\Delta_{\mathbf{Q}_2}|^2+|\Delta_{-\mathbf{Q}_2}|^2|\Delta_{\mathbf{Q}_3}|^2+|\Delta_{-\mathbf{Q}_3}|^2|\Delta_{\mathbf{Q}_1}|^2\right)\\
&+g_\phi^{(4)}\left[\left(\Delta_{\mathbf{Q}_1}^2\right)\left(\Delta_{-\mathbf{Q}_1}^2\right)^*+\left(\Delta_{\mathbf{Q}_2}^2\right)\left(\Delta_{-\mathbf{Q}_2}^2\right)^*+\left(\Delta_{\mathbf{Q}_3}^2\right)\left(\Delta_{-\mathbf{Q}_3}^2\right)^*+c.c.\right]\\
&+g_\theta^{(4)}\left[\left(\Delta_{\mathbf{Q}_1}\Delta_{-\mathbf{Q}_1}\right)\left(\Delta_{\mathbf{Q}_2}\Delta_{-\mathbf{Q}_2}\right)^*+\left(\Delta_{\mathbf{Q}_2}\Delta_{-\mathbf{Q}_2}\right)\left(\Delta_{\mathbf{Q}_3}\Delta_{-\mathbf{Q}_3}\right)^*+\left(\Delta_{\mathbf{Q}_3}\Delta_{-\mathbf{Q}_3}\right)\left(\Delta_{\mathbf{Q}_1}\Delta_{-\mathbf{Q}_1}\right)^*+c.c.\right],
\end{aligned}
$$

$$
\mathcal{F}^{(4)}=6\left(g_1^{(4)}+\frac{g_2^{(4)}}{2}+g_3^{(4)}+g_4^{(4)}\right)\Delta^4+2g_\phi^{(4)}\Delta^4\sum_{\alpha=1}^{3}\cos\left(2\phi_\alpha\right)+2g_\theta^{(4)}\Delta^4\left[\cos\left(2\theta_2-2\theta_1\right)+\cos\left(2\theta_3-2\theta_2\right)+\cos\left(2\theta_1-2\theta_3\right)\right]
$$

$$|\beta\xi_{\mathbf{k}}|\ll 1$$

$$2n_F(\xi_{\mathbf{k}})-1\simeq-\frac{\beta\xi_{\mathbf{k}}}{2}+\frac{\beta^3\xi_{\mathbf{k}}^3}{24}$$

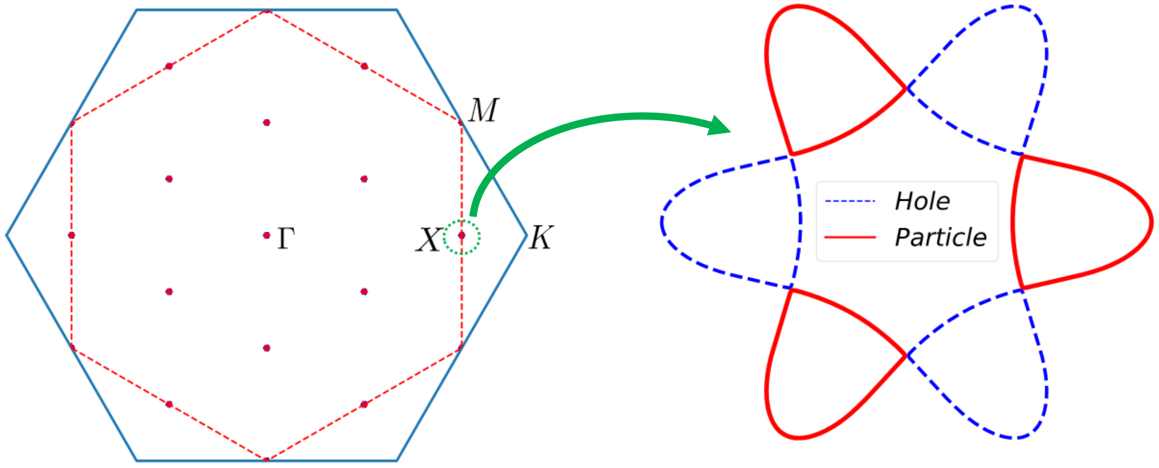
$g_\phi^{(4)}>0$ $\quad h_\phi=\cos(2\phi_1)+\cos(2\phi_2)+\cos(2\phi_3)$ $\quad\longrightarrow\quad$ $\phi_\alpha=\pm\dfrac{\pi}{2}$

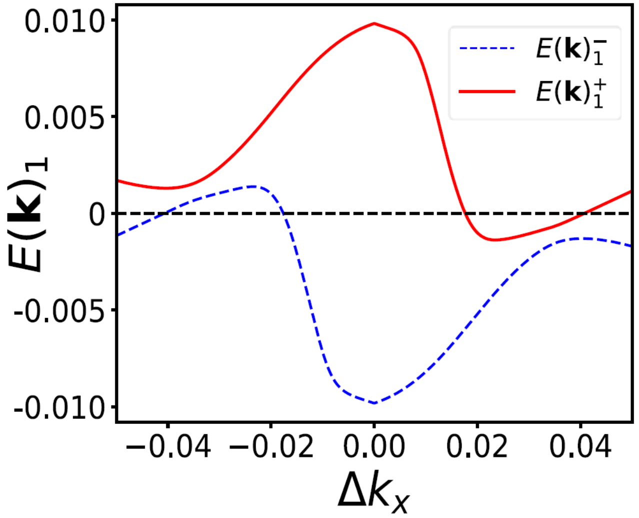
$g_\theta^{(4)}>0$ $\quad h_\theta=\cos(2\theta_2-2\theta_1)+\cos(2\theta_3-2\theta_2)+\cos(2\theta_1-2\theta_3)$ $\quad\longrightarrow\quad$ $\theta_2-\theta_1=\theta_3-\theta_2=\pm\dfrac{2\pi}{3}\mod\pi$

Density of states

$\Delta = 0.02, \Lambda = 0.1, \phi_\alpha = \frac{\pi}{2}, \theta_1 = 0, \theta_2 = \frac{2\pi}{3}, \theta_3 = -\frac{2\pi}{3}$

- $E(\mathbf{k})_1^+$ **(particle) and** $E(\mathbf{k})_1^-$ **(hole) Bogoliubov fermi surface**

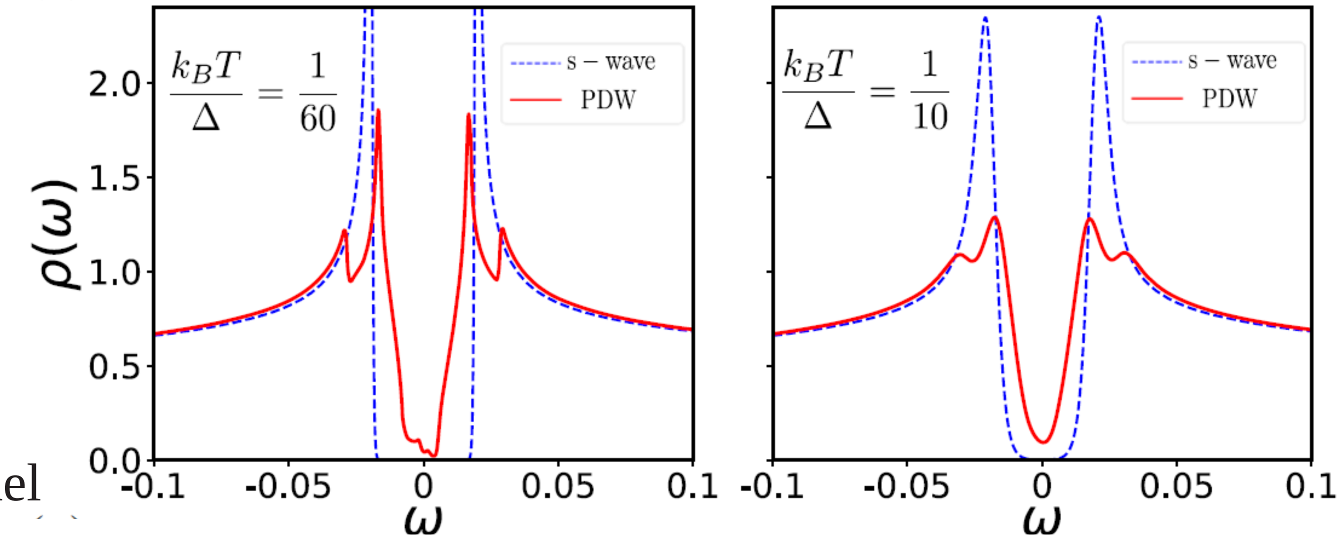




$k_y = 0$

- **DOS**

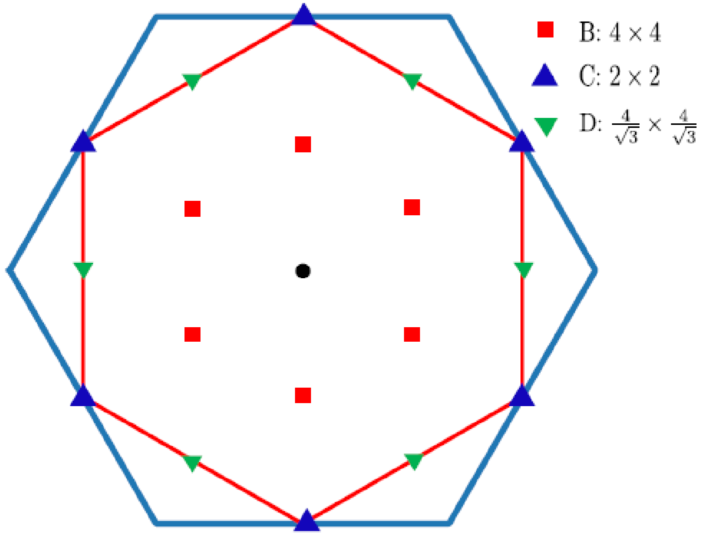
$$\rho(\omega) = -\frac{1}{8N}\sum_{\mathbf{k}}\sum_{i,j=1}^{16}\left[|u(\mathbf{k})_{ij}|^2\frac{\partial n_F\left(\omega - E(\mathbf{k})_j^+\right)}{\partial\omega} + |v(\mathbf{k})_{ij}|^2\frac{\partial n_F\left(\omega - E(\mathbf{k})_j^-\right)}{\partial\omega}\right]$$



'Multigap' V-shaped DOS from a single band model

➢ CDW order

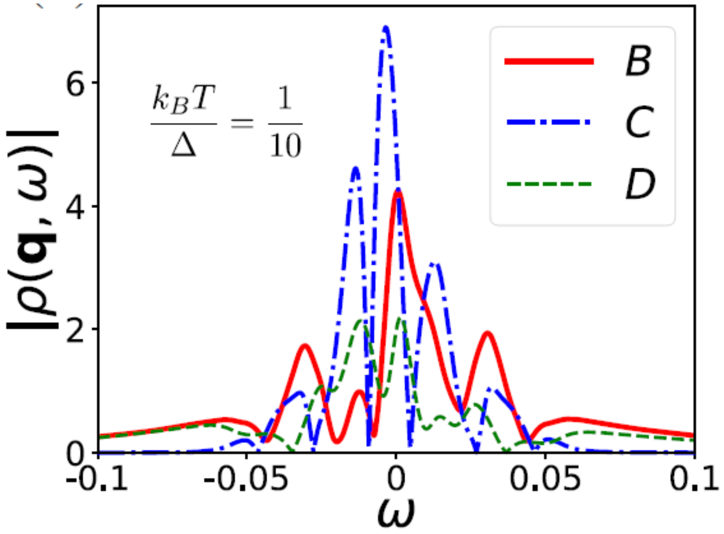
$$\rho(\mathbf{q}) = \frac{2}{N}\sum_{\mathbf{k}}\langle c^{\dagger}_{\mathbf{k},\uparrow} c_{\mathbf{k+q},\uparrow}\rangle$$



➢ Local DOS

$$\rho(\mathbf{q},\omega) = -\frac{2}{N}\sum_{\mathbf{k}}\sum_{j}\left[u(\mathbf{k})_{1j}u^*(\mathbf{k}+\mathbf{q})_{1j}\frac{\partial n_F\left(\omega - E(\mathbf{k})^+_j\right)}{\partial\omega} + v(\mathbf{k})_{1j}v^*(\mathbf{k}+\mathbf{q})_{1j}\frac{\partial n_F\left(\omega - E(\mathbf{k})^-_j\right)}{\partial\omega}\right]\bar{\delta}_{\mathbf{k},\mathbf{k+q}}$$

$$\rho_A(\omega) = |\rho(\mathbf{q}=\mathbf{0},\omega)|,$$
$$\rho_B(\omega) = |\rho(\mathbf{q}=\pm\mathbf{Q}_1,\omega)| = |\rho(\mathbf{q}=\pm\mathbf{Q}_2,\omega)| = |\rho(\mathbf{q}=\pm\mathbf{Q}_3,\omega)|,$$
$$\rho_C(\omega) = |\rho(\mathbf{q}=\pm2\mathbf{Q}_1,\omega)| = |\rho(\mathbf{q}=\pm2\mathbf{Q}_2,\omega)| = |\rho(\mathbf{q}=\pm2\mathbf{Q}_3,\omega)|,$$
$$\rho_D(\omega) = |\rho(\mathbf{q}=\pm(\mathbf{Q}_1-\mathbf{Q}_2),\omega)| = |\rho(\mathbf{q}=\pm(\mathbf{Q}_2-\mathbf{Q}_3),\omega)| = |\rho(\mathbf{q}=\pm(\mathbf{Q}_3-\mathbf{Q}_1),\omega)|$$



Nature **599**, 222–228 (2021)
Phys. Rev. X **11**, 031026 (2021)

Quasiparticle interference (QPI)

- In the presence of elastic scatterings, the LDOS will be modulated

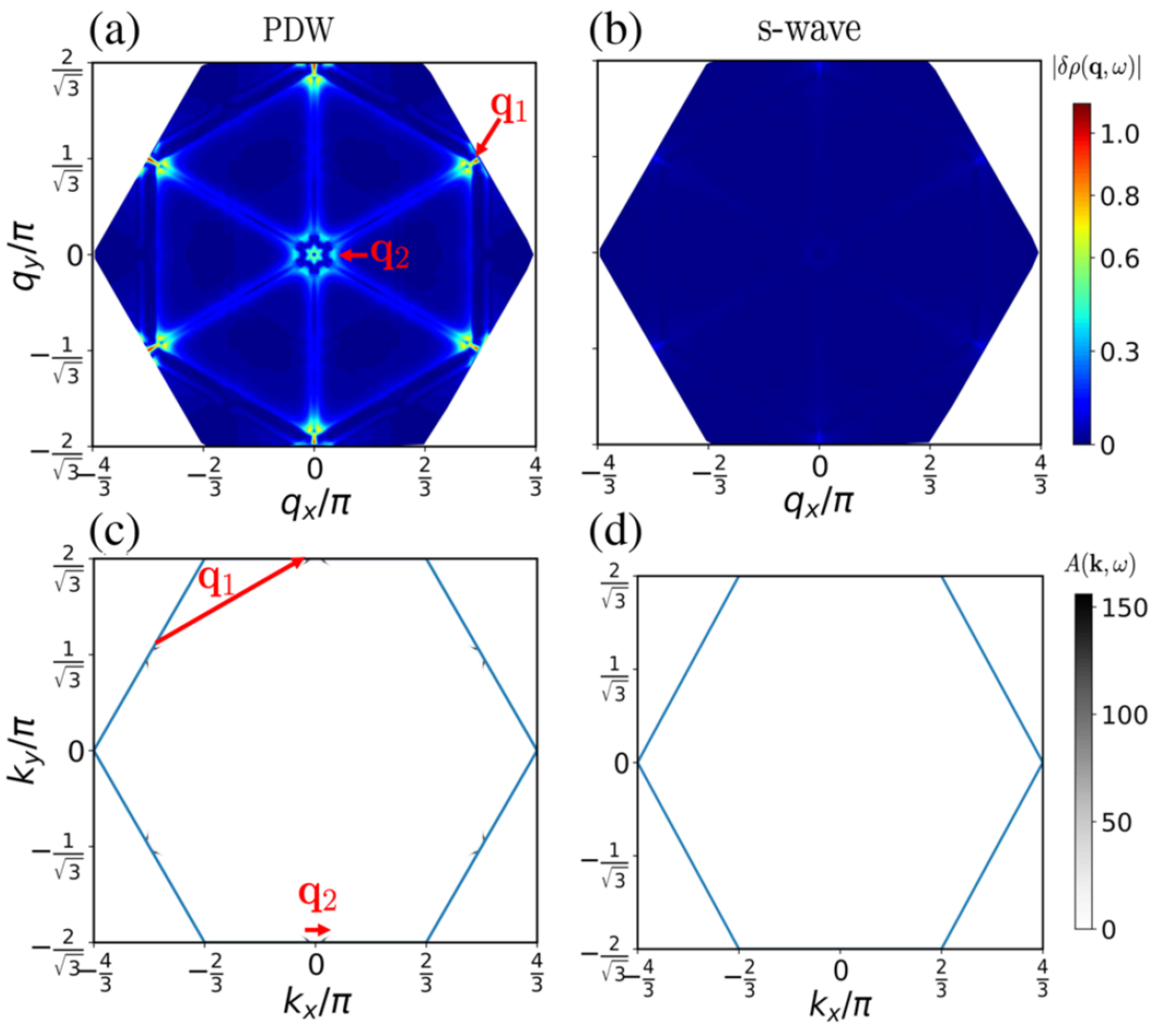
$$\delta\rho(\mathbf{q},\omega)$$
$$\equiv \rho_s(\mathbf{q},\omega) - \rho(\mathbf{q},\omega)$$
$$= -\frac{1}{16\pi N}\sum_{\mathbf{k}} \mathrm{Im}\tilde{\mathrm{Tr}}[\hat{\mathcal{G}}(\mathbf{k}+\mathbf{q},\omega+i\delta)\hat{T}(\omega)\hat{\mathcal{G}}(\mathbf{k},\omega+i\delta)]$$

◆ Scattering matrix

$$\hat{T}(\omega) = [(V_s\hat{\tau}_3)^{-1} - (1/N)\sum_{\mathbf{k}} \hat{\mathcal{G}}(\mathbf{k},\omega+i\delta)]^{-1}$$

- Electron spectrum function

$$A(\mathbf{k},\omega) = -(1/\pi)\mathrm{Im}[\hat{\mathcal{G}}(\mathbf{k},\omega+i\delta)]_{11}$$



PRB **67**, 020511(R) (2003)

$$\omega = 0.01 < \Delta$$

Summary & Outlook

Summary

- A phenomenological model related to PDW in AV_3Sb_5
- Inversion and time-reversal symmetry breaking ground state
- Bogoliubov fermi surfaces and V-shaped multi-gap DOS
- Local DOS and CDW
- Quasiparticle interference

Outlook

- Microscopic models and pairing mechanics of PDW in kagome materials
- Other descendant orders such as charge-4e SC order
- Similar models on other lattices with nested fermi surfaces

Thanks for your attention!